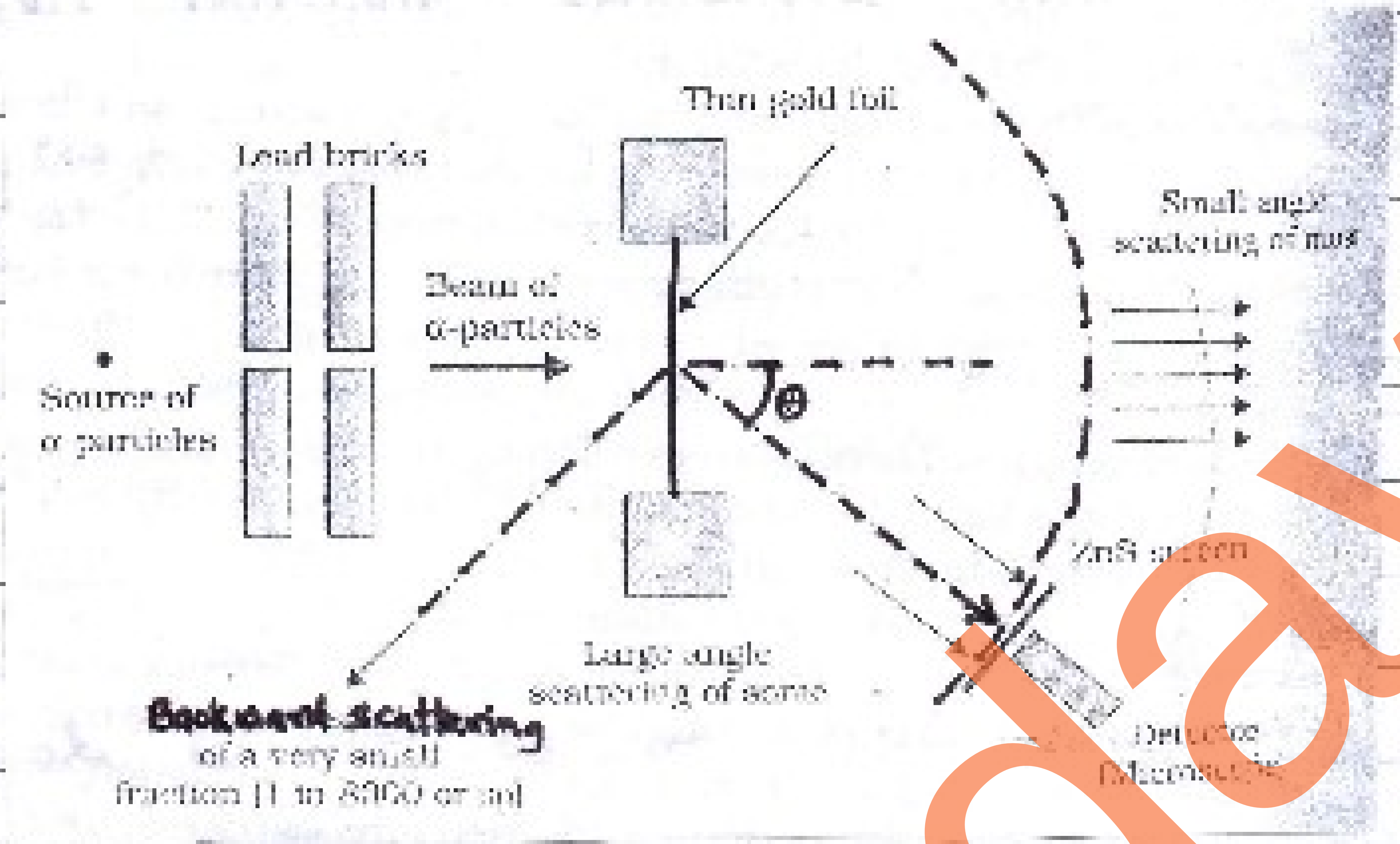
Atoms & Nuclei

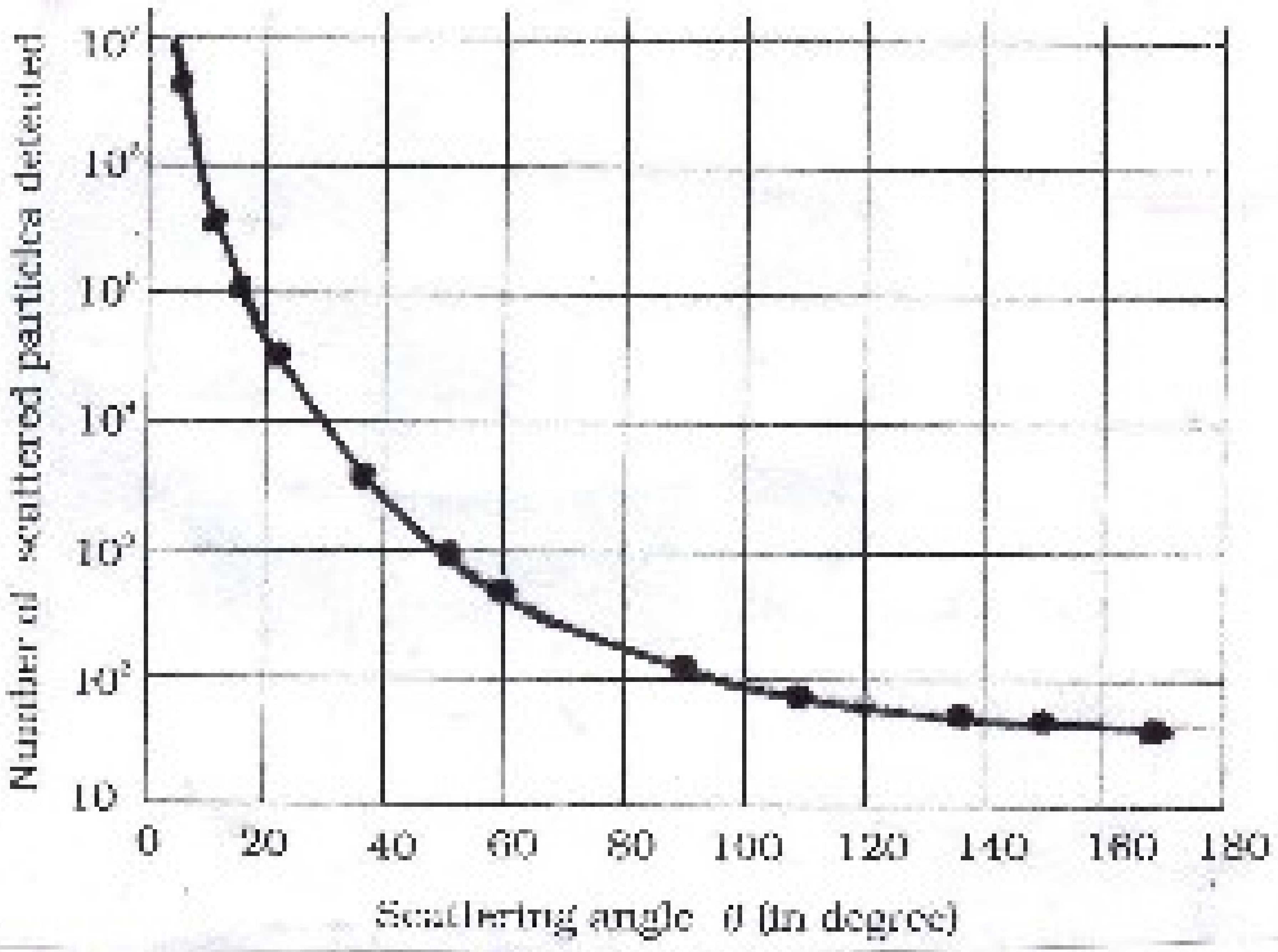
Rutherford's α-ray scattering experiment



- A beam of 5.5 MeV α-particles emitted from a $^{214}_{83}Bi$ (radioactive source) is directed towards a thin gold foil.
- α-particles (emitted by ^{214}Bi) were collimated into a narrow beam by their passage through lead bricks.
- The beam was allowed to fall on a thin gold foil.
- The scattered α-particles were observed through a rotatable detector consisting of ZnS screen & microscope
- The scattered α-particles on striking the screen produced scintillations, which may be observed through microscope
- The distribution of no of scattered particle is studied as a function of angle of scattering.

Observation

A graph is plotted betn no. of particle scattered & scattering angle θ.



- Most of α-particles pass through the gold foil without any deviation.
- Only about 0.14% of incident α-particles scatter by more than 1°.
- About 1 α-particle in every 8000 deflect by more than 90°.

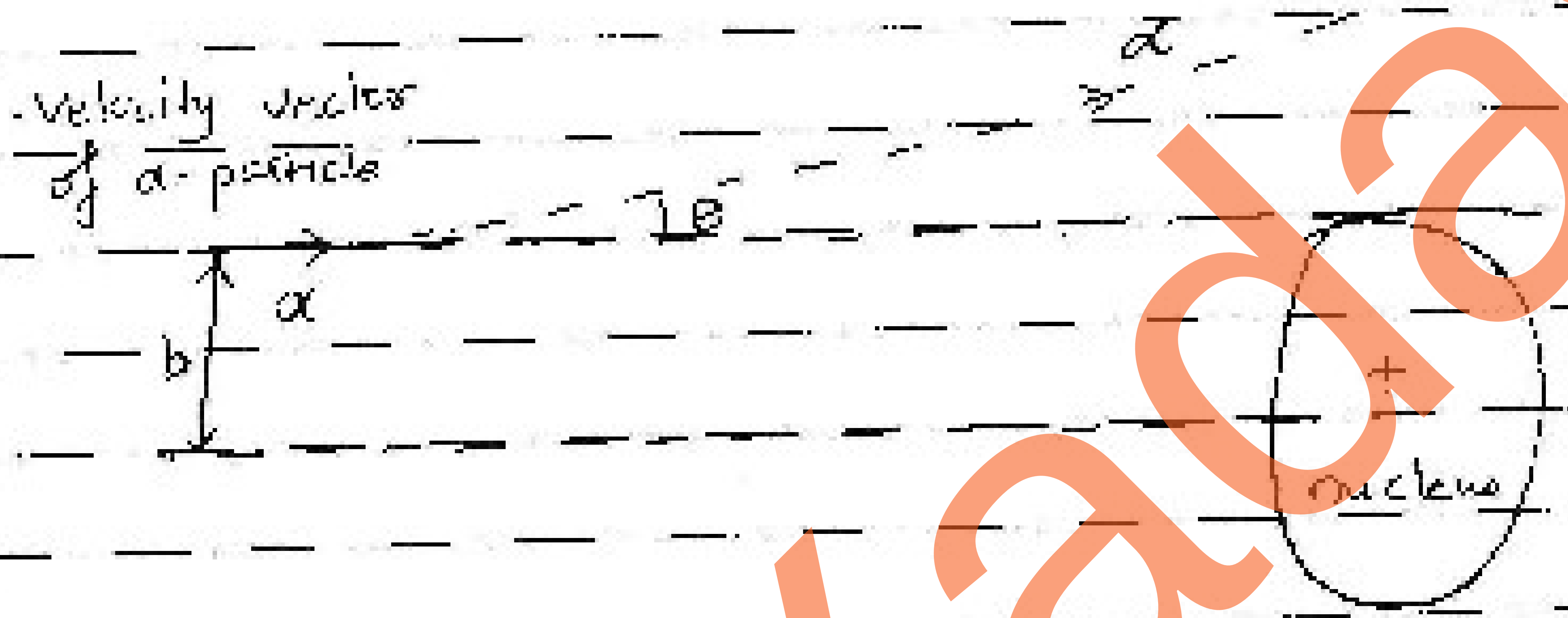
Conclusion

1. As most of the α-particles go undeviated it means most of the space is empty.
2. The scattering of α-particles by more than 1° is due to coulombic interaction forces bet^n α-particle & +ve charge present in the ~~nucleus~~ atom (i.e. nucleus) as electrons are too light to cause deviation.
   As only 0.14% α-particles suffered large deviation it means the volume occupied by this +ve charge is very small.
3. As very few α-particles were scattered by more than 90° because fast moving α-particles can only be deviated by large forces & heavy mass which is conc. in the dense central core called nucleus.
4. Rutherford suggested the size of the nucleus to be about 10^{-15} m – 10^{-14} m.

Impact parameter (b)

It is the ⊥r distance of the initial velocity vector of the α-particle from the centre of nucleus

$$b = \frac{1}{4\pi\epsilon_0} \cdot \frac{Ze^2 \cot\theta/2}{E}$$



- for α-particle close to nucleus
  b - small, large scattering
- for head on collision
  b - min, α-particle rebounds back ($\theta = \pi$)
- for α-particle away from nucleus
  b - large, approx. no deviation ($\theta \approx 0$)

→ As only a small fraction of no of incident particles rebound it means that no of α-particles undergoing head on collision is small

→ It means that the mass of atom is conc in a small volume.

→ Thus the scattering experiment is a powerful way to find the upper limit of size of nucleus

Distance of closest approach (d)

Consider an α-particle with initial K.E. (E) is directed towards the centre of nucleus of atom. Due to Coulomb's repulsion betⁿ nucleus & α-particle K goes on decreasing & electric potential energy (U) goes on increasing.

At a certain distance (d) from the nucleus, K reduces to zero & the particle stops. It is repelled by the nucleus & it retraces its path by 180°. This distance 'd' is called distance of closest approach.

At d, K is converted to U.

Now, charge on α-particle, $q_1 = +2e$

" " nucleus, $q_2 = +Ze$

So, $U = \frac{Ze \times 2e}{4\pi\epsilon_0 d}$

At d $K = U$

so, $= \frac{2Ze^2}{4\pi\epsilon_0 d}$

$$\therefore \boxed{d = \frac{2Ze^2}{4\pi\epsilon_0 K}}$$

* Read eg 12.1 & 12.2 from NCERT

Electron Orbits

If F_c - centripetal force required to keep a revolving electron in orbit

F_e - electrostatic force of attraction betⁿ the revolving electron & nucleus

For a dynamically stable orbit in hydrogen atom

$$F_c = F_e$$

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$\boxed{r = \frac{e^2}{4\pi\varepsilon_0 m v^2}}$$ — relation betn orbit radius & electron velocity

Now, $K = \frac{1}{2}mv^2$

$$K = \frac{e^2}{8\pi\varepsilon_0 r}$$

& $U = \frac{-e^2}{4\pi\varepsilon_0 r}$

∴ $E = K + U$

$$= \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\boxed{E = \frac{-e^2}{8\pi\varepsilon_0 r}}$$

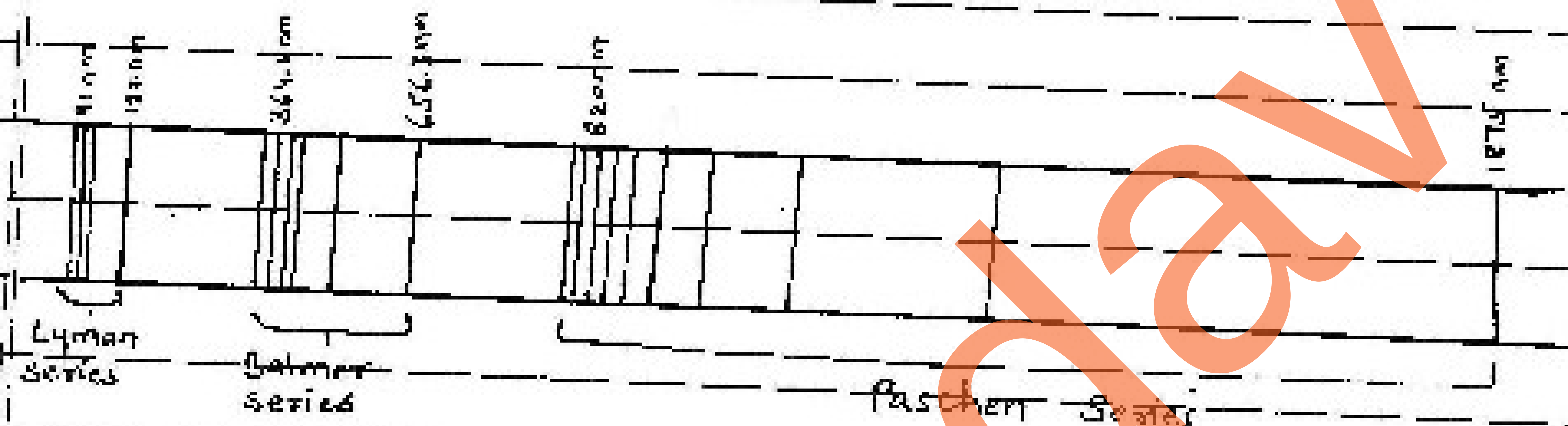
So the total energy of electron is negative which implies that the electron is bound to nucleus.

Atomic Spectra

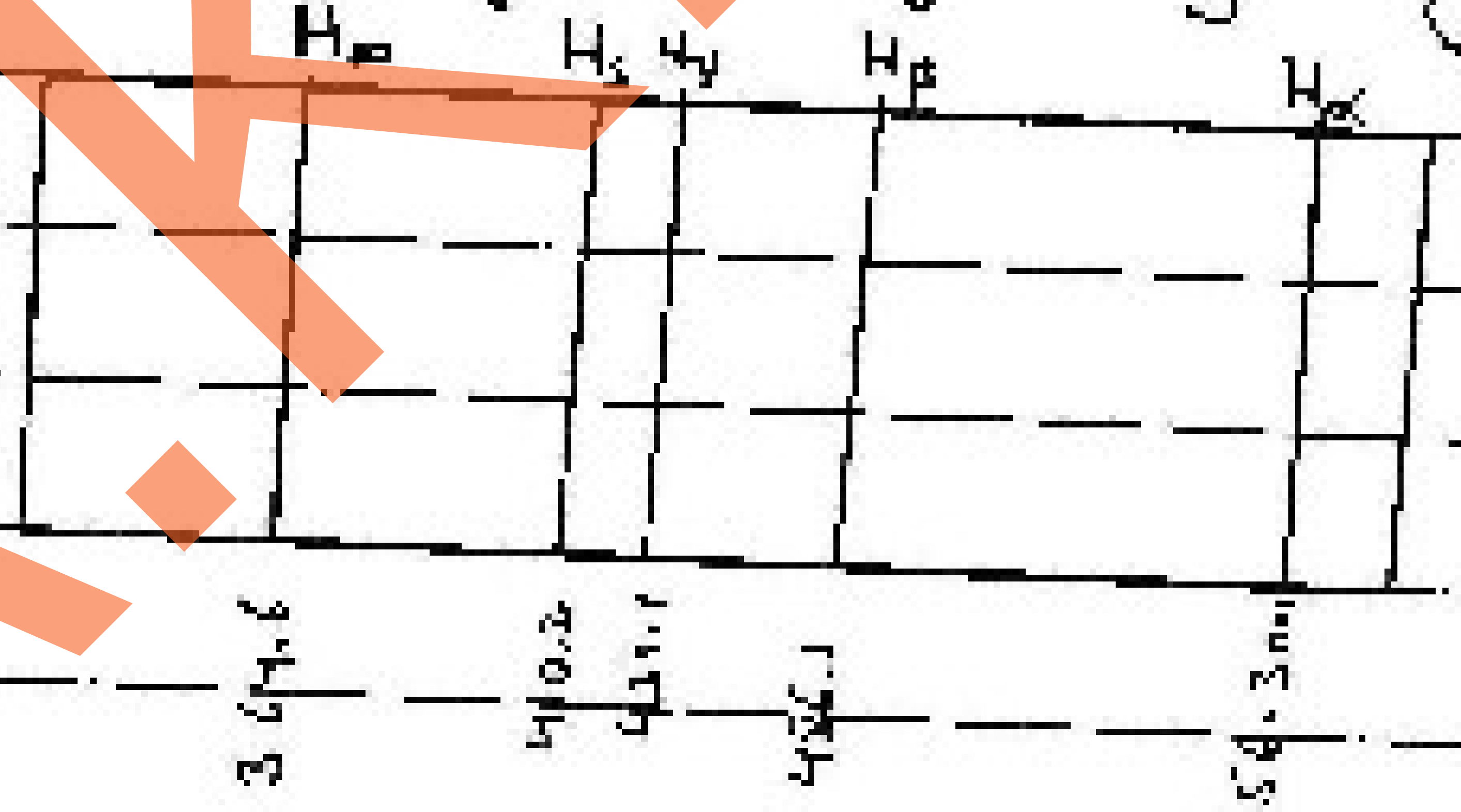
1) When an atomic gas at low pressure is excited by passing an electric current through it the gas emits radiations of certain specific wavelengths only.

A spectrum of this kind is called line emission spectrum & it consists of few bright lines on

2) When light (white) is passed through the same gas, we observe a bright background crossed by few dark lines signifying missing wavelength absorb by the gas. This kind of spectrum is called Line Absorption Spectrum



- From the fig. it is clear than spectral lines are in groups called spectral series.
- The spacing bet^n lines within certain sets of hydrogen spectrum decreases in a regular way.
- Balmer was the 1st to observe a spectral series in visible region of hydrogen spectrum.



Here, H_α line → Spectral line with largest λ (= 656.3) in red region
H_β → " " " λ = 486.1 nm in blue green region
H_γ → " " " λ = 434.1 nm in voilet region

Balmer found an empirical formula to account for these wavelengths

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

for $n = 3$, $\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2}\right) \Rightarrow \lambda = 656.3\,nm$

$n = 4$, $\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{4^2}\right) \Rightarrow \lambda = 486.1\,nm$

$n = \infty$, $\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) \Rightarrow \lambda = 364.6\,nm$.

$\lambda = 364.6\,nm$ is the limit of Balmer series. Beyond this limit, there is no further distinct lines, instead the spectrum becomes continuous (though faint).

* Lyman series – discovered in U-V region of hydrogen spectrum

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

* Paschen series – discovered in infra-red region of hydrogen spectrum

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$$

* Pfund series – discovered in far infra-red region of hydrogen spectrum

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$$

Limitations of Rutherford model

① In Rutherford's model an electron is revolving around the nucleus & is constantly experiencing a centripetal force. So, an electron has an accelerated motion.

Acc. to classical e-m theory, the electron must radiate energy in the form of e-m waves.
As the revolving electron loses energy continuously it spirals inwards & eventually fall into the nucleus.
But atom is stable & it doesn't collapse.

② Acc. to classical e-m theory

$$\left(\begin{array}{l}\text{frequency of e-m waves} \\ \text{emitted by revolving electron}\end{array}\right) = \left(\begin{array}{l}\text{frequency of revolution} \\ \text{of electron}\end{array}\right)$$

As revolving electrons spiral inwards their angular velocities & hence their angular frequencies of revolution would change continuously.

So, frequency of e-m waves emitted must change continuously & atoms should emit continuous spectrum.
But we observe only a line spectrum.

Bohr Model of Hydrogen Atom

① An electron in an atom can revolve in certain stable orbits without the emission of radiant energy.

② The electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $h/2\pi$
So, L of orbiting electron is quantised.

$$L = \frac{nh}{2\pi}$$

③ An electron might make a transition from one specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy diff. betⁿ initial & final states.

$$h\nu = E_i - E_f \qquad , E_i > E_f$$

Radii of Bohr's stationary orbits

For a dynamically stable orbits in hydrogen atom.

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\,\frac{e^2}{r^2}$$

$$r = \frac{e^2}{4\pi\varepsilon_0 m v^2} \quad \text{---} (1)$$

By Bohr's 2nd postulate

$$mvr = \frac{nh}{2\pi} \quad \text{---} (2)$$

$$v = \frac{nh}{2\pi(mr)} \quad \text{put in } (1)$$

$$\cancel{r} = \frac{e^2}{4\pi\varepsilon_0}\,\frac{1}{\cancel{m}\times n^2h^2} \times 4\pi^2 m^{\cancel{2}} r^{\cancel{2}}$$

$$\boxed{r = \frac{n^2h^2}{4\pi^2 m} \times \frac{4\pi\varepsilon_0}{e^2}}$$

The size of innermost orbit (n = 1) is

Bohr radius → $\boxed{a_0 = r_1 = \frac{h^2\varepsilon_0}{\pi m e^2}}$

On putting values of h, m, ε_0 , $a_0 = 5.29 \times 10^{-11}$ m.

Velocity of electron in Bohr's stationary orbit

from ①

$$mr = \frac{e^2}{4\pi\varepsilon_0} \times \frac{1}{v^2}$$

$$v^2 = \frac{e^2}{4\pi\varepsilon_0} \times \frac{1}{mr} \longleftarrow ③$$

Also $mr = \frac{nh}{2\pi v}$ put in ③

$$v^2 = \frac{e^2}{4\pi\varepsilon_0} \times \frac{2\pi v}{nh}$$

$$\boxed{v = \frac{1}{n} \times \frac{e^2}{4\pi\varepsilon_0} \times \frac{1}{(h/2\pi)}}$$

Total energy of electron in Bohr's stationary orbits

The total energy of electron in hydrogen atom is given by

$$E = \frac{-e^2}{8\pi\varepsilon_0 r}$$

$$= \frac{-e^2}{8\pi\varepsilon_0} \times \frac{4\pi^2 m}{n^2h^2} \times \frac{e^2}{4\pi\varepsilon_0}$$

$$E = \frac{-me^4}{8n^2\varepsilon_0^2h^2}$$

$$= \frac{-2.18 \times 10^{-18}}{n^2} \text{ J}$$

[on putting values of m, h, e, ε_0]

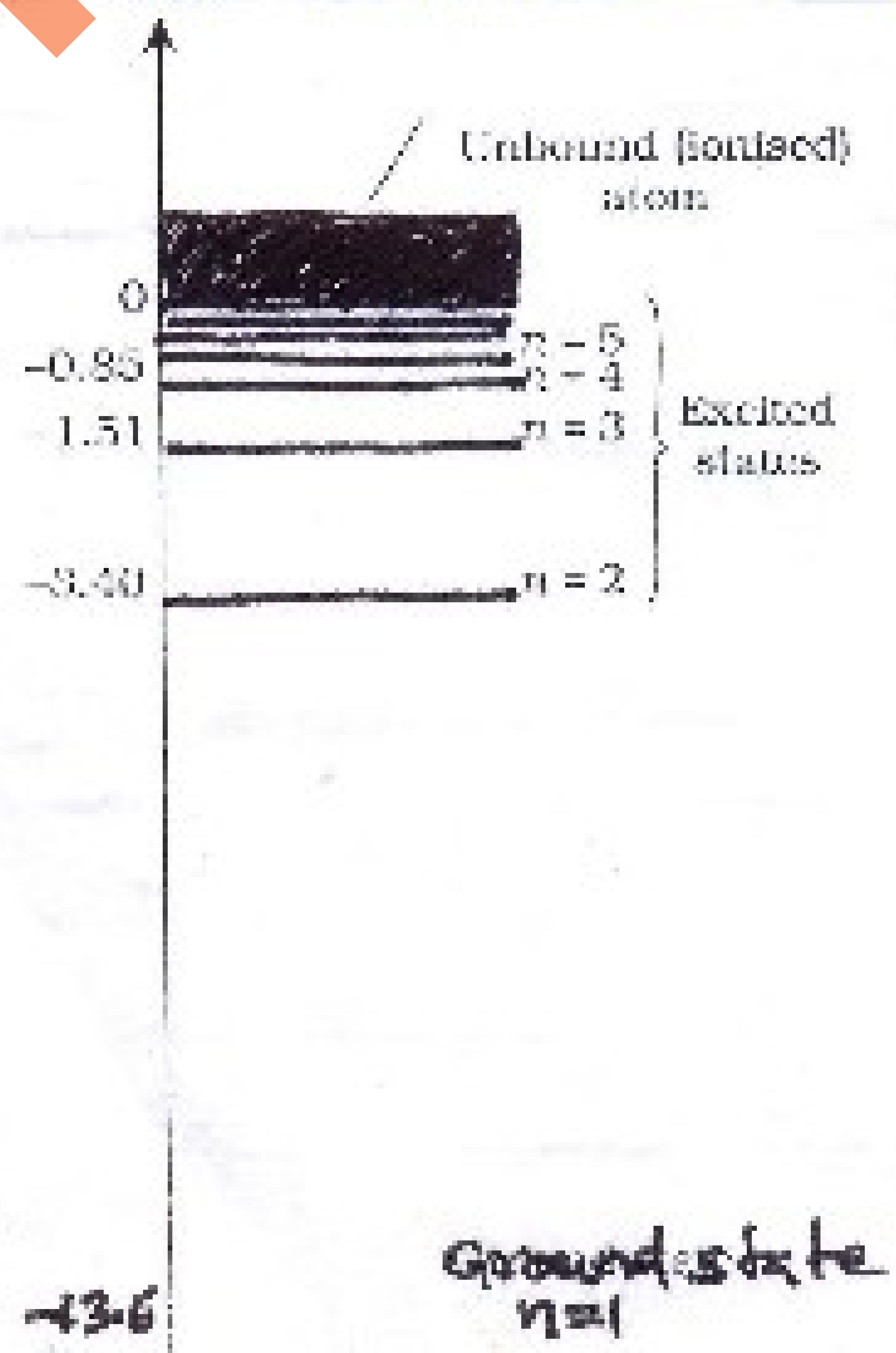
$$\boxed{E = \frac{-13.6}{n^2} \text{ eV}}$$

The -ve sign shows that electron is bound with the nucleus & energy would be required to remove the electron from the hydrogen nucleus.

Energy levels

- The energy of an atom is least when electron is revolving in an orbit closest to the nucleus $(n=1)$.
- So, the lowest state (called ground state) is that having lowest energy & revolving close to the nucleus.
- The energy of this state is $E_1 = -13.6\,eV$
- So, the minimum energy (called ionisation energy) required to remove the electron from ground state is 13.6 eV.
- At room temp, most hydrogen atoms are in ground state.
- When a hydrogen atom receives energy by electron collision, the atom acquire sufficient energy to raise the electron to higher energy ~~excited~~ state.

The atom is then said to be in excited state.



for $n=2$, $E_2 = -3.40\,eV$

∴ Energy required to excite an electron in hydrogen atom to its 1st excited state $= E_2 - E_1$

$= -3.40 - (-13.6)$

$= 10.2\,eV$

III^{ly} for 2nd excited state, $E = E_3 - E_1$

$= -1.51 - (-13.6)$

$= 12.09\,eV$

✡ for higher excited state $n = \infty$, $E_\infty = 0$

Line Spectra of Hydrogen Atom

Acc to Bohr's 3rd postulate

$$h\nu = E_i - E_f$$

$$= \frac{-me^4}{8n_i^2\varepsilon_0^2h^2} + \frac{me^4}{8n_f^2\varepsilon_0^2h^2}$$

$$\frac{hc}{\lambda} = \frac{me^4}{8\varepsilon_0^2h^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{1}{\lambda} = \frac{me^4}{8\varepsilon_0^2h^3c}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)}$$

$$R = \frac{me^4}{8\varepsilon_0^2h^3c} = 1.03 \times 10^7\ m^{-1}$$

↘ Rydberg's constant

[Very close to the theoretical value obtained by Balmer's empirical formula which shows agreement b/w theory & experimental values]

Bohr explained of the spectral lines as follows:

① Lyman Series – It is obtained when an electron jumps to 1st orbit ($n_i = 1$) from any other orbit ($n_f = 2, 3, 4 \ldots$)

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_f^2}\right]$$

Smallest wavelength, λ_s = 912 A° when $n_f = \infty$
Largest " , λ_ℓ = 1216 A° " = 2

i.e. all spectral lines lie in U-V region of spectrum

② Balmer Series – jumps to 2nd orbit ($n_i = 2$)

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_f^2}\right]$$

λ_s = 3646 A° when $n_f = \infty$
λ_ℓ = 6563 A° = 3

i.e. all spectral lines lie in visible region.

③ Paschen Series – 3rd orbit

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_f^2}\right]$$

λ_s = 822 A° when $n_f = \infty$
λ_ℓ = 18750 A° = 4

④ <u>Brackett Series</u> – to 4th orbit

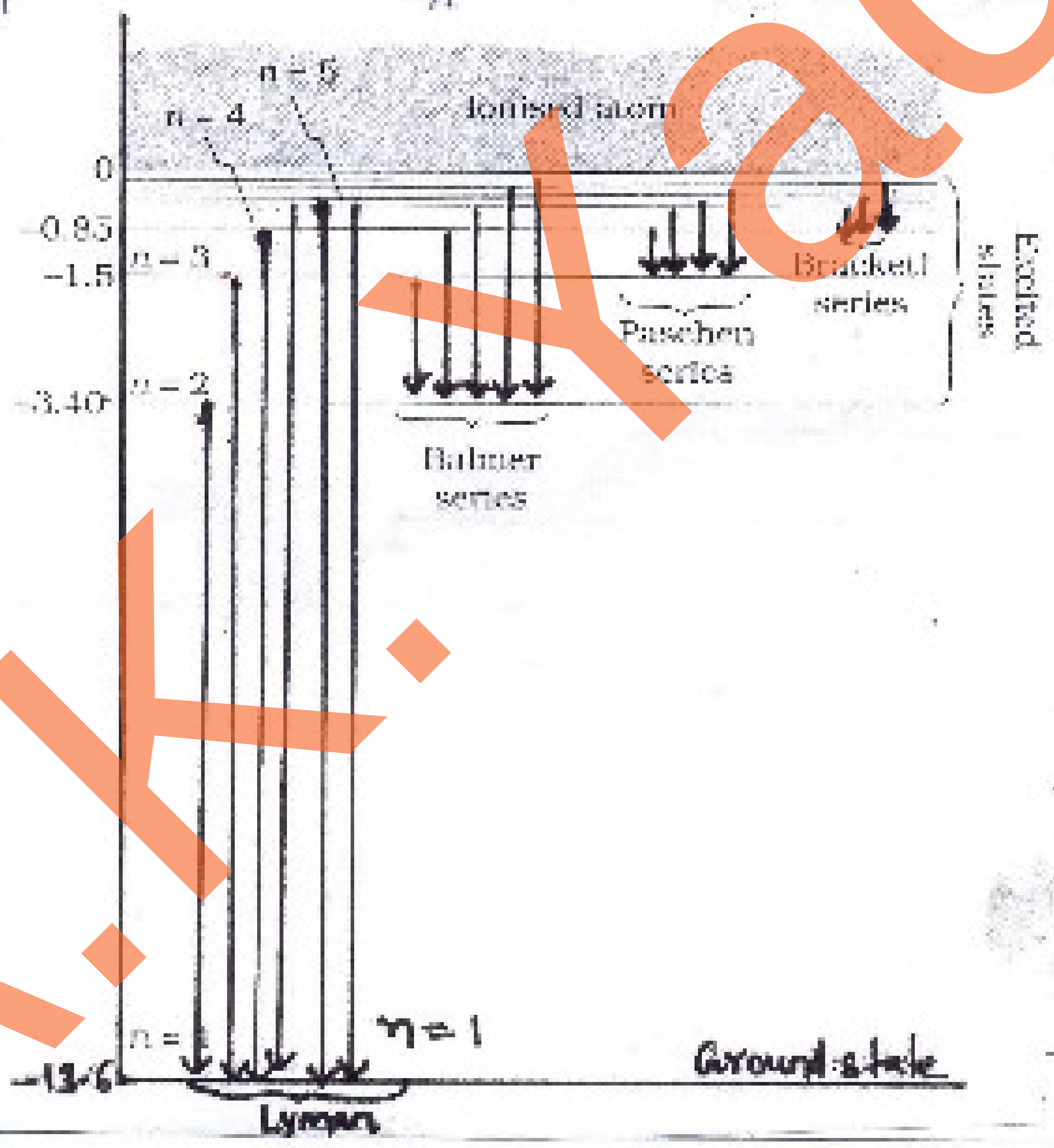
$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_f^2}\right)$$

infra-red region

⑤ <u>Pfund Series</u> – to 5th orbit

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n_f^2}\right)$$

infra-red region



- The various lines in the atomic spectra are produced when electrons jump from higher energy state to lower energy state & emit photons. These spectral lines are called <u>emission lines</u>.
- But when an atom absorbs a photon & moves to higher energy state, the process is called absorption.

So, if photons (with a continuous range of frequencies) pass through a gas & then analysed with a spectrometer, a series of dark spectral lines appear in continuous spectrum. The dark lines indicate the frequencies absorbed by the atoms of the gas.

Limitation of Bohr's Theory

① It is applicable only to hydrogenic (single electron) atoms.

Reason: Each electron interacts not only with +vely charged nucleus but also with all other electrons. The formulation of Bohr model: electrical force bet^n +vely charged nucleus & electron. It does not include the electrical forces bet^n electrons (which appear in multi-electron atoms).

② It can't explain relative intensities of the frequencies emitted.

③ It does not explain why orbits of electrons are circular & not elliptical.

Atoms & Nuclei

Nuclei

Atomic mass

- atomic mass unit (u) is used to measure atomic masses

- It is defined as 1/12 of the mass of ^{12}C atom

$$1u = \frac{1}{12} \times \text{mass of } 1\ ^{12}C \text{ atom}$$

$$= \frac{1}{12} \times 1.99 \times 10^{-26}\ kg$$

$$\boxed{1u = 1.66 \times 10^{-27}\ kg}$$

- Spectrometer is used to accurately measure atomic masses.

Isotopes

Atomic species of same element having same atomic number but different mass no. are called isotopes

eg Isotopes of Hydrogen → H_1^1, H_1^2, H_1^3

Chlorine → Cl_{17}^{35}, Cl_{17}^{37}

- All the known elements have one or more isotopes.

* Tritium (H_1^3) being unstable do not occur naturally & is produced artificially in lab

* C_{17}^{35} - 75.4 %, Cl_{17}^{37} - 24.6 % (abundance in nature)

$$\text{Average mass of } Cl = \frac{75.4 \times 35 + 24.6 \times 37}{100}$$

$$= 35.47u$$

Discovery of Neutron

- Deutrium & Tritium contains one proton each but their atomic masses are 2 & 3 resp. that means that in addition to protons, there exists some neutral matter inside the nucleus.
- This was verified by Chadwick

Chadwick's experiment

- When Be nuclei were bombarded with α-particles neutral radiations are emitted which can knock out protons from light nuclei like He, C & N.
- The only neutral radiation known at that time was photons.
- Applications of principle of conservation of energy & momentum showed that if the neutral radiation consisted of photons, the energy of photons would have to be much higher than the energy available from bombardment.
- So, solve this he assumed that the neutral radiations consists of a new type of neutral particles called neutrons having mass approx. equal to mass of proton.

* Z - atomic no.
A - mass no.

Isobars - same mass no. (A) different atomic no. (Z)

eg $^{3}_{1}H$ & $^{3}_{2}He$

Isotones - nuclides having same no. of neutrons

eg $^{198}_{80}Hg$ & $^{197}_{79}Au$

Nuclear Size

Experimentally volume of nucleus $\propto$ A

$$\frac{4}{3}\pi R^3 \propto A$$

$$R \propto A^{1/3}$$

$$\boxed{R = R_0 A^{1/3}}$$

$R_0 = 1.2 \times 10^{-15}$ m
empirical constant

* As A is different for different elements so atomic nuclei of different elements have different sizes.

Nuclear Density

$$N.D. = \frac{\text{mass of nucleus}}{\text{volume of nucleus}}$$

$$\rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A}$$

$$\boxed{\rho = \frac{3m}{4\pi R_0^3}}$$

As m & R_0 are const. so ρ is same for all element

* Putting the values of m & R_0, $\rho = 2.29 \times 10^{17}$ kg/m^3 which is very large as compared to density of ordinary matter. Hence matter in the nucleus is very densely packed.

Mass - Energy Equivalence

Einstein showed that mass is another form of energy & one can convert mass-energy into other forms of energy.

$$\boxed{E = mc^2}$$

Q. How much energy would be released when 1 a.m.u is completely converted into energy?

Ans

$$1 \text{ a.m.u} = 1.67 \times 10^{-27} \text{ kg}$$

$$E = mc^2$$

$$= 1.67 \times 10^{-27} \times (3 \times 10^8)^2$$

$$= 1.49 \times 10^{-10} \text{ J}$$

$$= \frac{1.49 \times 10^{-10}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 931.25 \times 10^6 \text{ eV}$$

$$\boxed{E = 931.25 \text{ MeV}}$$

Proton - electron hypothesis (1930)

→ It was put forward to account for the emission of α & β-particles from nuclei of radioactive elements.

→ Acc. to this

- nucleus of X_Z^A is made up of

  A - protons

  A-Z - electrons

  As every atom is electrically neutral so it must contain Z more electrons.
- electrons revolve around the nucleus in circular orbits

Rejection (causes)

① Acc. to de-Broglie hypothesis & heisenberg's uncertainty principle if an electron is to exist inside nucleus it should possess energy from 20 - 200 MeV

But energy emitted by electron in β-decay is atmost 2-3 MeV so existence not justified

② The presence of few electrons inside the nucleus & others revolving in orbits show dual role of electrons in atomic structure, which is difficult to visualize.

③ Experimentally magnetic moment of nuclei << magnetic moment of electrons

So electrons can't exist inside nucleus

Nuclear Forces

The strong forces of attraction which hold together the nucleons in the tiny nucleus of an atom, inspite of strong electrostatic forces of repulsion betn protons.

Characteristics

① Independent of charge

② strongest force in nature

③ very short range force (operative upto few fermi distances)

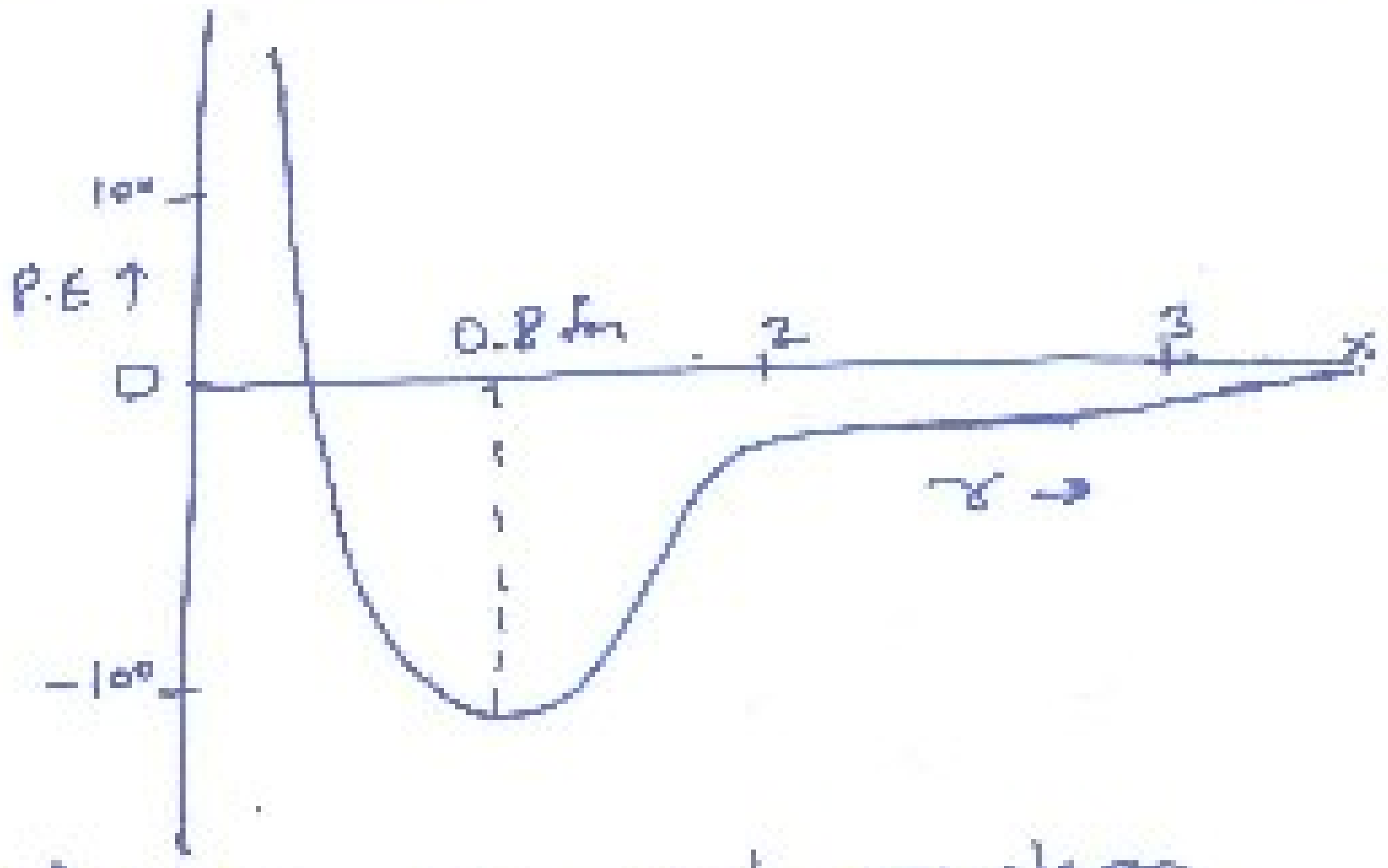
④ Variation of N.Force with distance

(i) N.F. negligible when $r > 10$ fermi

(ii) N.F. increases on decreasing distance, however they do not obey inverse square law.

(iii) $r < 0.8$ N.F strongly repulsive.

⑤ Variation of Potential energy betn a pair of nucleons with r



(i) P.E - minimum at $r = 0.8$ fm,
At this distance N.F = 0

(ii) $r > 0.8$ fm P.E.- decreases
N.F attractive

(iii) $r < 0.8$ fm P.E - decreases to zero & then positive

N.F. repulsive

⑥ Nuclear forces show saturation properties i.e. each nucleon interacts with its immediate neighbours only.

⑦ Nuclear forces are non-central. This shows that the distribution of nucleons in a nucleus is not spherically symmetric.

Nuclei

Mass defect (Δm)

The difference betⁿ the sum of the masses of neutrons & protons forming a nucleus & mass of the nucleus is called mass defect.

In a nucleus $_ZX^A$

∴ Z - atomic no. = no of protons (p)
A - mass no. = p + n

Let m_p - mass of proton
m_n - " " neutron
m_N - " " nucleus

$$\therefore \boxed{\Delta m = Zm_p + (A-Z)m_n - m_N}$$

Binding Energy

Binding energy of a nucleus is the energy with which nucleons are bound in the nucleus.

It is measured by the work required to separate the nucleons an infinite distance apart from the nucleus, so that they may not interact with each other.

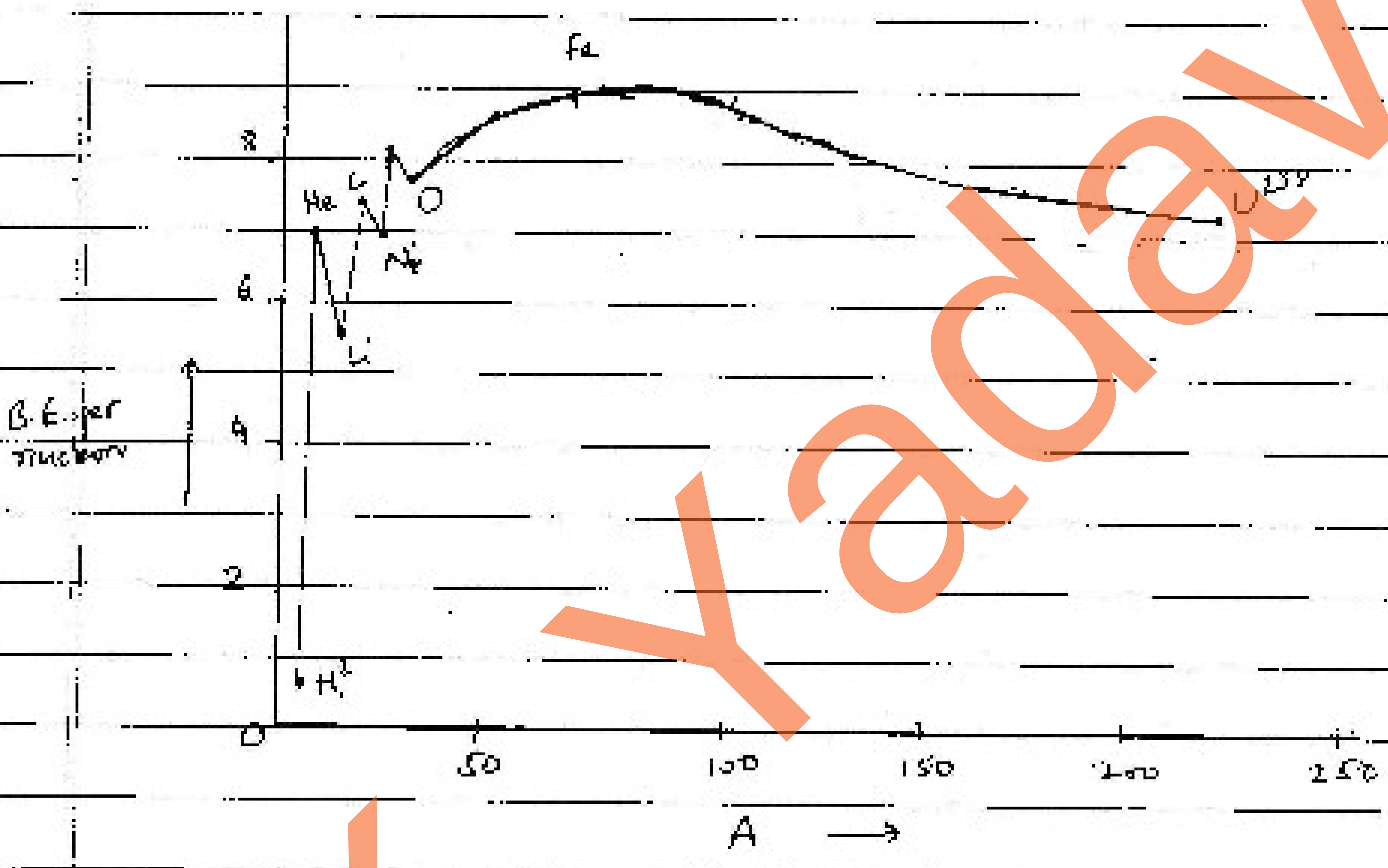
$$B.E. = \Delta m . c^2$$
$$= [Zm_p + (A-Z)m_n - m_N] . c^2$$

* It is this mass defect which appears in the form of B.E., responsible for binding the nucleons together in the nucleus.

B.E. per nucleon

The average energy per nucleon needed to separ a nucleus into its individual nucleons.

$$E_{bn} = \frac{E_b}{A}$$



Observation

1) E_{bn} is very small for very light nuclei like H_1^2, Li
2) Peak E_{bn} is 8.8 MeV for fe
3) Nuclides having mass no in the range of 30 to 120 show good stability because of the high E_{bn} corresponding to an average of 8.5 MeV per nucleon.
4) As the atomic number increases beyond 56, E_{bn} decreases & reaches 7.6 MeV for U^{238}.
This is so because in such large nuclei the attractive forces betⁿ the distant nucleons are smaller.

Conclusion

(a) A very heavy nucleus (A = 240) has lower E_{bn} compared to that of A = 120.
So, if A = 240 breaks into 2 A = 120 nuclei, nucleons get more tightly bound.
This implies energy would be released in the process. This process is called Nuclear Fission.

(b) Consider 2 very light nuclei (A ≤ 10) joining to form a heavier nucleus.
The E_{bn} of fused heavier nuclei is more than the E_{bn} of lighter nuclei
i.e. E_{bn} (fused heavier nuclei) > E_{bn} (lighter nuclei)

This implies that there will be gain in E_{bn} & hence release of energy.
This process is called Nuclear fusion.

Radioactivity

It is the property by virtue of which a heavy element disintegrates itself without being forced by any external agency.

Laws of radioactive decay

① It is a spontaneous process which does not depend upon external factors like tem etc.

② During disintegration of an atom, either an α-particle or a β-particle is emitted
[Neither both nor more than one particle]

③ The no. of atoms disintegrated per sec at any instant is directly proportional to the no. of radioactive atoms actually present in the sample at that instant

Let N_0 = no. of nuclei in sample at $t = 0$
N = " " " " " $t = t$
dN = " " " decaying in dt

So, acc. to law

$$\frac{dN}{dt} \propto -N$$

$$\frac{dN}{dt} = -\lambda N$$

λ - decay constant

$$\frac{dN}{N} = -\lambda dt$$

Integrating both sides

$$\int \frac{dN}{N} = -\lambda \int dt$$

$$\ln N = -\lambda t + c \quad - ①$$

c - constant of proportionality

At $t = 0$, $N = N_0$
∴ $c = \ln N_0$

So, eqn ① becomes

$$\ln N = -\lambda t + \ln N_0$$

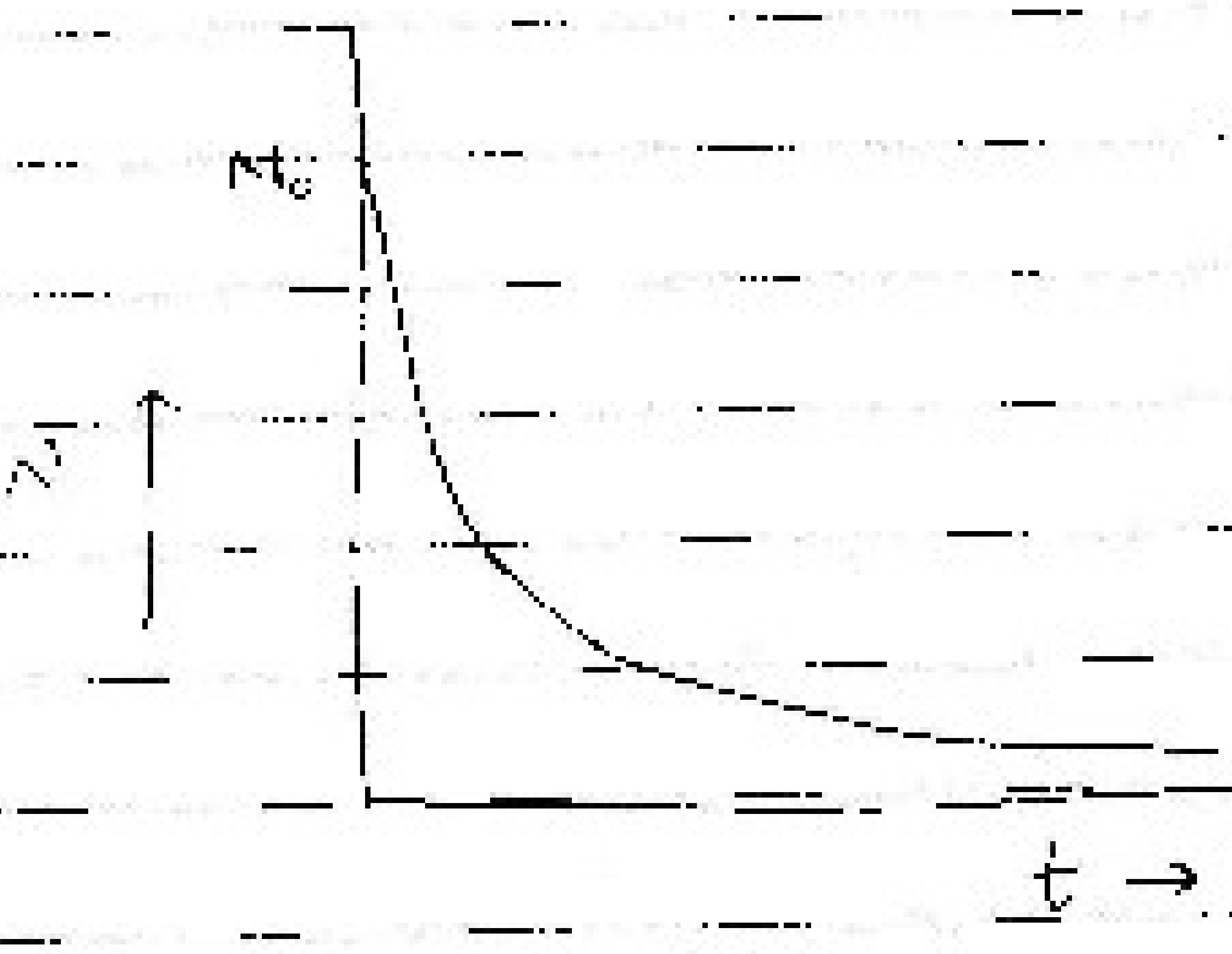
$$\ln N - \ln N_0 = -\lambda t$$

$$\ln \frac{N}{N_0} = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\boxed{N = N_0 e^{-\lambda t}}$$

* The same derivation can be done by replacing 'ln' with $\log_e$



Half-life ($t_{1/2}$ or T)
It is the time during which half the no. of radio-nuclei have undergone decay

$$N = N_0 e^{-\lambda t}$$

when $t = t_{1/2} = T$, $N = \frac{N_0}{2}$

$$\therefore \frac{N_0}{2} = N_0 e^{-\lambda T}$$

$$\frac{1}{2} = e^{-\lambda T}$$

$$2 = e^{\lambda T}$$

$$\log_e 2 = \lambda T$$

$$2.303 \log_{10} 2 = \lambda T$$

or $$\boxed{T = \frac{0.693}{\lambda}}$$

Note
At $t = [illegible]$, $N = \frac{N_0}{4}$

After 2 half lives

$$N = \frac{1}{2} \cdot \frac{N_0}{2} = \frac{N_0}{4} = N_0\left(\frac{1}{2}\right)^2$$

After 3 half lives

$$N = \frac{1}{2}\left(\frac{N_0}{4}\right) = \frac{N_0}{8} = N_0\left(\frac{1}{2}\right)^3$$

After 'n' half lives

$$\boxed{N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^{t/T}}$$

Q The half-life of Radon is 3.8 days. Calculate how much of 15 mg of Radon will remain after 38 days.

Ans $T = 3.8$ days, $t = 38$ days

$N_0 = 15$ mg, $N = ?$

$$n = \frac{t}{T} = \frac{38}{3.8} = 10$$

$$\therefore N = N_0\left(\frac{1}{2}\right)^n$$

$$= 15\left(\frac{1}{2}\right)^{10}$$

$$= 0.014 \text{ mg}$$

Activity (A)

It is defined as the no. of disintegrations per second occuring in that sample

$$A = \frac{dN}{dt}$$

* As $N = N_0 e^{-\lambda t}$, similarly, $A = A_0 e^{-\lambda t}$

** $\frac{A}{A_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

Units of Activity

① Becquerel (Bq) → one decay per sec. — SI unit

② Rutherford → 10^6 decay per sec.

③ Curie → 3.7×10^{10} decay per sec.

Q Find the half life period of a radioactive material if its activity drops to $\frac{1}{16}$ of its initial value in 30 yrs.

Ans

$$\frac{A}{A_0} = \frac{1}{16}, \quad t = 30 \text{ yrs}, \quad T = ?$$

Now, $$\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$$

$$\frac{1}{16} = \frac{1}{2^n}$$

$$2^n = 2^4$$

$$n = 4$$

Now, $$n = \frac{t}{T}$$

$$T = \frac{t}{n} = \frac{30}{4} = 7.5 \text{ yrs.}$$

Average or Mean Life (τ)

It is the arithmetic mean of the lives of all the nuclei present initially.

Consider a radioactive sample containing N_0 atom initially.

Let N - no. of active nuclei present in sample after time t.

Suppose dN atoms further disintegrate in time dt

So, average life of dN atoms lie bet^n t & t+dt.

If dt is very small, then the average life of dN atoms is t.

∴ Total life of dN atoms = t.dN

So, total life of all the atoms in the sample

$$= \int_0^{N_0} t.dN$$

$$\therefore \quad \tau = \frac{\int_0^{N_0} t.dN}{N_0} \longrightarrow ①$$

Now, $N = N_0 e^{-\lambda t}$

$dN = -\lambda N_0 e^{-\lambda t} dt$ put in ②

The limits of N are replaced by ∞ & 0

because N = 0 at t = ∞

= N_0 at t = 0

$$\therefore \quad \tau = \frac{-1}{N_0}\int_\infty^0 t \lambda N_0 e^{-\lambda t} dt$$

$$= \lambda \int_0^\infty t e^{-\lambda t} dt$$

$$\tau = \lambda \left[\left[\frac{t e^{-\lambda t}}{-\lambda} \right]_0^\infty - \int_0^\infty \frac{e^{-\lambda t}}{-\lambda} dt \right]$$

$$= \lambda \left[0 + \frac{1}{\lambda} \int_0^\infty e^{-\lambda t} dt \right]$$

$$= \int_0^\infty e^{-\lambda t} dt$$

$$= \left[\frac{e^{-\lambda t}}{-\lambda} \right]_0^\infty$$

$$\boxed{\tau = \frac{1}{\lambda}}$$

As $T = \frac{0.693}{\lambda}$

So $\boxed{\tau = \frac{T}{0.693}}$

Alpha Decay

The phenomenon of emission of an α-particle from a radioactive nucleus

$$_{Z}X^{A} \longrightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4} + Q$$

where Q - disintegration energy

Eg $_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}$

Disintegration energy (Q)

The difference bet^n initial mass energy & total mass energy of decay products is called disintegration energy.

In α-decay

$$Q = (m_X - m_Y - m_{He})c^2$$

This energy is shared by daughter nuclei & α-particle.

Beta Decay

It is the phenomenon of emission of an electron from a radioactive nucleus.

$${}_{Z}X^{A} \longrightarrow {}_{Z+1}Y^{A} + {}_{-1}e^{0} + Q$$

Types of β-decay

① β⁻ decay – electron is emitted
antineutrino ($\bar{\nu}$) is emitted

$${}^{32}_{15}P \longrightarrow {}^{32}_{16}S + e^{-} + \bar{\nu}$$

Basic nuclear process → conversion of neutron to proton

$$n \rightarrow p + e^{-} + \bar{\nu}$$

② β⁺ decay → positron (e^+) & neutrino (ν) emitted

$${}^{22}_{11}Na \longrightarrow {}^{22}_{10}Ne + e^{+} + \nu$$

Basic nuclear process → conversion of proton to neutron

$$p \rightarrow n + e^{+} + \nu$$

* Read neutrino hypothesis from Pradeep.

Imp Q A free neutron decays to proton, the decay of proton to neutron is possible only inside the nucleus. Why?

Ans Because proton has smaller mass than neutron

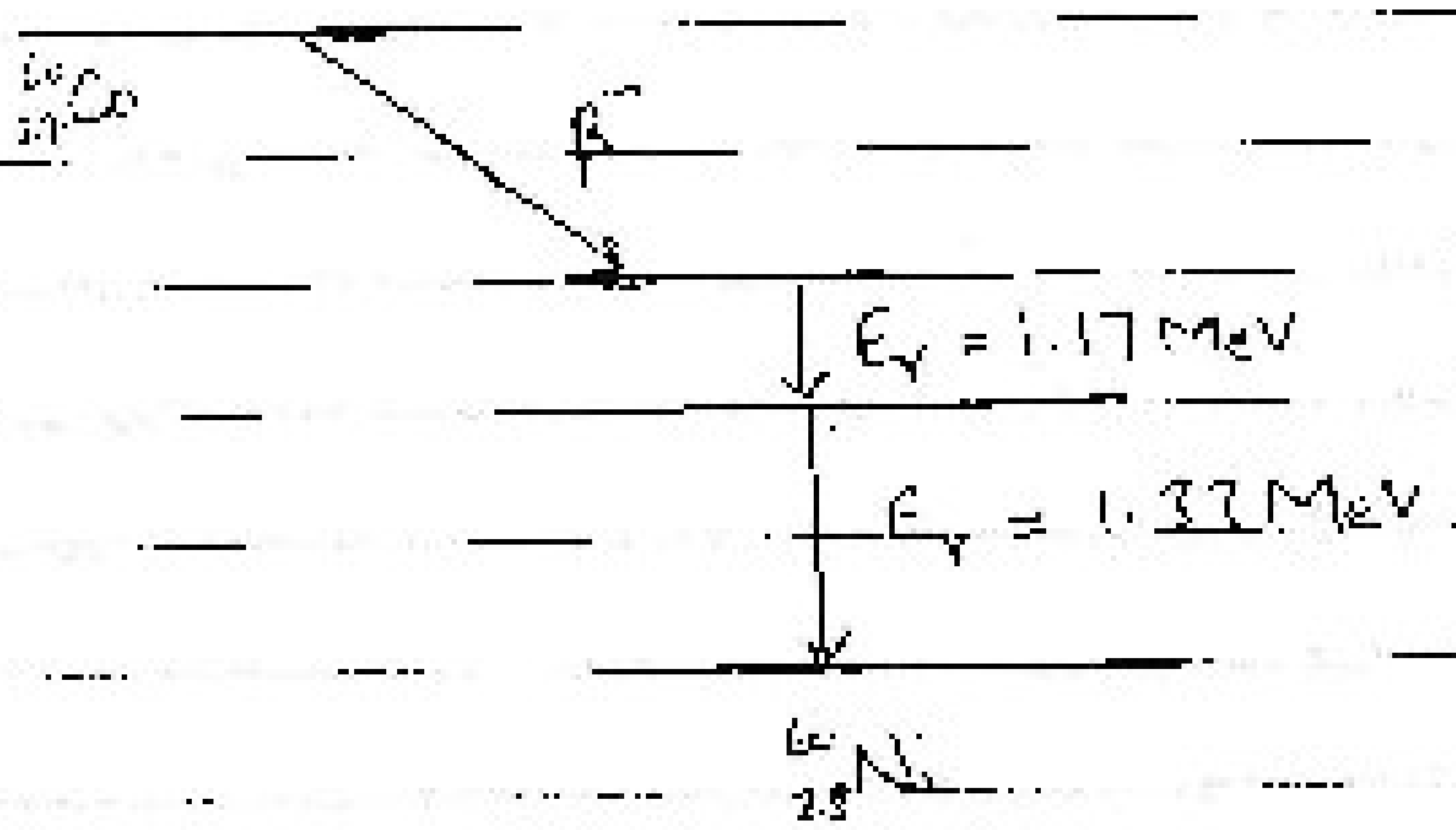
* Neutrino

- neutral particle with negligible mass
- weak interaction with other particles
- very difficult to detect as they can penetrate large quantity of matter without any interaction.

Gamma decay

- The nucleus has discrete energy levels of the order of MeV
- When a nucleus in excited state spontaneously decays to a lower energy state, a photon is emitted with energy equal to difference in the 2 energy levels of nucleus
This is called gamma decay.
- The energy corresponds to radiation of very short wavelength (shorter than X-ray)
- Gamma decay almost always follows α & β- decay because the processes always leave the nucleus in an excited state.

eg.



Nuclear Fusion

It is the phenomenon of fusing 2 or more lighter nuclei to form a single heavy stable nu.

eg ① $ {}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu + 0.42 \text{ MeV} $

② $ {}^2_1H + {}^2_1H \rightarrow {}^3_2He + n + 3.27 \text{ MeV} $

③ $ {}^2_1H + {}^2_1H \rightarrow {}^3_1H + {}^1_1H + 4.03 \text{ MeV} $

- In all these reactions, 2 +vely charged partic! combine to form a heavier nucleus.
- Coulomb repulsion betn these charges prohibits t to come close within the range of their attract nuclear forces & fuse. [Coulomb barrier]
- The height of Coulomb barrier depends on charg & their radii
- The essential condition for carrying out nucle fusion is to raise the tem. of the system so that particles have enough K.E to penetrate the Coulomb barrier. This is called thermonuclear fus.
- Thermonuclear fusion is the source of energy output in interior of stars

Energy in the sun (p-p cycle)

$ {}^1_1H + {}^1_1H \rightarrow {}^2_1H + e^+ + \nu + 0.42 \text{ MeV} $ --- ①

$ e^+ + e^- \rightarrow \gamma + \gamma + 1.02 \text{ MeV} $ --- ②

$ {}^2_1H + {}^1_1H \longrightarrow {}^3_2He + \gamma + 5.49 \text{ MeV} $ --- ③

$ {}^3_2He + {}^3_2He \longrightarrow {}^4_2He + {}^1_1H + {}^1_1H + 12.86 \text{ MeV} $ --- ④

eqⁿ ④ will occur only if eqⁿ ①, ② & ③ occur twice.

Now, 2① + 2② + 3③ + ④ gives

$$4\,{}^1_1H + 2e \longrightarrow {}^4_2He + 2\nu + 6\gamma + 26.7\,MeV$$

or $$[4\,{}^1_1H + 4e^-] \longrightarrow ({}^4_2He + 2e^-) + 2\nu + 6\gamma + 26.7\,MeV$$

Thus, 4 hydrogen atoms combine to form an helium atom with release of 26.7 MeV energy.

When all the hydrogen is converted into helium, the core starts to cool. The star begins to collapse under its own gravity which increases the tem. of core. When tem increases to 10^8 K fusion of helium into carbon takes place.

Nuclear Fission

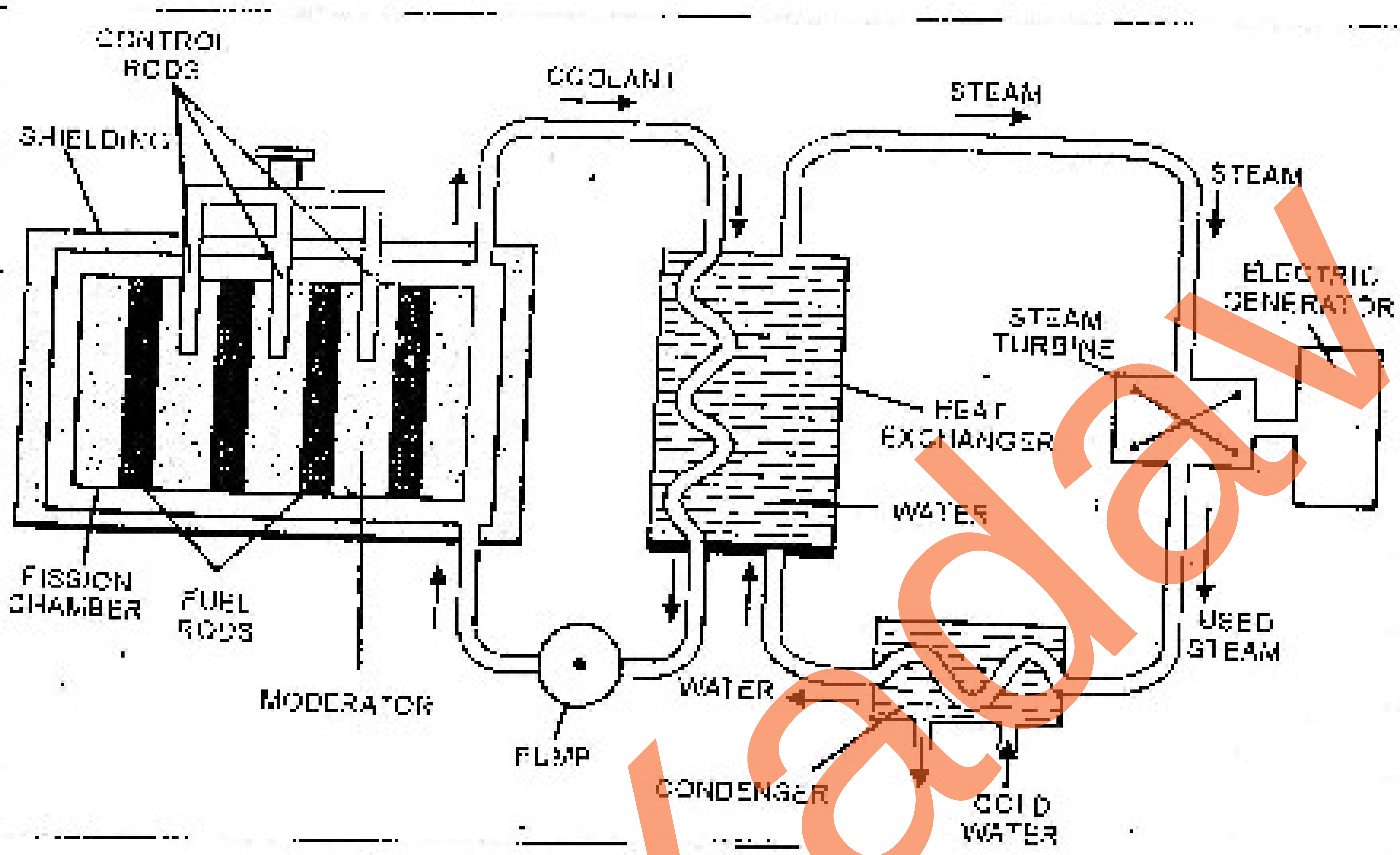
In this type of reaction a heavy unstable nucleus split into 2 light weight nuclei with the release of tremendous amount of energy.

eg ① $U^{235}_{92} + {}^1_0n \longrightarrow Ba^{144}_{56} + Kr^{89}_{36} + 3\,{}^1_0n$

- In place of Ba & Kr we can also have (Sb & Nb) or (Xe, Sr).

- The energy released is of the order of 200 MeV.

Nuclear Reactor



Construction

① Nuclear Fuel - fissionable material used for fission process.

commonly used fuel → U^{235}, Pu^{239}

② Moderator - To slow down the speed of fast moving neutrons.

eg: Water, heavy water (D_2O), graphite

③ Control rods - To absorb the extra neutrons produced in fission reaction so that the reaction remains controlled.

eg. Boron or Cadmium rods.

In addition to control rods, safety rods are also used to reduce K to less than 1.

④ Coolant – To remove the heat produced & transfer it from the core to surrounding.
eg – water, D_2O (at ordinary tem), liquid Na (at high tem.)

⑤ Shielding – The whole reactor is protected with concrete walls to ~~shield~~ prevent the escape of harmful radiation.

Working
- Initially controlling rods are kept out & a neutron is projected to collide with U^{235}
- When sufficient no. of neutrons are produced controlling rods are inserted to absorb extra neutrons.
- The fast moving neutrons are slowed down with the help of moderator which results in the generation of heat.
- This heat energy is absorbed by coolant & it eventually leads to moving of turbine.

Multiplication factor (K)
It is the measure of growth rate of neutrons in the reactor

$$K = \frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$$

If K = 1 – chain reaction sustained
size of fissionable material is critical
> 1 – chain reaction accelerates resulting in explosn
size of material super-critical
< 1 – chain reaction stops
size of material sub-critical

Breeder Reactor

The reactor to convert unfissionable material like U^{238} into fissionable material like Pu^{239}

$$U^{238}_{92} + {}^{1}_{0}n \longrightarrow U^{239}_{92} \longrightarrow Np^{239}_{93} \xrightarrow{-\beta} Pu^{239}_{92}$$

Thermal neutrons

The slow moving neutrons which can cause fission reason

Q. Why neutron needs to be slowed down?

Ans. Because otherwise it will escape from the reactor without colliding with Uranium.