Viscosity

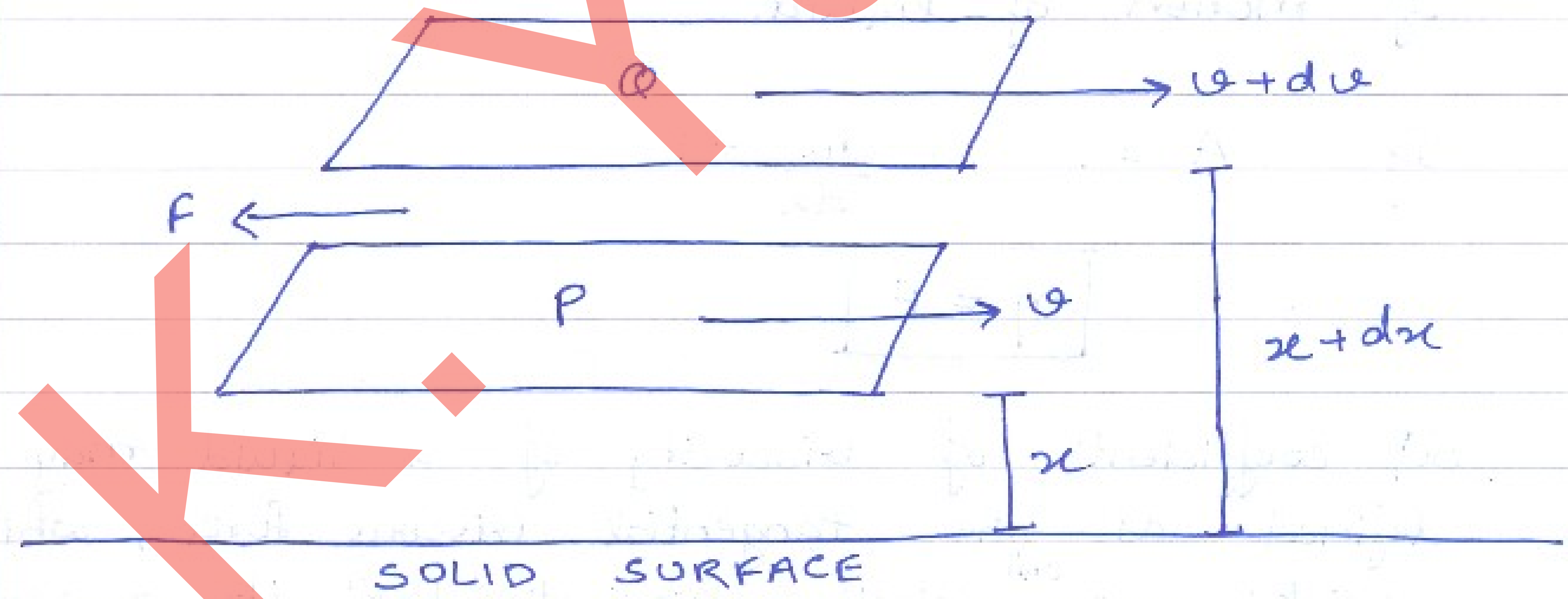
Viscosity

The property of fluids by virtue of which an internal resistance or friction comes into play, when a fluid is in motion is called viscosity.

OR

The property of fluids by virtue of which a backward dragging force acts tangentially on the layers of the fluid in motion & it tries to stop the motion is called viscosity.

Coefficient of Viscosity (η)



Consider 2 parallel layers P & Q at distances x, $x + dx$ from the solid surface moving with velocities v & $v + dv$ resp.

The relative motion bet^n the 2 layers can take place only if some external force acts bet^n them.

Due to viscosity, a force F acts in opposite direction to destroy the relative motion

Acc. to Newton, the viscous force F depends on

(a) area of layers in contact

$$F \propto A$$

(b) velocity gradient betn layers

$$F \propto \frac{dv}{dx}$$

$$\therefore \quad F \propto A\frac{dv}{dx}$$

$$\boxed{F = -\eta A\frac{dv}{dx}}$$

−ve sign shows that the viscous force is directed in a direction opp. to the direction of motion of liquid.

If $A = 1$ & $\frac{dv}{dx} = 1$

$$\boxed{\eta = F}$$

So, coefficient of viscosity of a liquid may be defined as the tangential viscous force, which maintains a unit velocity gradient betn its 2 parallel layers, each of unit area.

Unit of η

S.I. unit − poiseuille (Pl)

$$1\ Pl = 1\ Nm^{-2}s$$

c.g.s unit − poise

$$1\ poise = 1\ dyne\ cm^{-2}s$$

$$1\ Pl = 1\ Nm^{-2}s = 10^5\ dyne \times (100cm)^{-2}s$$

$$= 10\ dyne\ cm^{-2}s$$

$$\boxed{1\ Pl = 10\ poise}$$

Similarity betn solid friction & viscosity

1. Both opposes motion
2. comes into play whenever there is relative motion
3. Both are due to molecular attractions.

Differences

1. Solid friction - independent of area in contact
   viscosity - proportional to surface area
2. solid friction - independent of tem.
   viscosity - decreases with tem.

Q. How is the knowledge of η helps in selecting a suitable lubricant for a machine?

Ans The knowledge of η of different liquids & its variation with tem. helps in selecting a suitable lubricant for a machine.

Poiseuille's formula for flow of liquid through a capillary tube

Poiseuille found that volume of liquid flowing per sec. (V) is

$V \propto p$ [pressure diff. at 2 ends]
$V \propto r^4$ [radius of tube]
$V \propto \frac{1}{l}$ [length of tube]

$V \propto \frac{1}{\eta}$

So, $V \propto \frac{pr^4}{\eta l} \Rightarrow \boxed{V = \frac{\pi}{8}\frac{pr^4}{\eta l}}$

Stoke's law

- When a spherical body falls through the fluid, layer in immediate contact with it, sticks to it, & moves along the body with same velocity
- The layer next to it has lesser velocity, the next still lesser & so on.
- As a result of this, a backward dragging force called viscous force (F) is set up, which opposes the motion of the body.
- As the velocity goes on increase, F also increases & becomes equal to weight of the body.
- Then the body moves with a constant velocity called terminal velocity (v)
- Acc. to Stoke

"When a body attains terminal velocity, then the viscous force on the body is

$$F = 6\pi\eta r v$$

<u>Proof</u>:

Let $F \propto \eta^a r^b v^c$

$$F = k\eta^a r^b v^c$$

$$[MLT^{-2}] = k[ML^{-1}T^{-1}]^a [L]^b [LT^{-1}]^c$$

$$= [M]^a [L]^{-a+b+c} [T]^{-a-c}$$

$a = 1$

$-a+b+c = 1$

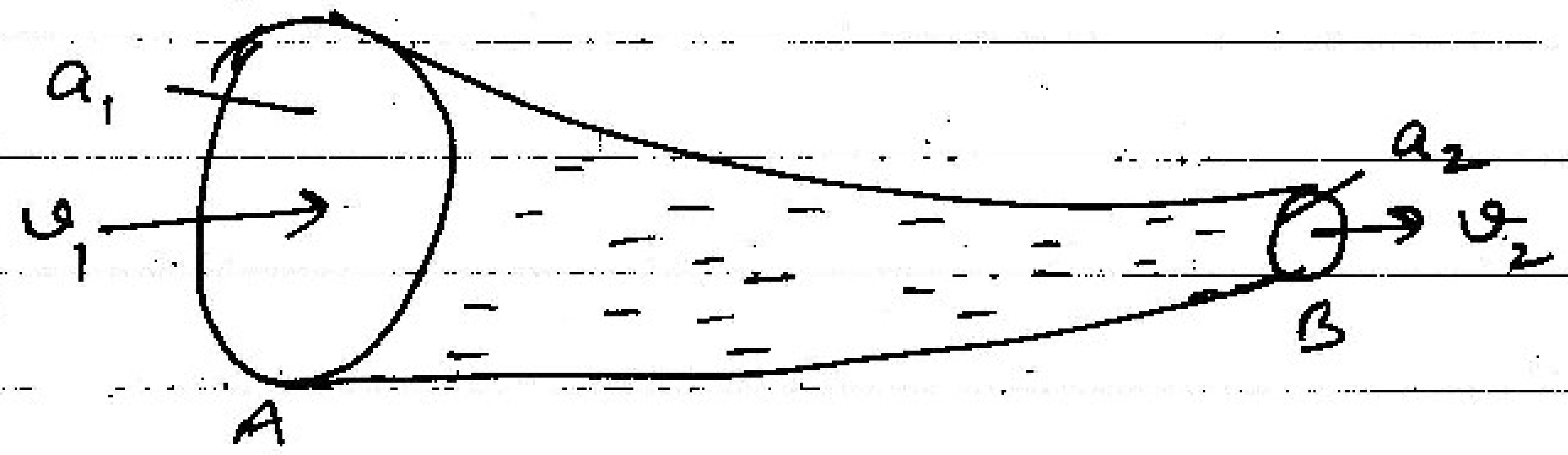
$-a-c = -2 \Rightarrow c = 1$

$b = 1$

$F = k\eta r v \Rightarrow \boxed{F = 6\pi\eta r v}$

Fluid Flow

Eqn of continuity



Consider a liquid moving through a pipe AB of varying cross-section.

Let a_1 – cross-sectional area of pipe at A
a_2 – " " " " B
v_1 – ~~exact~~ entering velocity of liquid at A
v_2 – exist " " " B.
ρ_1 – density of liquid at A
ρ_2 – " " " B

Volume of liquid entering per sec. at A = $a_1 v_1$

∴ Mass of liquid entering per sec. at A = $a_1 v_1 \rho_1$
" " " " " " B = $a_2 v_2 \rho_2$

As there is no source or sink of liquid along the length of pipe, the mass of liquid crossing any section of pipe is same

i.e. $a_1 v_1 \rho_1 = a_2 v_2 \rho_2$

Now, $\rho_1 = \rho_2$ (as the liquid is incompressible)

$$\boxed{a_1 v_1 = a_2 v_2}$$

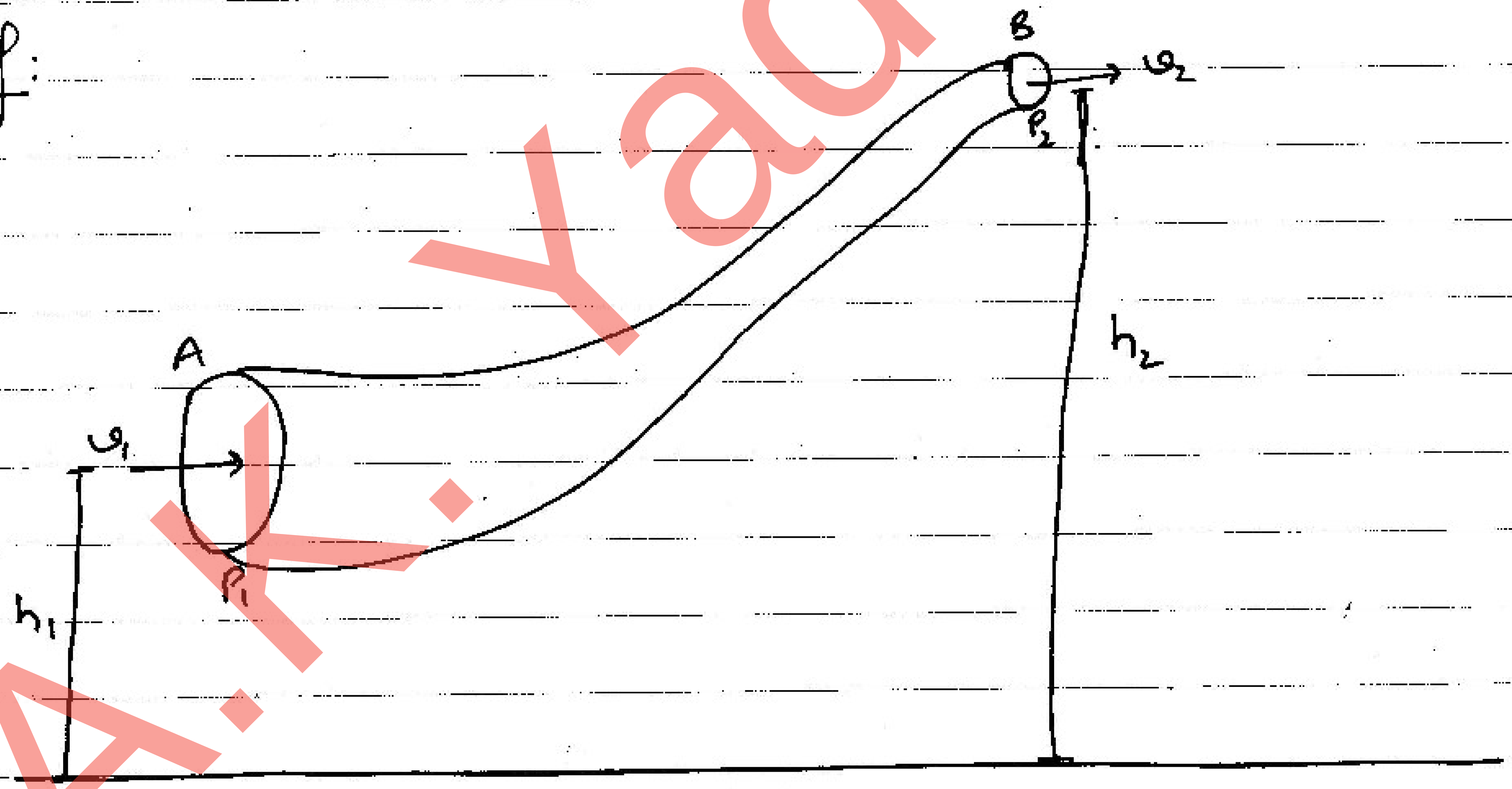
Bernoulli's theorem

The total energy (pressure energy, potential energy & kinetic energy) of an incompressible and non-viscous fluid in steddy flow through a pipe remains constant throughout the flow, provided there is no source or sink of fluid along the length of the pipe.

Mathematically, for a unit mass of fluid

$$\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{const.}$$

Proof:



Consider a tube AB of varying cross-section through which an ideal liquid is in streamline flow.

Let P_1, P_2 – pressure at A & B

a_1, a_2 – area of cross-section of tube at A & B

h_1, h_2 – mean height of A & B from ground

v_1, v_2 – velocity of liquid at A & B

ρ – density of ideal liquid flowing through the tube.

Also, $P_1 > P_2$

Acc. to eq^n of continuity, the mass m of the liquid crossing per sec. through any section of the tube is

$$a_1 v_1 \rho = a_2 v_2 \rho = m$$

$$a_1 v_1 = a_2 v_2 = \frac{m}{\rho} = V$$

As $a_1 > a_2$ so $v_2 > v_1$

Work done per sec. on the liquid by the pressure energy at A $= P_1 a_1 \times v_1 = P_1 V$

Work done per sec. ~~by~~ the liquid against the pressure energy at B $= P_2 a_2 \times v_2 = P_2 V$

Net work done per sec. on the liquid by the pressure energy in moving the liquid from A to B
$= P_1 V - P_2 V$

Increase in P.E./sec of liquid from A to B $= mgh_2 - mgh_1$

" " K.E./sec. " " " " $= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$

Acc. to work energy theorem
Work done/sec. on the liquid by pressure energy $=$ increase in P.E./sec. + increase in K.E./sec.

$$P_1 V - P_2 V = (mgh_2 - mgh_1) + \left(\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2\right)$$

$$P_1 V + mgh_1 + \frac{1}{2} m v_1^2 = P_2 V + mgh_2 + \frac{1}{2} m v_2^2$$

$$\frac{P_1 V}{m} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2 V}{m} + gh_2 + \frac{1}{2}v_2^2$$

$$\frac{P_1}{\rho} + gh_1 + \frac{1}{2}v_1^2 = \frac{P_2}{\rho} + gh_2 + \frac{1}{2}v_2^2$$

$$\boxed{\frac{P}{\rho} + gh + \frac{1}{2}v^2 = \text{const.}}$$

Limitations of Bernoulli's theorem

1. It was incorrectly assumed that velocity of liquid across any cross-section of tube is uniform.
2. The viscous drag force comes into play when the liquid is in motion, has not been taken into account.
3. While deriving the eqⁿ, it is wrongly assumed that there is no loss of energy when the liquid is in motion.
4. If the liquid is flowing along a curved path, the energy due to centrifugal force should also be considered.

Date __/__/__

Applications of Bernoulli's theorem

① **Atomiser**

Air blowing out of the bulb creates low pressure so liquid rises up from high pressure region to low pressure acc. to Bernoulli's theorem.

② **Lift on aeroplane wing (aerofoil)**

- The upper surface is more curved than the lower surface & head is more thicker than tail.
- As upper surface is more curved than lower surface so speed of air above the wings is more.
- According to Bernoulli's theorem pressure above the wings will be less.
- Due to this pressure difference on the two sides of the wings, a vertical lift acts on the aeroplane.

③ **Magnus effect**

- When a ball is spinning as well as moving linearly, the streamlines at the top of the ball are opposite to each other and those below are in same direction.
- As a result, the velocity of air flow is greater below than above the ball.
- Acc. to Bernoulli's theorem, the pressure on upper side of ball becomes more than the pressure on the lower side of the ball.
- Due to it, a resultant force acts on

the ball at right angles to the linear motion in the downward direction, resulting the ball to move along a curved path.

- This dynamic force due to spinning of ball is called Magnus effect.

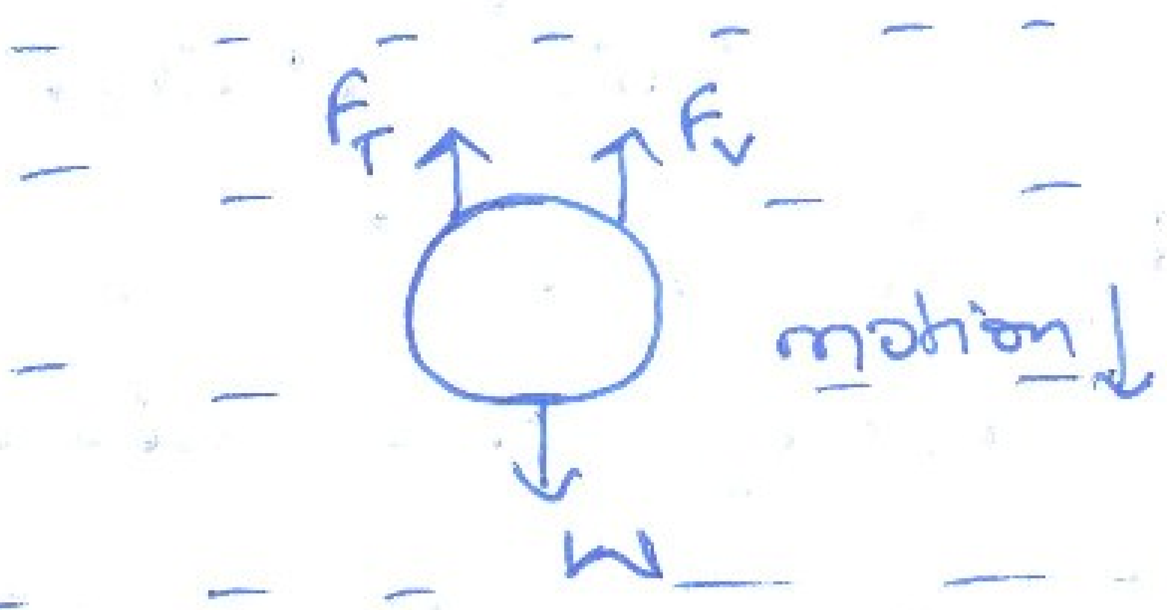
(4) Blowing off the roofs during storm.

(5) Motion of two parallel boats.

Terminal velocity

It is the max. constant velocity acquired by the body while falling freely in a viscous medium.

Consider a small spherical body falling freely in a viscous medium.



The forces acting on the body are

(i) Weight (W) of the body acting downwards

(ii) Upthrust (F_T) equal to weight of liquid displaced.

(iii) Viscous drag force (F_V) acting upwards

Let ρ - density of material of the spherical body
r - radius of spherical body
σ - density of medium

Weight of the body, $W = V \times \rho \times g$
$$= \frac{4}{3}\pi r^3 \rho g$$

Upthrust due to buoyancy, $F_T = V \times \sigma \times g$
$$= \frac{4}{3}\pi r^3 \sigma g$$

Upward viscous drag force, $F_v = 6\pi\eta r v$

When body attains terminal velocity

$$F_T + F_V = W$$

$$\frac{4}{3}\pi r^3 \sigma g + 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho$$

$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \sigma)$$

$$\boxed{v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}}$$

Conclusion

(i) If $\rho < \sigma$, v = negative

- Body will move up with a constant velocity.
- Example - gas bubbles rise up through soda water bottle.

(ii) If $\rho > \sigma$, v = positive

- Body will attain terminal velocity in downward direction

(iii) If $\rho = \sigma$, $v = 0$

- Body remains suspended in the medium.

Variation of Viscosity

① Increase in tem.

viscosity of liquids - decreases
" " gases - increases

② Increase in pressure

viscosity of liquids - increases
" " water - decreases
" " gases - unchanged

Streamline, Laminar & Turbulent Flow

Streamline flow - The flow in which every particle of the liquid ~~flow~~ follows exactly the path of its preceding particle & has the same velocity in magnitude & direction as that of its preceding particle while crossing through that point.

Laminar flow - Flow in which the liquid moves in layers.

Turbulent flow - When a liquid moves with a velocity greater than its critical velocity, the motion of the particles of liquid becomes disorderly. Such a flow is called turbulent flow.

Reynold's number (N_R)

It is a pure number which determines the nature of flow of liquid through a pipe.

$$N_R = \frac{\rho D v_c}{\eta}$$

where
- ρ - density of liquid
- D - diameter of tube
- v_c - critical velocity
- η - coefficient of viscosity of liquid

- $N_R = 0$ to 2000 - stream line or laminar
- $= 2000 - 3000$ - unstable
- > 3000 - turbulent.

- N_R is independent of the system of units used for the measurement of various quantities.