SUNCITY SCHOOL

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VECTORS

1 (a) $\hat{i}$ & $\hat{j}$ are unit vectors along x and y axis. Find the magnitude and direction of $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$

(b) Find the components of vector $A = 2\hat{i}+3\hat{j}$ along the vector $\hat{i}+\hat{j}$ and $\hat{i}-\hat{j}$.

Ans. (a)

$$\left|\hat{i}+\hat{j}\right| = \sqrt{1^2+1^2} = \sqrt{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45^0$$

$$\left|\hat{i}-\hat{j}\right| = \sqrt{1^2+(-1)^2} = \sqrt{2}$$

$$\cos\theta = \frac{1}{\sqrt{2}}$$

$$\theta = 45^0$$

2 At what angle the forces $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ act so that their resultant is $(3A^2 + B^2)^{1/2}$ assume that $\vec{A}$ and $\vec{B}$ are collinear vectors

Ans.

$$R^2 = \left(\vec{A}+\vec{B}\right)^2 + \left(\vec{A}-\vec{B}\right)^2 + 2\left(\vec{A}+\vec{B}\right)\left(\vec{A}-\vec{B}\right)\cos\theta$$

$$3A^2 + B^2 = A^2 + B^2 + 2AB\cos\theta + A^2 + B^2 - 2AB\cos\theta + 2(A^2 - B^2)\cos\theta$$

$$3A^2 + B^2 = 2A^2 + 2B^2 + 2(A^2 - B^2)\cos\theta$$

$$A^2 - B^2 = 2(A^2 - B^2)\cos\theta$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^0$$

3 A vector $\vec{X}$ when added to two vectors $\vec{A} = 3\hat{i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ gives a unit vector along y axis as their resultant. Find the vector $\vec{X}$

Ans.

$$\vec{A} = 3\hat{i} - 5j + 7k$$

$$\vec{B} = 2\hat{i} + 4j - 3k$$

$$ATQ$$

$$\vec{A} + \vec{B} + \vec{X} = j$$

$$3\hat{i} - 5j + 7k + 2\hat{i} + 4j - 3k + \vec{X} = j$$

$$5\hat{i} - j + 4k + \vec{X} = j$$

$$\vec{X} = j - (5\hat{i} - j + 4k) = -5\hat{i} + 2j - 4k$$

4 a) Can 2 vectors of different magnitude be combined to give zero resultant? Can 3 vectors do?

b) Under what condition/s the magnitude of the sum of 2 vectors is equal to the magnitude of difference between them?

Ans. (a) Only vectors with same magnitude acting in opposite direction can give zero resultant in case of two vectors.
In case of 3 vectors, vectors of different magnitude can give zero resultant.

(b)

$$\left|\vec{P}+\vec{Q}\right| = \left|\vec{P}-\vec{Q}\right|$$

$$\sqrt{P^2+Q^2+2PQ\cos\theta} = \sqrt{P^2+Q^2-2PQ\cos\theta}$$

$$4PQ\cos\theta = 0$$

$$\cos\theta = 0$$

$$\theta = 90^0$$

5 The vector sum of two vectors $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If vector $\vec{Q}$ is reversed, the resultant becomes $\vec{S}$. Then prove that $R^2 + S^2 = 2(P^2 + Q^2)$.

Ans. $\vec{R} = \vec{P}+\vec{Q}$

$$R^2 = P^2+Q^2+2PQ\cos\theta \qquad \rightarrow (1)$$

$$\vec{S} = \vec{P}-\vec{Q}$$

$$S^2 = P^2+(-Q)^2+2P(-Q)\cos\theta$$

$$S^2 = P^2+Q^2-2PQ\cos\theta \qquad \rightarrow (2)$$

$$(1)+(2)$$

$$R^2+S^2 = 2(P^2+Q^2)$$

6 If unit vectors A and B are inclined at an angle θ, then show that ($A - B$) =2 sin (θ/2).

Ans. $\left(A-B\right)^2 = \left(A-B\right).\left(A-B\right)$

$$= A.A-2A.B+B.B$$

$$=1-2\cos\theta+1$$

$$=2-2\cos\theta$$

$$=2(1-\cos\theta)$$

$$\left(A-B\right)^2 = 2\left(2\sin^2\frac{\theta}{2}\right)$$

$$\left(A-B\right) = 2\sin\frac{\theta}{2}$$

7 Find the value of n if vectors $2\hat{i}+4\hat{j}-n\hat{k}$ and $3\hat{i}-4\hat{j}-2\hat{k}$ are orthogonal

Ans. $\left(2\hat{i}+4\hat{j}-n\hat{k}\right).\left(3\hat{i}-4\hat{j}-2\hat{k}\right)=0$

$$6-16+2n=0$$

$$2n=10$$

$$n=5$$

8 If $\vec{A}=3\hat{i}+4\hat{j}$ & $\vec{B}=7\hat{i}+24\hat{j}$, find a vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$.

Ans. For parallel to $\vec{A}$, find unit vector of $\vec{A}$ and multiply it with magnitude of $\vec{B}$ to get the answer.

$\vec{A} = 3\hat{i} + 4\hat{j}$

$A = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$A = \frac{\vec{A}}{A} = \frac{3\hat{i} + 4\hat{j}}{5}$

$\vec{B} = 7\hat{i} + 24\hat{j}$

$B = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$

$BA = 25\frac{3\hat{i} + 4\hat{j}}{5} = 5(3\hat{i} + 4\hat{j}) = 15\hat{i} + 20\hat{j}$

9 Find the unit vector perpendicular to each of these vectors $\vec{P} = 3\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{Q} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

Ans. $\vec{P} = 3\hat{i} + \hat{j} + 2\hat{k}$

$\vec{Q} = 2\hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{R} = \vec{P} \times \vec{Q}$

$$\vec{P} \times \vec{Q} = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{pmatrix} = \hat{i}\left(4-(-4)\right) - \hat{j}(12-4) + \hat{k}(-6-2) = 8\hat{i} - 8\hat{j} - 8\hat{k}$$

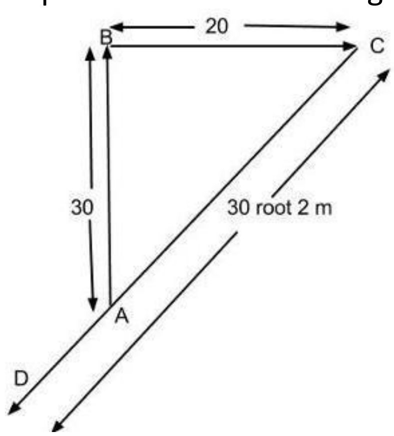
$\vec{R} = 8\hat{i} - 8\hat{j} - 8\hat{k}$

$\left|\vec{R}\right| = \sqrt{8^2 + (-8)^2 + (-8)^2} = 8\sqrt{3}$

$R = \frac{\vec{R}}{\left|\vec{R}\right|} = \frac{8\hat{i} - 8\hat{j} - 8\hat{k}}{8\sqrt{3}} = \frac{\hat{i} - \hat{j} - \hat{k}}{\sqrt{3}}$

10 A person moves 30 m north, then 20 m east and then 30√2 m south west. Find his displacement from the original positions

Ans.



$AC = \sqrt{30^2 + 20^2} = \sqrt{900 + 400} = 10\sqrt{13} = 36m$

$AD = CD - AC = 30\sqrt{2} - 10\sqrt{13} = 42.42 - 36 = 6.42m$

$\text{Displacement} = 30 + 6.42 = 36.42m$

11 Three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are such that $\vec{A} = \vec{B} + \vec{C}$ and their magnitudes are 5, 4 and 3 respectively. Find the angle between $\vec{A}$ and $\vec{C}$.

Ans. $\vec{A} = \vec{B} + \vec{C}$

$\vec{A}.\vec{A} = \left(\vec{B} + \vec{C}\right).\left(\vec{B} + \vec{C}\right)$

$A^2 = B^2 + C^2 + 2BC\cos\theta$

$5^2 = 4^2 + 3^2 + 2(4)(3)\cos\theta$

$25 = 25 + 24\cos\theta$

$24\cos\theta = 0$

$\cos\theta = 0$

$\theta = 90^0$

12 A man can swim with a speed of 4 km/h in still water. How long does he takes to cross a river 1 km wide, if the river flow steadily at 3 km/h and he makes his strokes normal to the river current? How far down the river does he go, when he reaches the other bank?

Ans. $v_m = 4kmhr^{-1}$

$v_r = 3kmhr^{-1}$

$t = \frac{s}{v_m} = \frac{1}{4}hr$

$d = v_r \times t = 3 \times \frac{1}{4} = 0.75km$

13 A boatman can row with speed of 10 km/h in still water. If the river flows steadily 5 km/h, in which direction should the boatman row in order to reach a point on the other bank directly opposite to the point from where he started? The width of the river is 2 km.

Ans. $v_b = 10kmhr^{-1}$

$v_r = 5kmhr^{-1}$

$v^2 = v_b^2 - v_r^2$

$= 100 - 25$

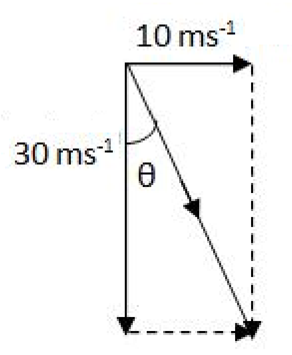
$v = \sqrt{75} = 5\sqrt{3}kmhr^{-1}$

$\cos\theta = \frac{5}{10} = \frac{1}{2}$

$\theta = 60^0 (against\ the\ flow)$

14 On a certain day rain was falling vertically with a speed of 30 m/s. If wind starts blowing with a speed of 10 m/s in the direction from north to south, find the direction in which a boy should hold his umbrella in order to protect himself from the rain?

Ans.



$$\tan\theta = \frac{10}{30}$$

$$\theta = \tan^{-1}\left(\frac{1}{3}\right)$$