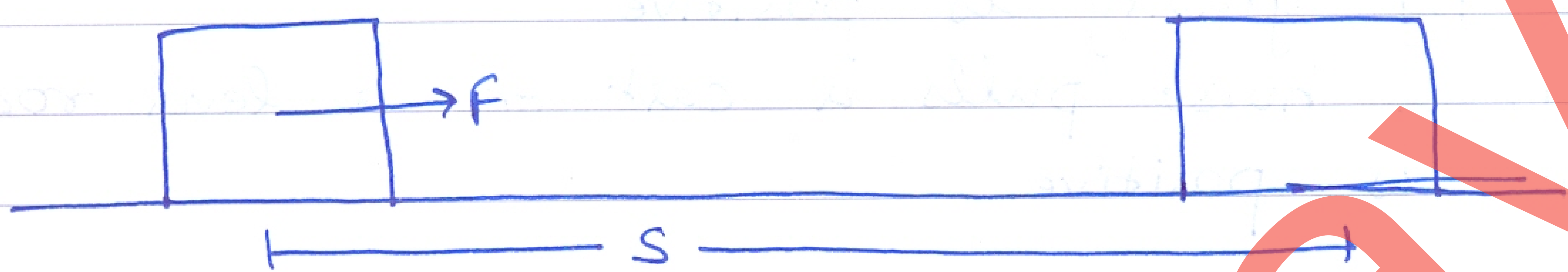
Work, Energy & Power

Work

Work is said to be done, if a force acting on a body is able to actually move it through some distance in the direction of force.



$$W = \vec{F} \cdot \vec{S} = FS\cos\theta$$

Dimensional formula : $[ML^2T^{-2}]$

Unit of work

(a) Absolute unit

S.I. unit – Joule

cgs " – erg

1 Joule – Work done is said to be 1 joule, if a force of 1 N displaces a body through 1 m in the direction of force.

$$1J = 1N \times 1m$$

* $1J = 10^7$ erg

(b) Gravitational unit

S.I. unit – kgm

cgs – g cm

1 kg-m – Work done is said to be 1 kgm, if a force of 1 kgf displaces a body through 1 m in the direction of force

$$1\,kgm = 1\,kgf \times 1m = 9.8\,J$$

Types of work done

(1) **Positive work** ($\theta < 90^\circ$)

Examples

(i) When a body falls freely under gravity, the work done by gravity is positive

(ii) When a horse pulls a cart on a level road, work done is positive.

(2) **Negative work** ($\theta > 90^\circ$)

Examples

(i) When a body slides over a rough surface, the work done by frictional force is negative.

(ii) When 2 like charges move towards one another, work done by electrostatic force is negative.

~~(iii)~~

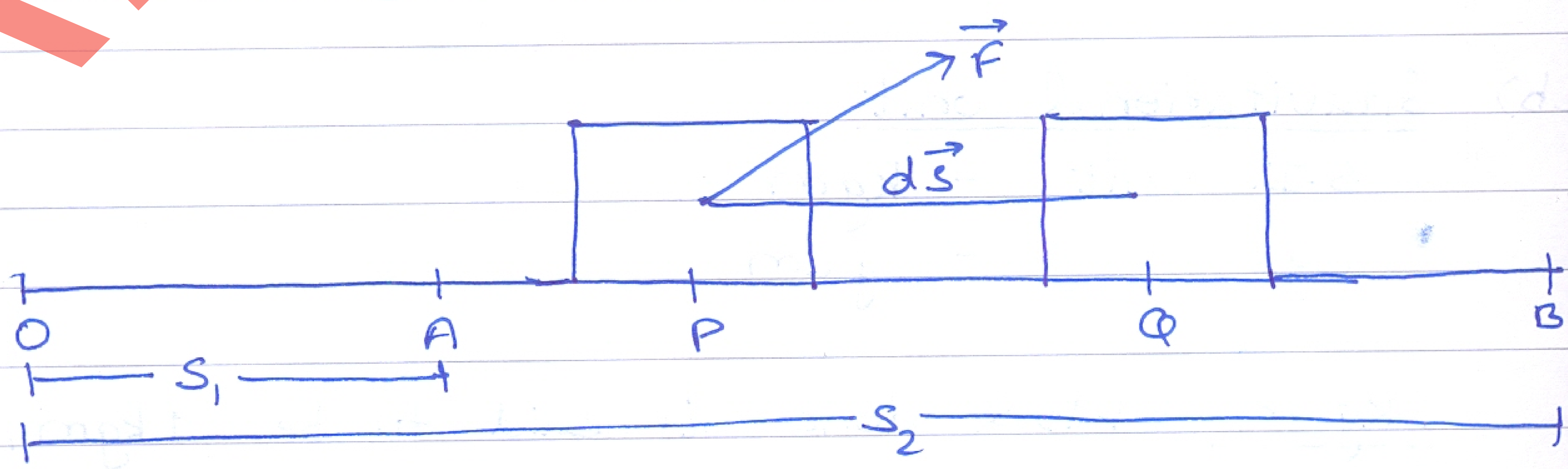
(3) **Zero work** ($\theta = 90^\circ$)

Examples

(i) When a coolie travels on a platform with load on his head, work done by coolie is zero.

(ii) When a body is moved along a circular path, the work done by tension in the string is zero.

Work done by a varying force



Suppose that at any time, the body is at pt. P & the force on the body is $\vec{F}$ due to which it moves from $\vec{P}$ to Q $(PQ = ds)$

[* The force varies bet^n P & Q but due to small value of $d\vec{s}$, it is taken as constant]

Small amount of work done by $\vec{F}$ bet^n P & Q is

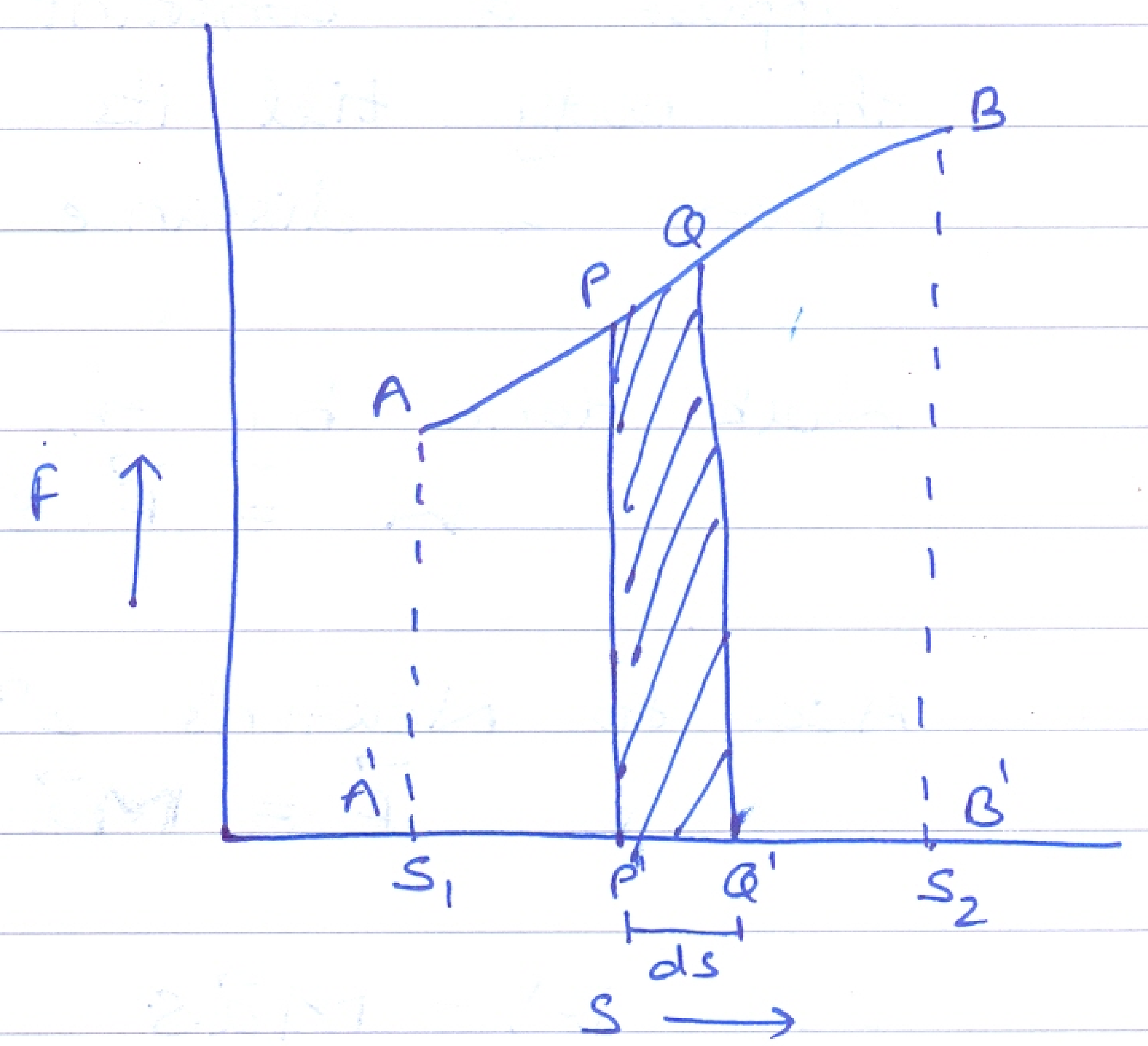
$$dW = \vec{F} \cdot d\vec{s}$$

Total work done in moving the body from A to B is

$$W = \int_A^B \vec{F} \cdot d\vec{s}$$

$$\boxed{W = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s}}$$

Graphical treatment



Small work done bet^n P & Q

$dW = F\, ds$

$= PP' \times P'Q'$

$=$ area of $PQQ'P'$

Total work done between A & B is

$$W = \int_{s_1}^{s_2} F\, ds$$

$=$ Area $ABB'A'$

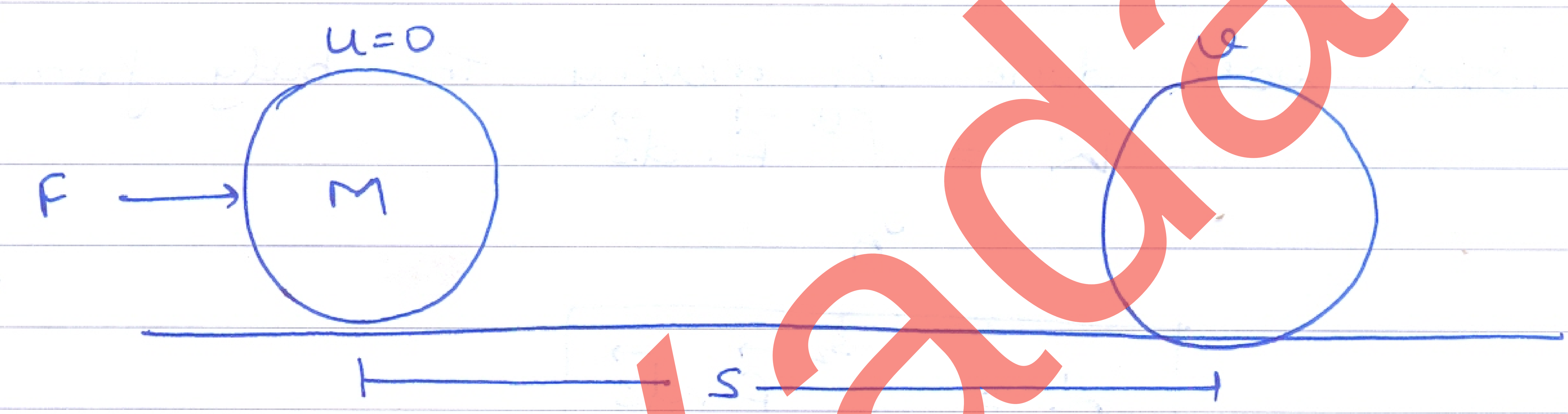
Energy

It is the capacity to do work.

S.I. unit - Joule , cgs - erg

Kinetic Energy

The energy possessed by a body by virtue of its motion is called kinetic energy.



Consider a body of mass M initially at rest. Suppose a constant force $\vec{F}$ is applied on the body till its velocity becomes $\vec{v}$ & it covers a distance 's'.

Work done by the force is

$$W = \vec{F}.\vec{s} = Fs$$

Acc. to Newton's 2^{nd} law

$$\vec{F} = M\vec{a}$$

$$\therefore \quad W = M\vec{a}s \quad \text{——— (1)}$$

Acc. to 3^{rd} eqn of motion

$$v^2 - u^2 = 2as$$

$$v^2 - 0^2 = 2as$$

$$\frac{v^2}{2} = as \quad \text{put in (1)}$$

$$\therefore \quad W = \frac{1}{2} M v^2$$

This work done is a measure of K.E. acquired by the body, so,

$$W = K.E = \frac{1}{2} M v^2$$

Calculus method

Consider a body of mass m initially at rest. Let a force 'F' be applied on the body to displace it through 'ds' in its own direction.

Work done by the force on the body is

$$dW = \vec{F} \cdot \vec{ds}$$
$$= F\, ds \cos 0^\circ$$
$$= F\, ds$$
$$= m a\, ds$$
$$= m \frac{dv}{dt} ds$$
$$= m \left(\frac{ds}{dt} \right) dv$$
$$= m v\, dv$$

Work done in increasing the velocity of the body from 0 to v is

$$W = \int_0^v m v\, dv$$
$$= m \int_0^v v\, dv$$
$$= m \left[\frac{v^2}{2} \right]_0^v$$
$$= \frac{1}{2} m v^2$$

Work-Energy Theorem

Work done by net force in displacing a body is equal to change in kinetic energy of the body.

Consider a body of mass 'm' moving with initial velocity u.
Let a force F be applied on the body so that it acquires an acceleration 'a' such that

$$F = ma$$

Work done in moving the body through a small displacement 'ds' is

$$dW = F\,ds$$
$$= ma\,ds$$
$$= m\frac{dv}{dt}\,ds$$
$$= m\left(\frac{ds}{dt}\right)dv$$
$$= mv\,dv$$

Total work done on the body by the force in increasing the velocity from u to v is

$$W = \int_u^v mv\,dv$$
$$= m\left[\frac{v^2}{2}\right]_u^v$$
$$= \frac{1}{2}m\left[v^2-u^2\right]$$
$$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$
$$= \text{change in K.E.}$$

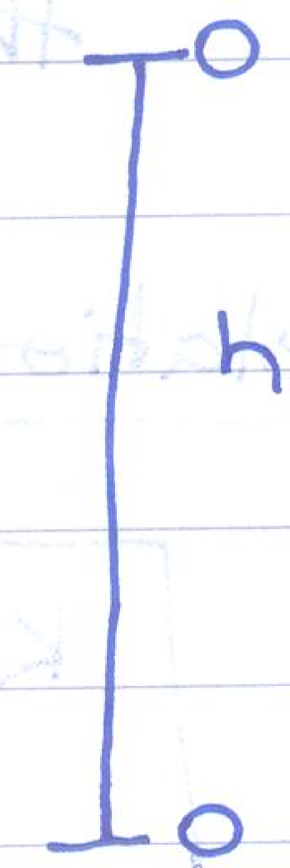
Conservative & Non-conservative forces

Conservative forces

A force is said to be conservative if work done or against the force in moving a body depends only on the initial & final positions of the body, & not on the nature of path followed bet^n the initial & final positions.

eg Gravitational force

Case I



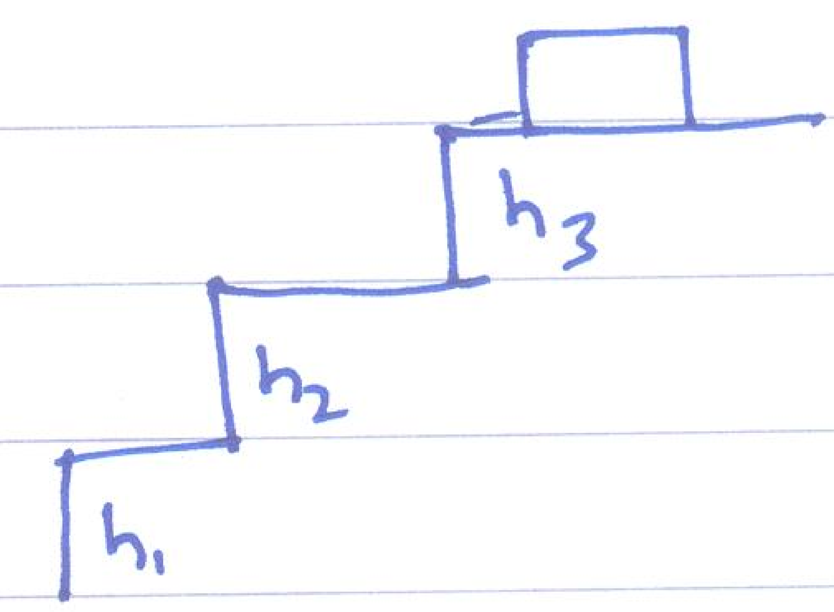
Work done $= \vec{F}.\vec{s}$

$= Fs\cos\theta$

$= mgh\cos 0$

$= mgh$

Case II

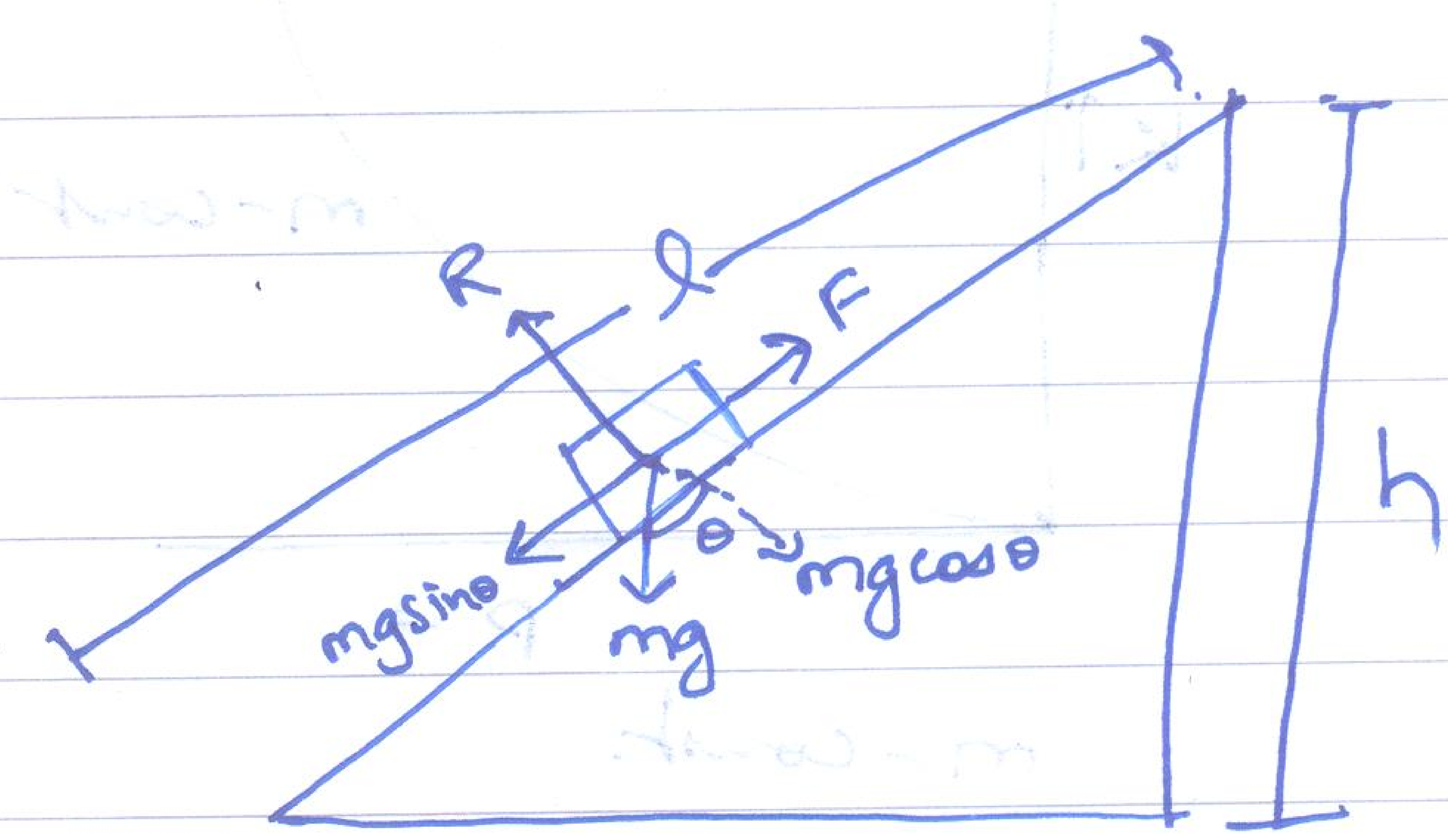


$W = mgh_1 + 0 + mgh_2 + 0 + mgh_3$

$= mg(h_1 + h_2 + h_3)$

$= mgh$

Case III



$W = \vec{F}.\vec{s}$

$= F \times l$

$= mg\sin\theta \times l$

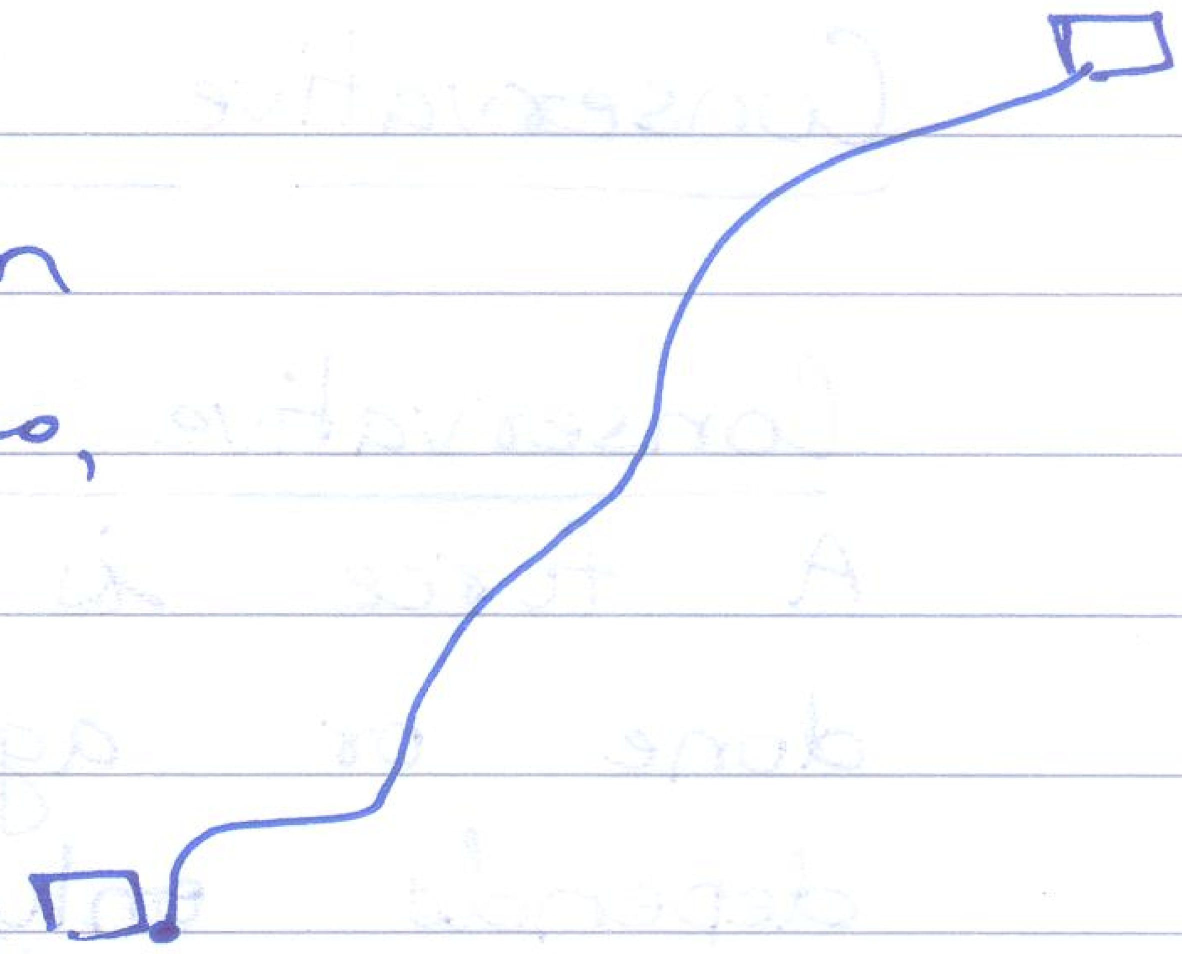
$= mg \times \frac{h}{\cancel{l}} \times \cancel{l}$

$= mgh$

Case 4

It is same as case 2 when seen through a microscope so,

$W = mgh$

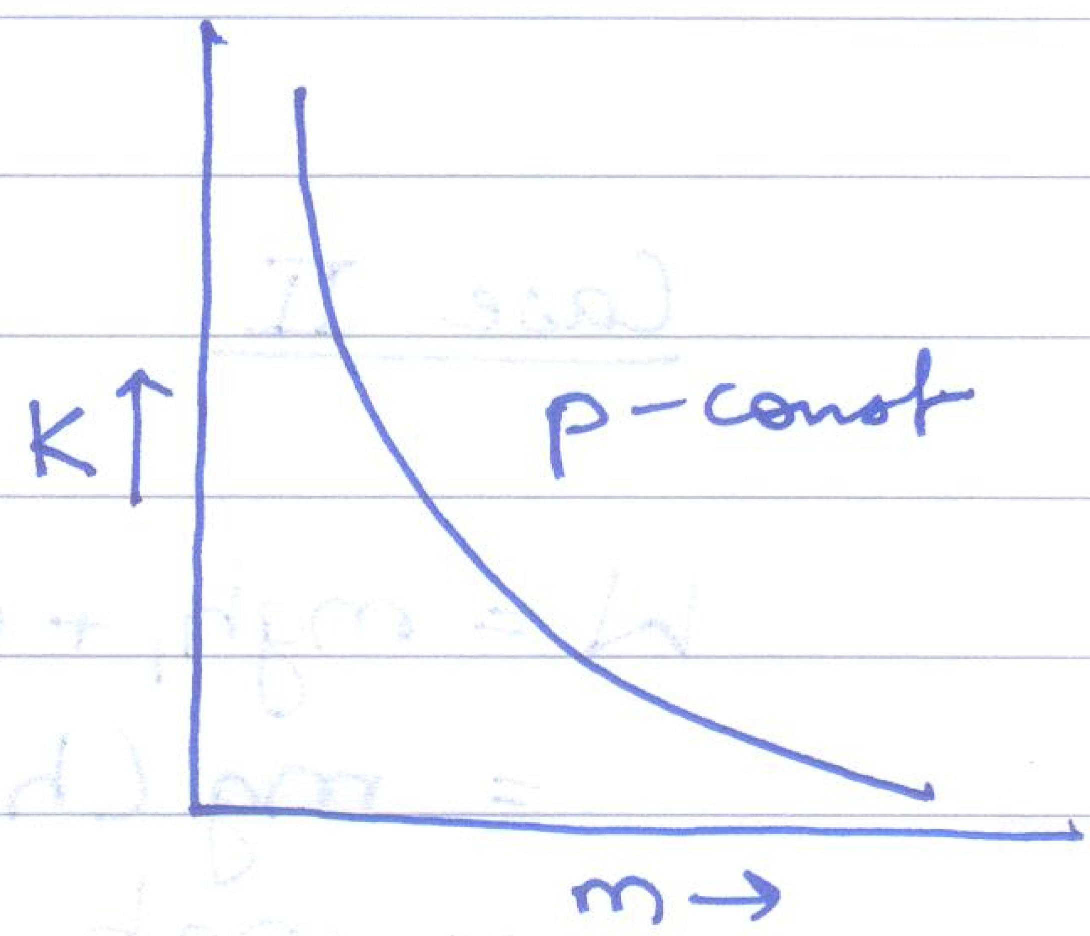


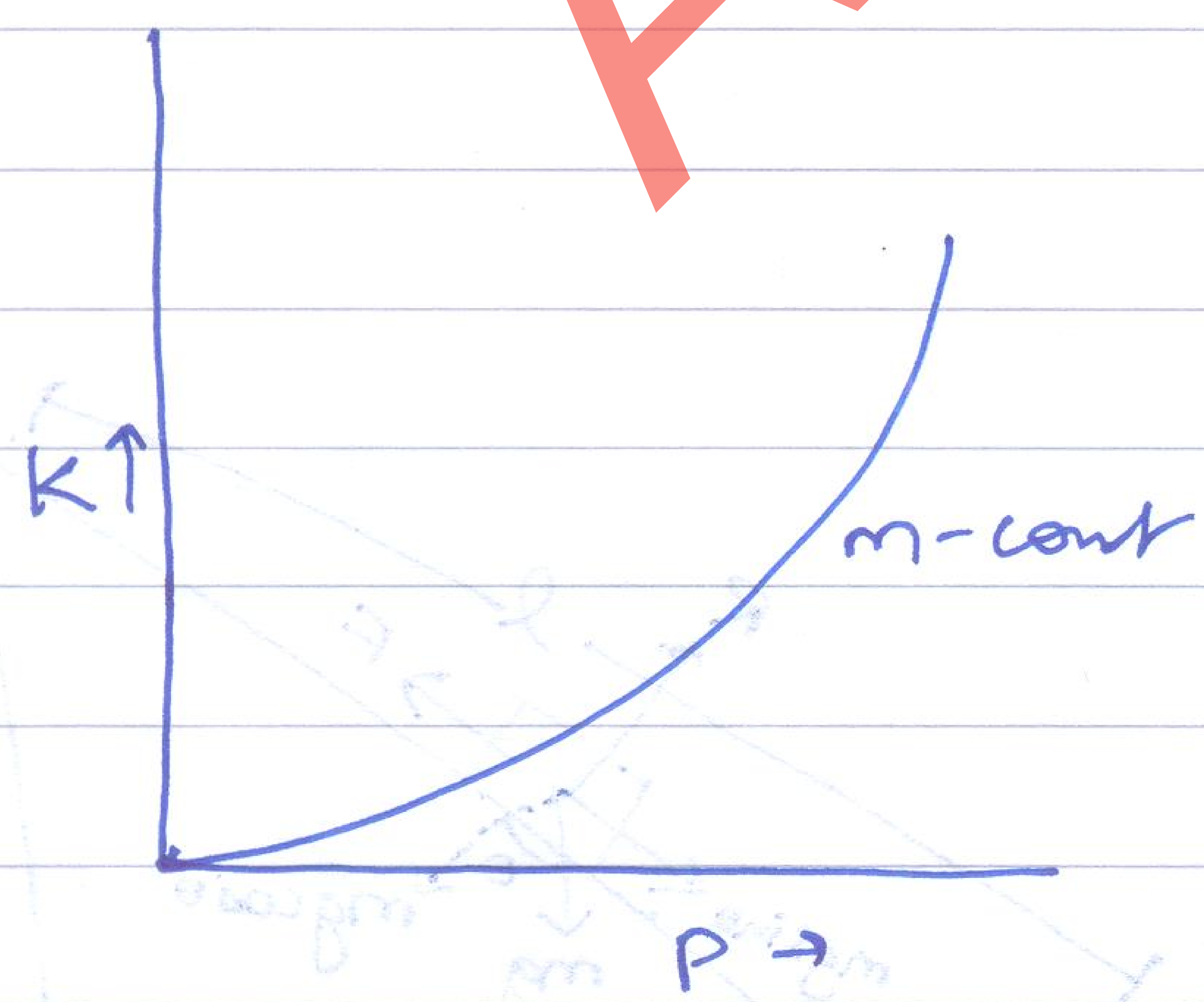
From the above discussion we find that the work done by gravitational energy is independent of the path i.e. it is conservative.

Relation betn K.E. & momentum (p)

$$\boxed{K.E = \frac{1}{2}mv^2 = \frac{1}{2}mv^2 \times \frac{m}{m} = \frac{p^2}{2m}}$$

If p – constant, $K.E. \propto \frac{1}{m}$

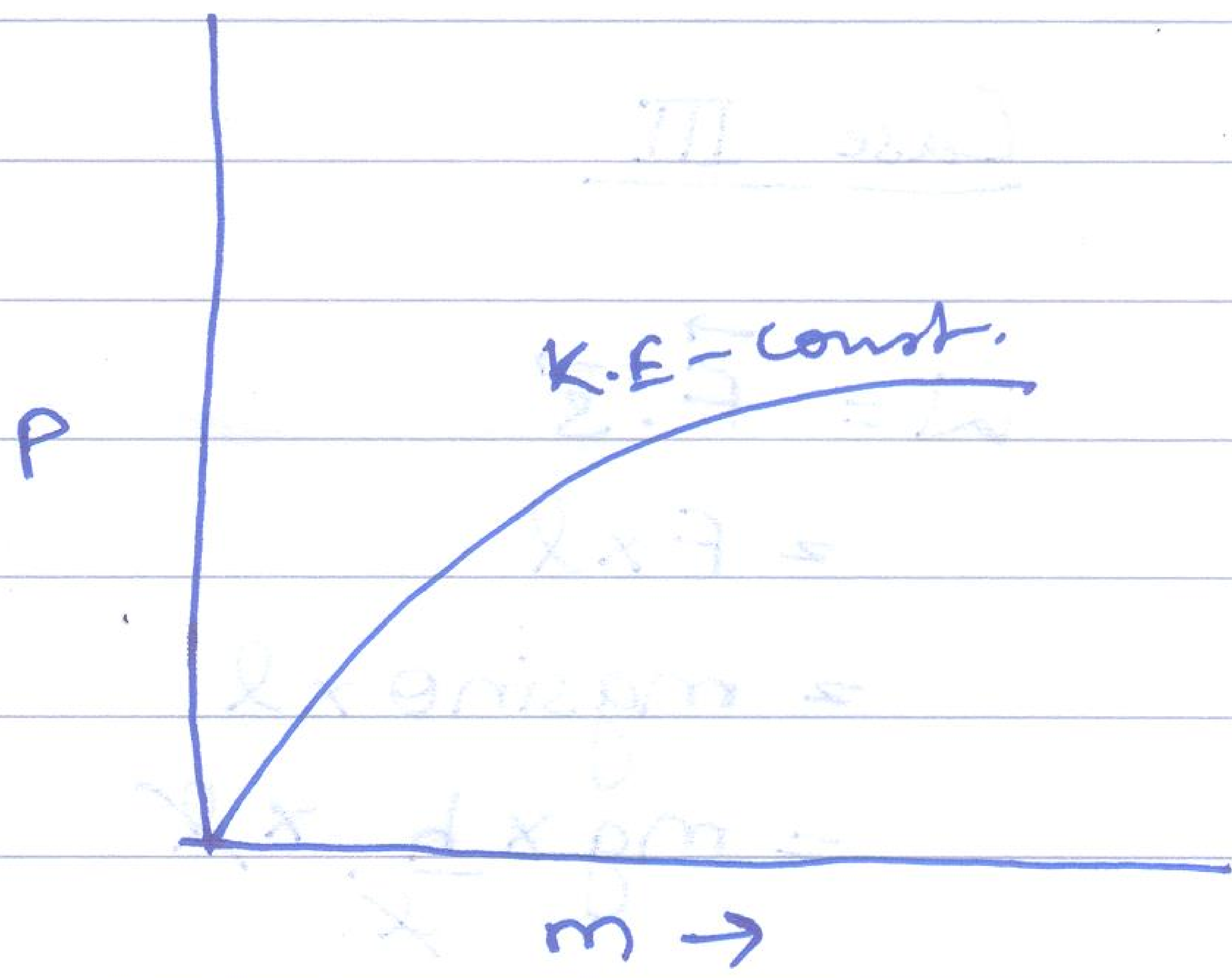




m – const.

$p^2 \propto K.E$

$p \propto \sqrt{K.E.}$



K.E – const

$p^2 \propto m$

$p \propto \sqrt{m}$

Collisions

A collision is an isolated event in which 2 or more colliding bodies exert relatively strong forces on each other for a relatively short time.

Types of collision

(a) **Elastic collision**

A collision in which there is absolutely no loss of K.E. is called an elastic collision.

eg 1. Collision betn atomic & subatomic particles
2. " " 2 ivory balls

Basic characteristics

(i) Linear momentum is conserved.
(ii) K.E. " "
(iii) Total energy " "
(iv) Forces involved are conservative forces.

(b) **Inelastic collision**

A collision in which there occurs some loss of K.E. is called inelastic collision.

* Perfectly inelastic - If the 2 bodies stick to one another after collision.

eg 1. Mud thrown on the wall.
2. Arrow struck in target & they move together.
3. Meteorite collides head on with earth, & gets buried in earth.

Basic characteristics

(i) Linear momentum is conserved.
(ii) K.E not conserved.
(iii) Total energy conserved.
(iv) Some or all forces involved may be conservative.

Coefficient of restitution (e)

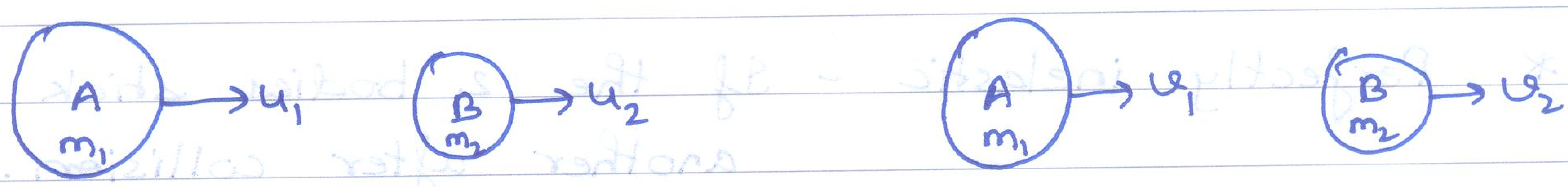
It is the ratio of relative velocity of separation after collision to the relative velocity of approach before collision.

$$\boxed{e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}} = \frac{v_2 - v_1}{u_2 - u_1}}$$

For perfectly elastic collision, $e = 1$
" " inelastic ", $e = 0$
other collisions, $0 < e < 1$

Elastic collision in one dimension

Consider 2 balls A & B of masses m_1 & m_2 moving initially along the same straight line with velocities u_1 & u_2 $(u_1 > u_2)$ resp.



Before collision After collision

The 2 balls collide & let the collision be perfectly elastic. After collision, let v_1 & v_2 ~~be~~ the velocities of A & B resp. $(v_2 > v_1)$ along the same straight line.

Linear momentum before collision $= m_1 u_1 + m_2 u_2$

" " after " $= m_1 v_1 + m_2 v_2$

K.E. before " $= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$

" after " $= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

As linear momentum is conserved

$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$m_2 (v_2 - u_2) = m_1 (u_1 - v_1) \quad \text{—— (1)}$$

As K.E is also conserved

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

$$m_2 (v_2^2 - u_2^2) = m_1 (u_1^2 - v_1^2) \quad \text{—— (2)}$$

(2) ÷ (1)

$$\frac{\cancel{m_2} (v_2 + u_2) \cancel{(v_2 - u_2)}}{\cancel{m_2} \cancel{(v_2 - u_2)}} = \frac{\cancel{m_1} (u_1 + v_1) \cancel{(u_1 - v_1)}}{\cancel{m_1} \cancel{(u_1 - v_1)}}$$

$$v_2 + u_2 = u_1 + v_1$$

$$\boxed{v_2 - v_1 = u_1 - u_2} \quad \text{—— (3)}$$

from (3) $v_2 = u_1 - u_2 + v_1$ put in (1)

$$m_2 (u_1 - u_2 + v_1 - u_2) = m_1 (u_1 - v_1)$$

$$m_2 (u_1 - 2u_2 + v_1) = m_1 (u_1 - v_1)$$

$$m_2 u_1 - 2 m_2 u_2 + m_2 v_1 = m_1 u_1 - m_1 v_1$$

$$m_1 v_1 + m_2 v_1 = m_1 u_1 - m_2 u_1 + 2 m_2 u_2$$

$$(m_1 + m_2) v_1 = (m_1 - m_2) u_1 + 2 m_2 u_2$$

$$\boxed{v_1 = \frac{(m_1 - m_2) u_1 + 2 m_2 u_2}{m_1 + m_2}} \quad \text{—— (4)}$$

from ③ $v_1 = v_2 - u_1 + u_2$ put in ①

$$m_2(v_2 - u_2) = m_1(u_1 - v_2 + u_1 - u_2)$$
$$m_2(v_2 - u_2) = m_1(2u_1 - u_2 - v_2)$$
$$m_2 v_2 - m_2 u_2 = 2m_1 u_1 - m_1 u_2 - m_1 v_2$$
$$m_2 v_2 + m_1 v_2 = 2m_1 u_1 - m_1 u_2 + m_2 u_2$$
$$(m_1 + m_2) v_2 = 2m_1 u_1 + (m_2 - m_1) u_2$$

$$\boxed{v_2 = \frac{2m_1 u_1 + (m_2 - m_1) u_2}{m_1 + m_2}} \quad ⑤$$

Special cases

1. When masses of the 2 bodies are equal

$$m_1 = m_2 = m$$

$$\boxed{v_1 = \frac{2mu_2}{2m} = u_2} \quad , \quad \boxed{v_2 = \frac{2mu_1}{2m} = u_1}$$

2. When the target body B is at rest $(u_2 = 0)$

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} \quad , \quad v_2 = \frac{2m_1 u_1}{m_1 + m_2}$$

(a) When masses are equal, $m_1 = m_2$

$$v_1 = 0 \quad , \quad v_2 = u_1$$

Body A comes to rest while body B starts moving with the initial velocity of A.

(b) When body B is very heavy, $m_2 >> m_1$ i.e. $m_1 = 0$

$$v_1 = \frac{-m_2}{m_2} u_1 = -u_1 \quad ; \quad v_2 = 0$$

So, when a light body A collides against a heavy body B at rest, A rebounds with its own velocity & B continues to be at rest.

eg Rebounding of a ball to the same height (from which it was thrown) on striking a floor.

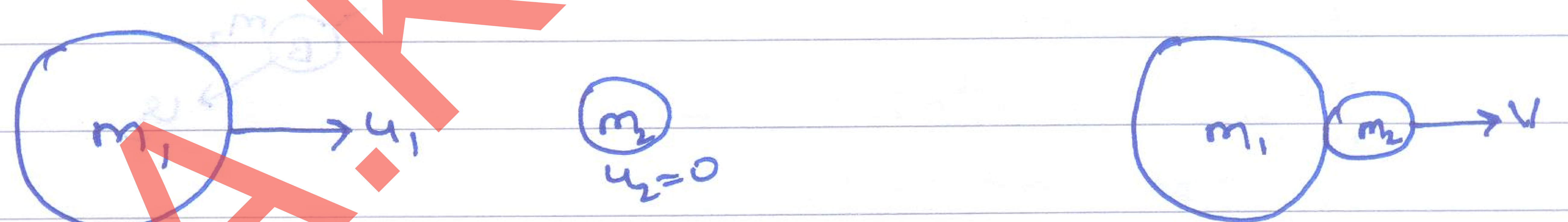
(c) When body B has negligible mass, $m_2 << m_1$ i.e. $m_2 = 0$

$$v_1 = \frac{m_1}{m_1} u_1 = u_1 \quad ; \quad v_2 = \frac{2m_1u_1}{m_1} = 2u_1$$

So, after collision A keeps on moving with the same velocity & B starts moving with double the velocity of A.

Perfectly inelastic collision in one dimension

Consider 2 bodies with masses m_1 & m_2 with the first body moving with velocity u_1 & second body at rest.
After collision both move with common velocity V.



Before collision   After collision.

Also the momentum is conserved

$$m_1u_1 + m_2u_2 = (m_1 + m_2)V$$
$$m_1u_1 = (m_1 + m_2)V$$

$$\boxed{V = \frac{m_1u_1}{m_1 + m_2}}$$

K.E. before collision $= \frac{1}{2} m_1 u_1^2$

" after " $= \frac{1}{2}(m_1 + m_2)V^2 = \frac{m_1^2 u_1^2}{2(m_1 + m_2)}$

$$\text{Loss of K.E.} = \frac{1}{2} m_1 u_1^2 - \frac{m_1^2 u_1^2}{2(m_1+m_2)}$$

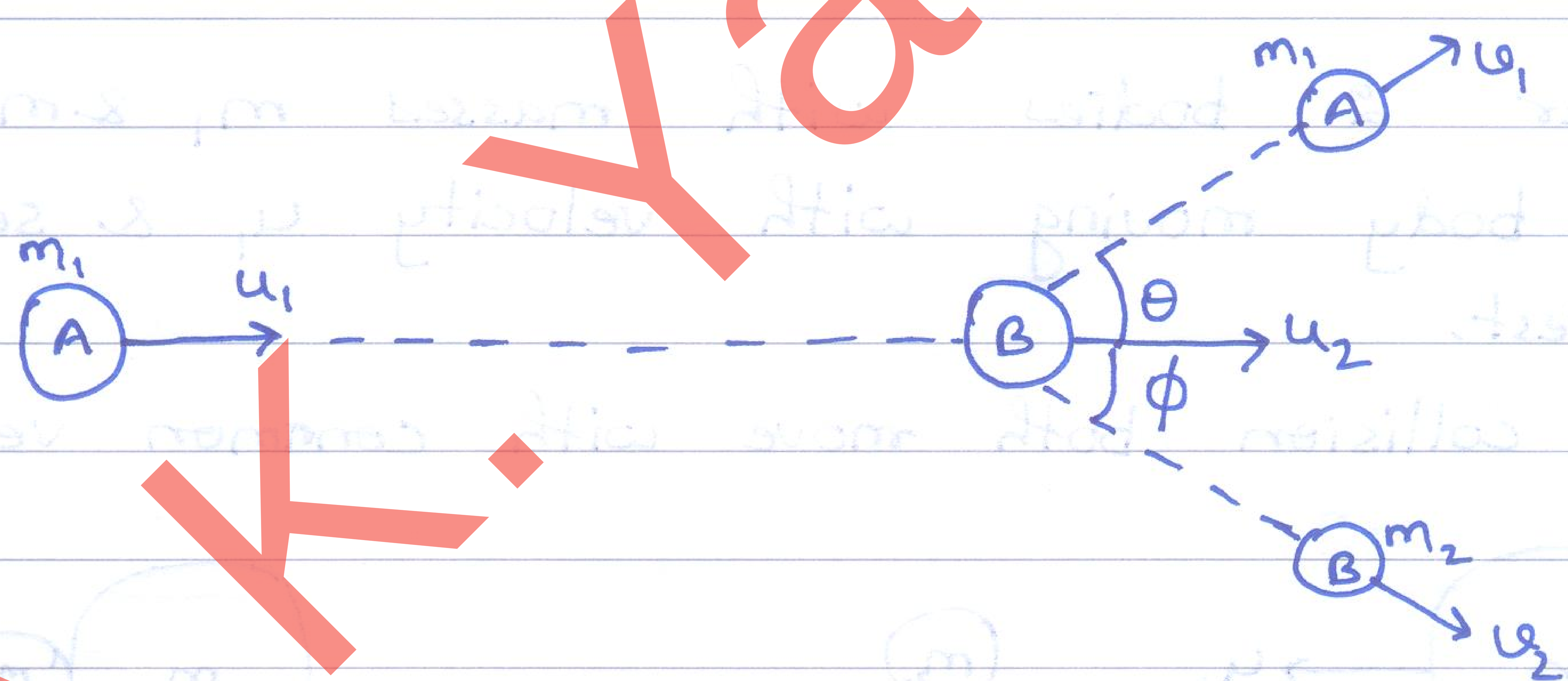
$$= \frac{m_1 u_1^2 (m_1+m_2) - m_1^2 u_1^2}{2(m_1+m_2)}$$

$$= \frac{m_1 u_1^2 (m_1+m_2-m_1)}{2(m_1+m_2)}$$

$$= \frac{m_1 m_2 u_1^2}{2(m_1+m_2)} = +ve$$

$\therefore$ Some K.E. is always lost in an inelastic collision.

Elastic collision in two dimensions



Consider 2 bodies A & B having masses m_1 & m_2 moving with velocities u_1 & u_2 $(u_1 > u_2)$ along X-axis. After collision the 2 bodies move with velocities v_1 & v_2 making angles θ & ϕ resp. with x-axis.

As the linear momentum is conserved

Along X-axis

$$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\theta + m_2 v_2 \cos\phi \quad \text{—} (1)$$

Along Y-axis

$$0 = m_1 v_1 \sin\theta - m_2 v_2 \sin\phi \quad \text{—} (2)$$

$$m_1 v_1 \sin\theta = m_2 v_2 \sin\phi \quad \text{—} (2)$$

K.E. is also conserved

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$m_1 u_1^2 + m_2 u_2^2 = m_1 v_1^2 + m_2 v_2^2 \quad \text{—} (3)$$

Let $m_1 = m_2 = m$

$u_1 = u$

$u_2 = 0$

$\therefore$

$$u = v_1 \cos\theta + v_2 \cos\phi \quad \text{—} (4)$$

$$v_1 \sin\theta = v_2 \sin\phi \quad \text{—} (5)$$

$$u^2 = v_1^2 + v_2^2 \quad \text{—} (6)$$

$$(v_1 \cos\theta + v_2 \cos\phi)^2 = v_1^2 + v_2^2$$

$$v_1^2 \cos^2\theta + v_2^2 \cos^2\phi + 2 v_1 v_2 \cos\theta \cos\phi = v_1^2 + v_2^2$$

$$\begin{aligned}
2 v_1 v_2 \cos\theta \cos\phi &= v_1^2 + v_2^2 - v_1^2 \cos^2\theta - v_2^2 \cos^2\phi \\
&= v_1^2 (1 - \cos^2\theta) + v_2^2 (1 - \cos^2\phi) \\
&= v_1^2 \sin^2\theta + v_2^2 \sin^2\phi \\
&= v_1^2 \sin^2\theta + v_1^2 \sin^2\theta
\end{aligned}$$

$$2 v_1 v_2 \cos\theta \cos\phi = 2 v_1^2 \sin^2\theta$$

$$\cos\theta = \frac{v_1}{v_2} \frac{\sin^2\theta}{\cos\phi}$$

Now, $\cos(\theta + \phi) = \cos\theta \cos\phi - \sin\theta \sin\phi$

$$= \frac{v_1}{v_2} \frac{\sin^2\theta}{\cos\phi} \cdot \cos\phi - \sin\theta \frac{v_1}{v_2} \sin\theta$$

$$= 0$$

$$= \cos 90^\circ$$

$$\therefore \boxed{\theta + \phi = 90^\circ}$$