Electromagnetic Induction

Magnetic Flux (ϕ)

The magnetic flux through any surface held in a magnetic field $\vec{B}$ is the total number of magnetic field lines crossing the surface

$$\boxed{\phi = \vec{B}\cdot\vec{A} = BA\cos\theta}$$

*When m.f. is touching surface tangentially $\theta = 90°$, $\phi = 0$
normal to surface $= 0°$, $\phi = BA$

Unit – Weber (S.I), maxwell (cgs)

1 Weber – It is the amount of magnetic flux over an area of $1m^2$ held normal to a uniform m.f. of 1 tesla

$$1 \text{ weber} = 1 \text{ tesla} \times 1m^2$$

* $1\,Wb = 10^8\,Mx$

Faraday's Laws of Electromagnetic Induction

First law

Whenever the amount of magnetic flux linked with a circuit changes, an e.m.f is induced in the circuit. The induced e.m.f lasts so long as the change in flux continues.

Second law

The magnitude of e.m.f induced in a circuit is directly proportional to the rate of change of magnetic flux linked with the circuit.

$$e \propto -\frac{d\phi}{dt}$$

$$\boxed{e = -\frac{d\phi}{dt}} \quad \text{or} \quad e = \frac{-(\phi_2 - \phi_1)}{t}$$

→ –ve sign indicates that induced e.m.f always opposes any change in magnetic flux associated with the circuit.

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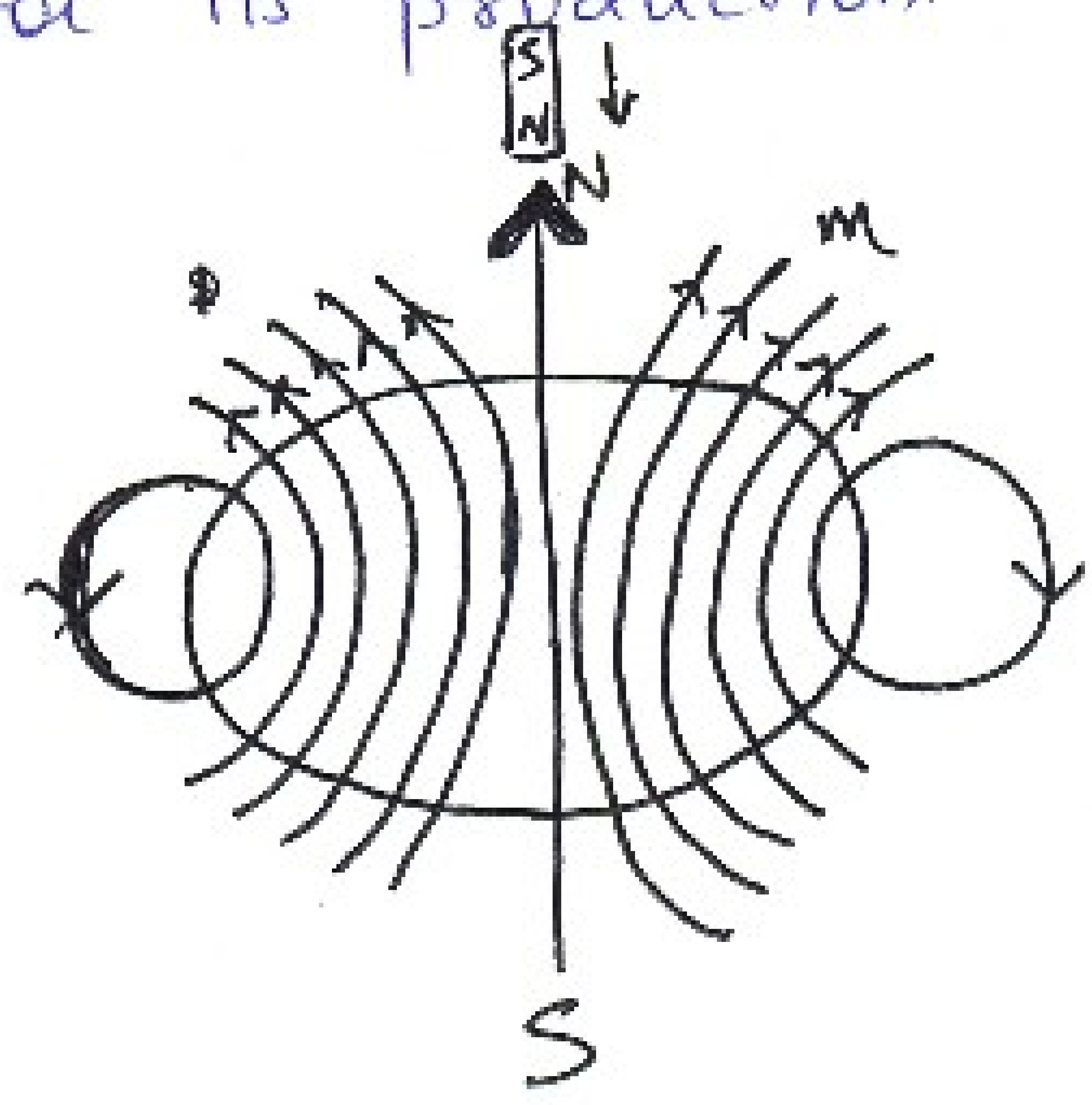
Len's Law

According to Lenz's law, the polarity of the induced e.m.f is such that it opposes the change in magnetic flux responsible for its production.

Example :



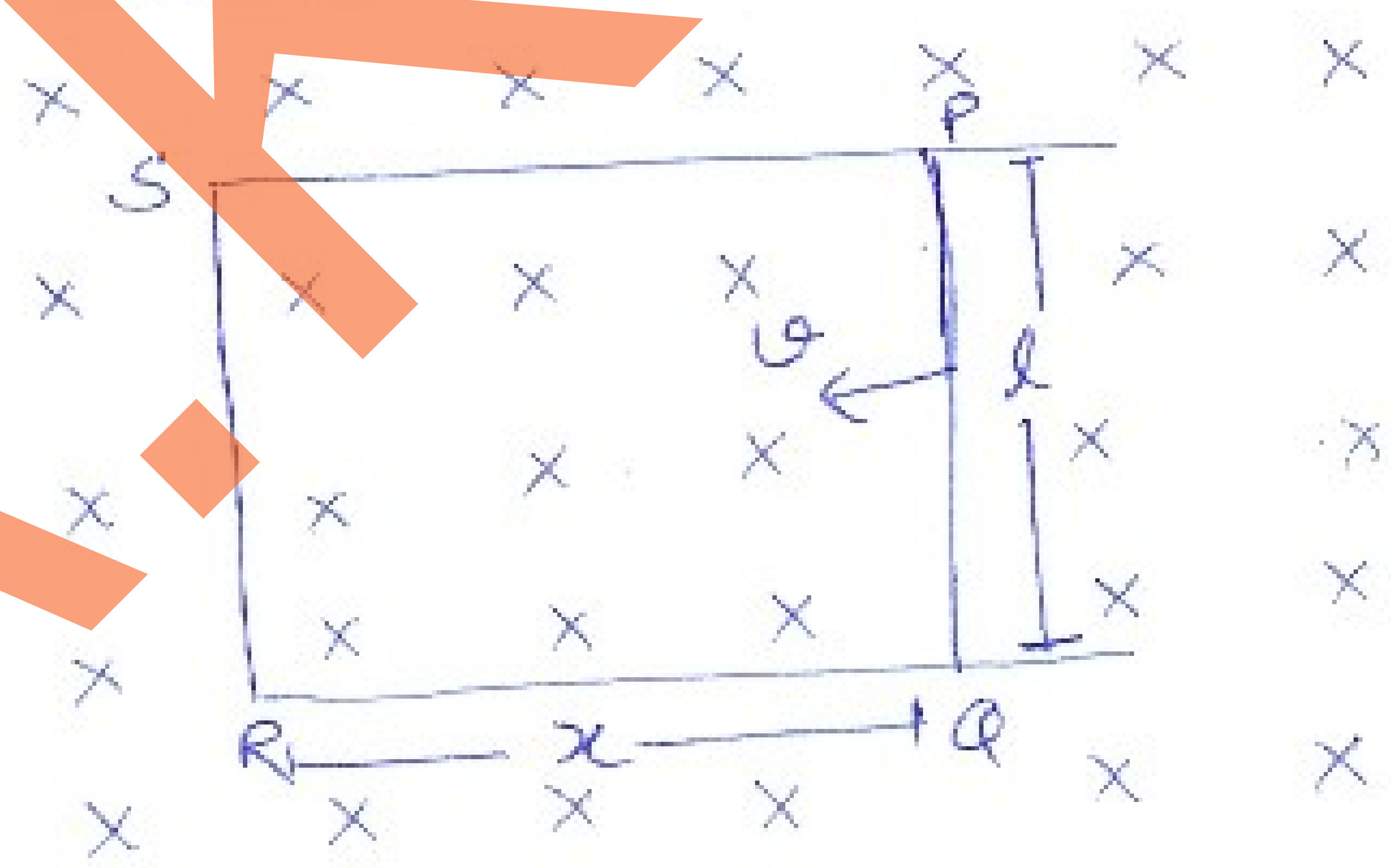
→ When north pole of a bar magnet is being pushed towards the coil, the amount of magnetic flux (ϕ) linked with the coil increases.

→ Current is induced in the coil in such a direction that it opposes the increase in flux.

→ This is possible only when current induced in the coil is in anticlockwise direction w.r.t. an observer on the side of the bar magnet.

→ Vice-versa in opposite case

Motional Electromotive force



Consider a rectangular conducting loop PQRS in the plane of paper with arm PQ free to move.
The crosses represent a uniform magnetic field B perpendicular to the plane of paper & directed inwards.

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Let the conductor PQ is moved towards left with a constant velocity v.

The area enclosed by PQRS decreases & hence the amount of magnetic flux linked with loop decreases. & an e.m.f is induced.

$$\phi = BA = Blx$$

& $$e = -\frac{d\phi}{dt} = \frac{-d}{dt}(Blx)$$

$$= Bl\left(-\frac{dx}{dt}\right)$$

$$\boxed{e = Blv}$$

where $v = -\frac{dx}{dt}$

is velocity of PQ towards left

e - motional e.m.f

Acc. to Lenz's law, direction of induced e.m.f is QRSP.

OR

Consider any arbitrary charge (+q) in arm PQ which is moving towards left with v in B)

Lorentz force on the charge, $F = qvB$

Work done in moving the charge from P to Q is

$$W = F \times l$$

$$= qvB \times l$$

So, the e.m.f is

$$e = \frac{W}{q}$$

$$= \frac{qvB \times l}{q}$$

$$\boxed{e = Blv}$$

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Energy consideration in motional e.m.f

When a conductor of length l is moved with velocity v in a $\perp^r$ magnetic field, the motional e.m.f produced in the conductor is

$$e = Blv$$

Let 'r' be the resistance of arm PQ, then current induced in the loop is

$$I = \frac{e}{r} = \frac{Blv}{r}$$

[Resistance in other arms is negligible]

The magnitude of force on PQ is

$$F = BIl$$
$$= B\left(\frac{Blv}{r}\right)l$$
$$= \frac{B^2l^2v}{r}$$

Power required to push the conductor

$$P = Fv$$
$$= \frac{B^2l^2v}{r} \times v = \frac{B^2l^2v^2}{r}$$

As the conductor is pushed mechanically, the mechanical energy dissipated per second is

$$P = I^2r = \frac{B^2l^2v^2}{r}$$

which is the same as the power required to push the conductor.

Hence, mechanical energy required to move PQ is converted into electrical energy first (i.e. induced e.m.f) & then to thermal energy.

$$\therefore \boxed{\text{Heat energy produced/sec} = \frac{B^2l^2v^2}{r}}$$

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Eddy Currents

Currents induced in the bulk pieces of conductors when the amount of magnetic flux linked with the conductor changes.

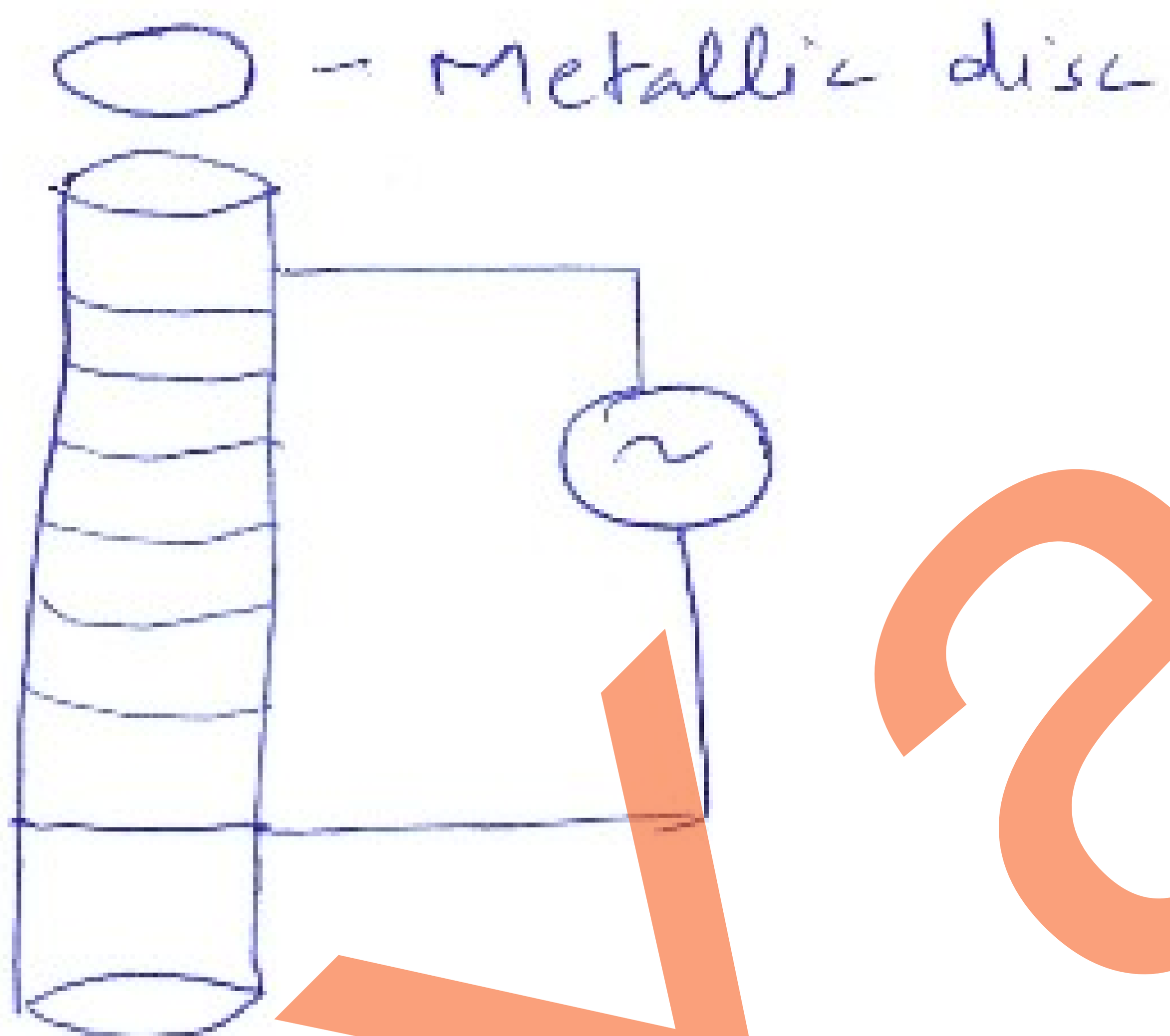
$$i = \frac{e}{R} = \frac{-d\phi/dt}{R}$$

OR

Eddy currents are the currents induced in a conductor, when placed in a changing magnetic field.

Experimental demonstration

(1)



→ Introduce a soft iron core inside a solenoid & connect it to source of alternating e.m.f.

→ Place a metallic disc over the face of solenoid.

→ When the circuit is switched on, the metallic disc is thrown up into air.

Explanation

circuit on → current starts growing → magnetic field (& hence magnetic flux) increases through the disc → induced cullent produced in disc (disc converted into a small magnet)

If the upper face of soft iron core aquires North polarity, then the lower face of disc also acquires North polarity acc. to Lenz's law

As N-N repel so, disc is thrown up.

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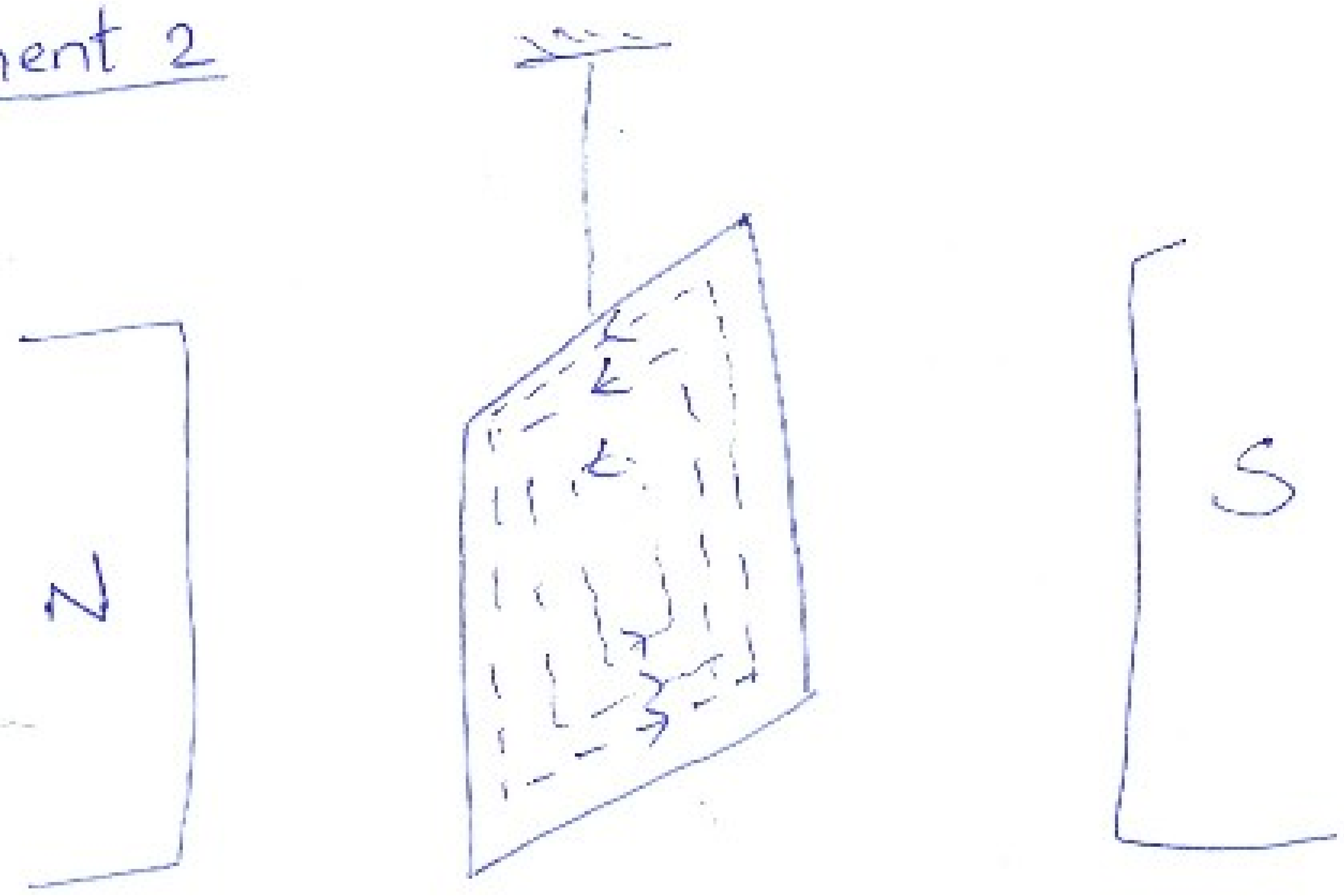
Experiment 2



→ Suspend a flat metallic plate betⁿ the 2 poles of electromagnet.
→ When m.f. OFF, plate oscillates freely & for a long time
ON, " comes to rest quite soon.

Explanation (ON case)

When plate is in equilibrium position, ϕ (linked with plate) - max.
" " " moving towards extreme position

Area betⁿ poles decreases → ϕ decreases → results in production of induced currents in the plate in the form of closed loops.
↓
Acc. to Lenz's law, the induced currents oppose the plate from moving towards its extreme position
↓
Similarly, when the metallic plate returns from its extreme position, again induced or eddy currents are set up which again oppose the motion of the plate.
↓
In either case, the motion of the plate are opposed and hence damped.

* Electromagnetic Damping

The production of eddy currents in a metallic plate oscillating inside the magnetic field produces damping effect. It is called electromagnetic damping.
It is used in making dead-beat galvanometer, induction motor etc.

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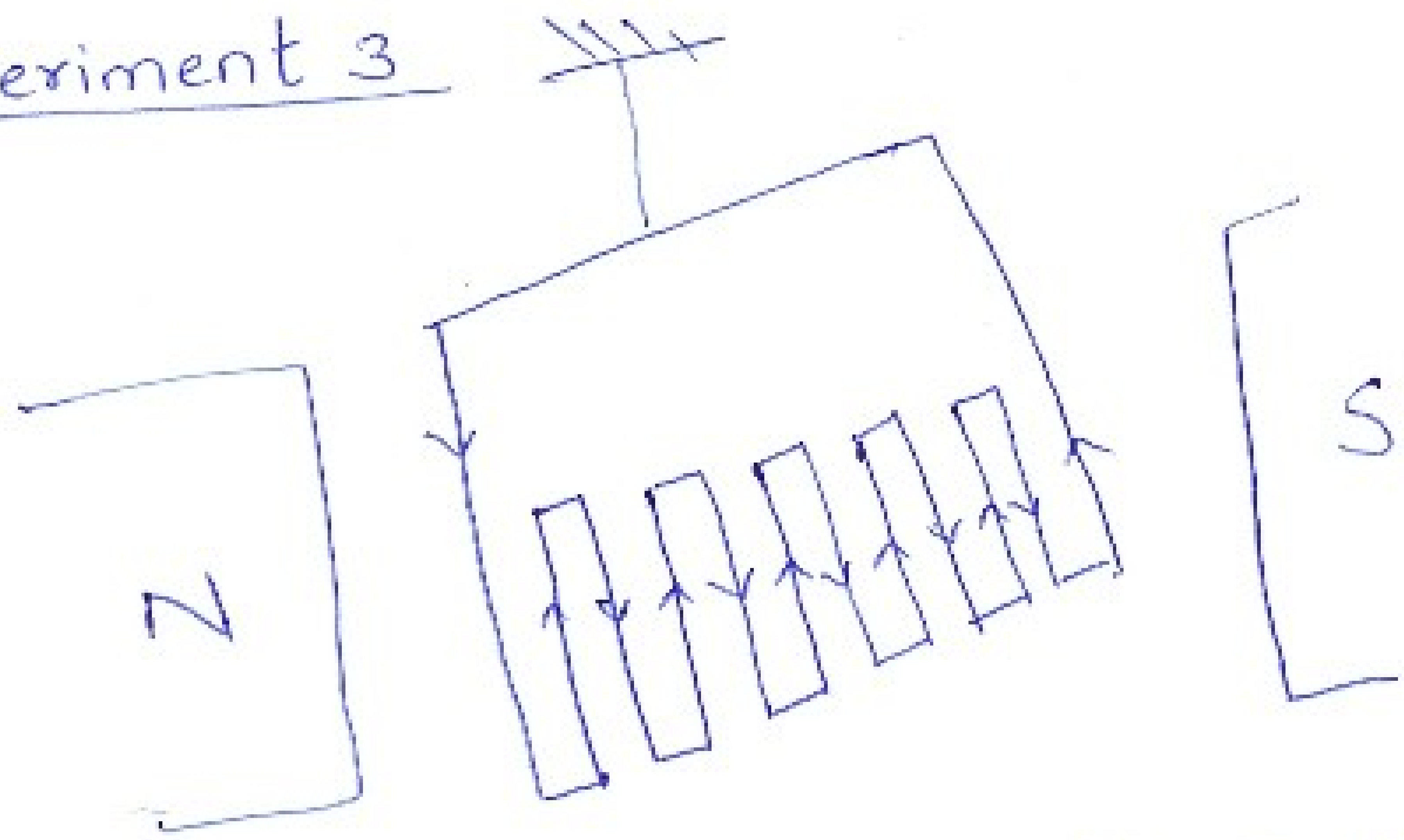
Experiment 3



Here less damping i.e. eddy currents reduced. This is because magnetic moments of the induced currents depends upon area enclosed by currents ($\vec{M} = i\vec{A}$)

Applications of eddy currents

① Magnetic brakes in trains

- Strong electromagnets are situated above the rails in electrically powered trains.
- When electromagnets are activated, the eddy currents induced in the rails oppose the motion of the train.

② Electromagnetic damping

- Certain galvanometers have a fixed core made of non-magnetic metallic material.
- When the coil oscillates, the eddy currents generated in the core oppose the motion & bring the coil to rest quickly.

③ Induction furnace

[used to produce high tem. & to prepare alloys]

- A high frequency a.c. is passed through a coil which surrounds the metals to be melted.
- The eddy currents generated in the metals produce high tem. (sufficient to melt them)

④ Electric meter → shiny metal disc rotates due to eddy current (e.c. produced in the disc by m.f. produced by a.c.)

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Undesirable effects of eddy currents

1) opposes relative motion
2) loss of energy in the form of heat
3) excessive heating may break the insulation in the appliances & reduce their life.

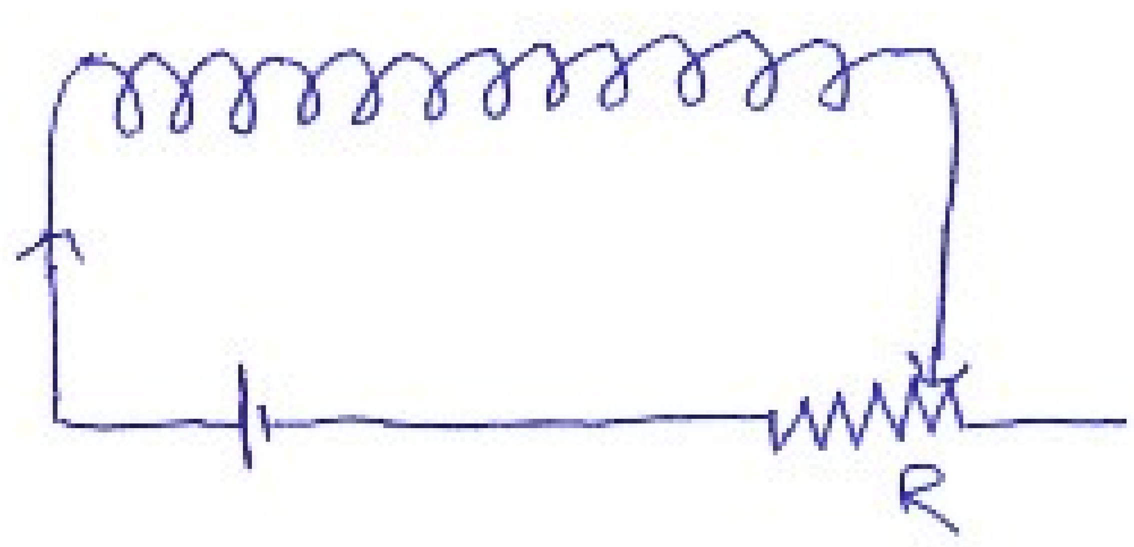
To minimise eddy currents

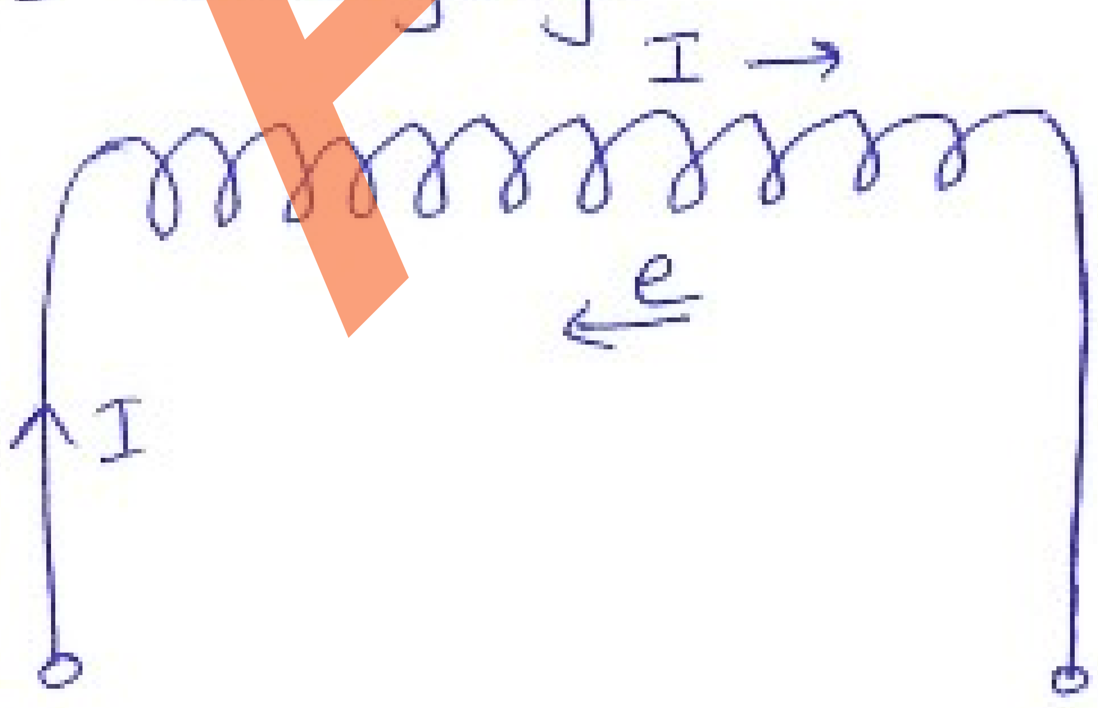
- Metal core to be used in appliance like dynamo, transformer is taken in the form of thin sheets.
- Each sheet is electrically insulated from one another. Such a core is called laminated core.
- The planes of these sheets are arranged parallel to the magnetic field so that they cut across the paths of eddy currents.
- Large resistance betⁿ the thin sheets confines the eddy currents to individual sheets.
- Hence eddy currents are reduced to large extents.

Self Induction [Inertia of electricity]

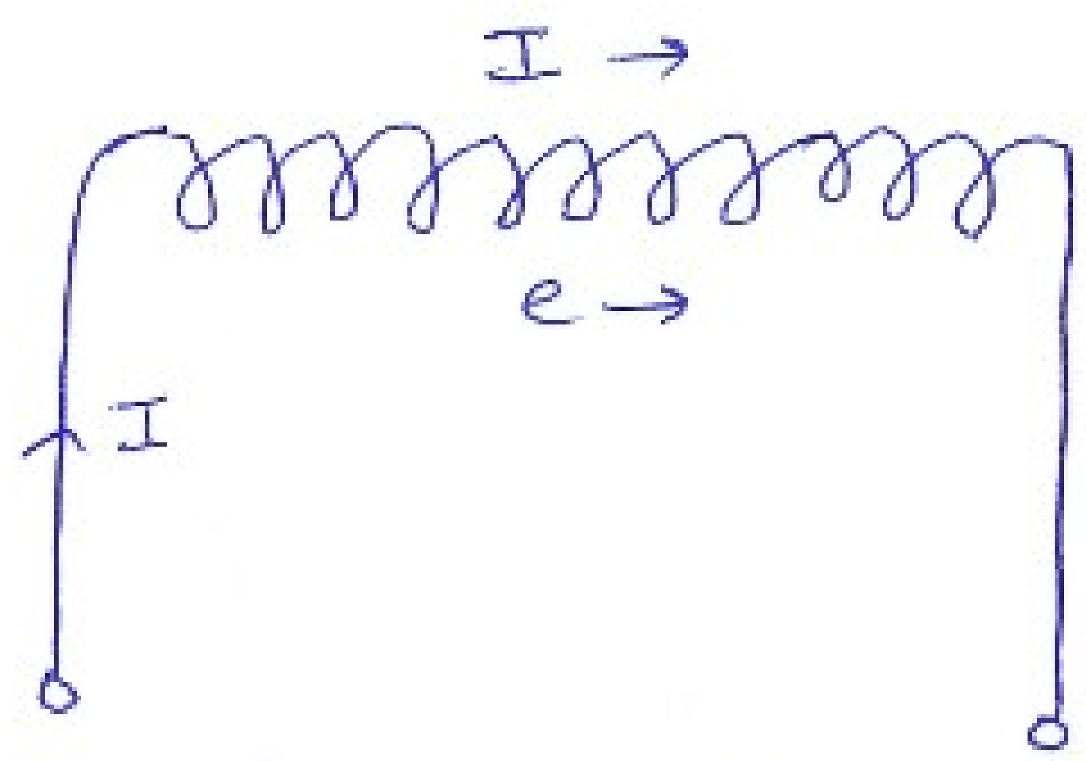
It is the property of a coil by virtue of which, the coil opposes any change in the strength of current flowing through it by inducing an e.m.f in itself.

If current in coil L is changed by varying the contact position on variable resistor, a self induced e.m.f appears in the coil - while current is changing.





Increasing



Decreasing

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Coefficient of self induction

It is found that $\phi \propto I$ (for a coil)

$$\boxed{\phi = LI}$$

L - coefficient of self induction

If $I = 1$ $\boxed{L = \phi}$

so, L is numerically equal to the amount of magnetic flux linked with the coil when unit current flows through the coil.

Now, $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(LI) = -L\frac{dI}{dt}$

if $\frac{dI}{dt} = 1$, then $\boxed{e = -L}$

so, L is equal to the e.m.f induced in the coil when rate of change of current through the coil is unity.

S.I. unit - henry (H)

$$1 \text{ henry} = \frac{1 \text{ volt}}{1 \text{ A/s}} = \frac{1 \text{ volt-sec}}{\text{ampere}} = \frac{1 \text{ Wb}}{1 \text{ A}}$$

so, self inductance of a coil is said to be one henry when a current change at the rate of 1 A/s through the coil induces an e.m.f. of 1 V in the coil

Dimensional formula

$$L = \frac{e\,dt}{dI} = \frac{W}{q} \times \frac{dt}{dI} = \frac{[ML^2T^{-2}][T]}{[AT][A]} = [ML^2T^{-2}A^{-2}]$$

* Inductance is a scalar quantity
* L depends on -
  (i) no. of turns
  (ii) area of cross-section
  (iii) nature of material of core on which the coil is wound

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Self inductance of a long solenoid

Consider a long solenoid of length l, area A & no. of turns per unit length n.

The magnetic field inside the solenoid is

$$B = \mu_0 n I$$

The magnetic flux passing through each turn is

$$\phi = B \times \text{area of each turn}$$
$$= \mu_0 n I A$$

Total magnetic flux linked with the solenoid

$$\phi = \text{m.flux linked with one turn} \times \text{total no. of turns}$$

$$= \mu_0 n I A \times n l$$

$$= \mu_0 n^2 l A I$$

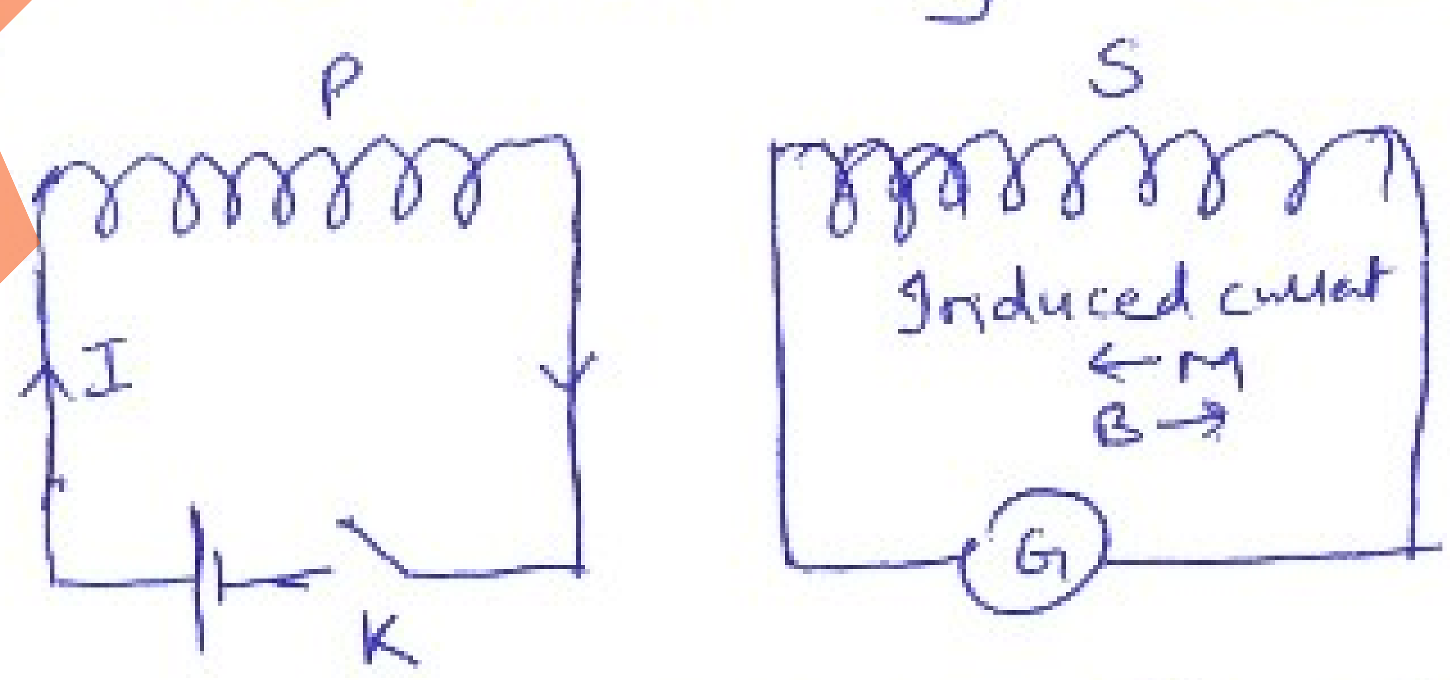
Also, $\phi = LI$, so

$$\boxed{L = \mu_0 n^2 l A}$$

* A wire can't act as a inductor as magnetic flux linked with wire of negligible area of cross-section is zero.

Mutual Induction

It is the property of 2 coils by virtue of which each opposes any change in the strength of current flowing through the other by developing an induced e.m.f.



On pressing or releasing K, galvanometer shows a sudden temporary deflection due to mutual induction

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On pressing K

Current in P increases → magnetic flux linked with P increases → → As S is nearby, magnetic flux linked with S also increases →

An e.m.f induced in S

↓

Acc. to Lenz's law, the induced current in S would oppose increase in current in P by flowing in a direction opp. to current in P.

On releasing K (opp. of 'on pressing')

Coefficient of Mutual induction (M)

Suppose I - strength of current in one coil
ϕ - total magnetic flux linked with all the turns of neighbouring coil.

It is found that $\phi \propto I$

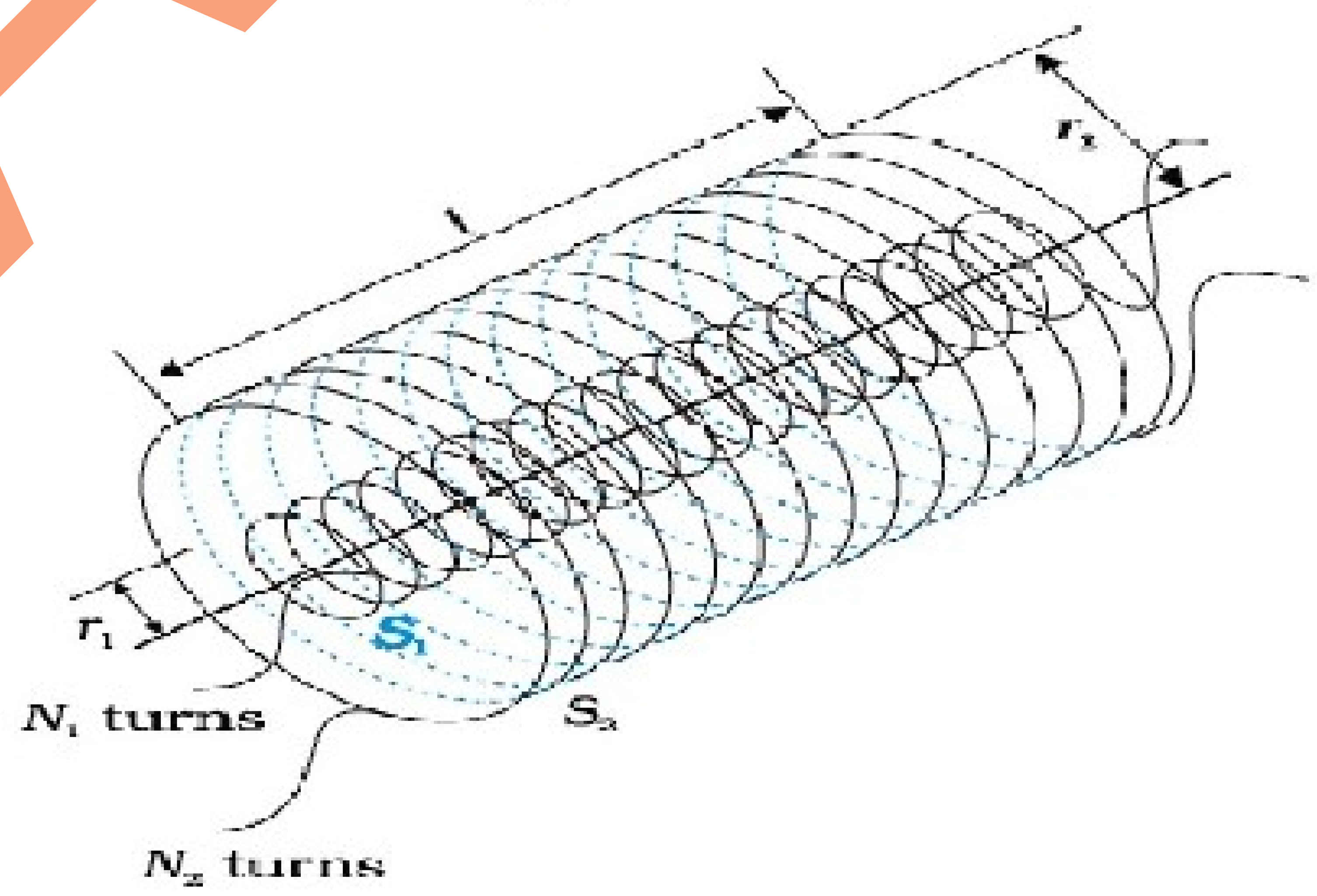
$$\boxed{\phi = MI}$$

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(MI) = -M\frac{dI}{dt}$$

S.I. unit - henry

* M depends upon
(i) geometry of 2 coils (shape, size, no. of turns)
(ii) distance betⁿ 2 coils
(iii) orientation of the 2 coils.

Mutual Inductance of 2 long co-axial solenoids



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Consider 2 long co-axial solenoids each of length 'l'.

Let
r_1 - radius of inner solenoid S_1
n_1 - no. of turns per unit length of solenoid S_1
r_2 - radius of outer solenoid S_1
n_2 - no. of turns per unit length of solenoid S_2
N_1 - Total no. of turns of S_1
N_2 - Total no. of turns of S_2
$l^2 >> r_2$

Case I

When current I_2 is set up through S_2, it sets up a magnetic flux through S_1.

$$\phi_1 = M_{12} I_2 \quad \text{--- (1)} \quad [M_{12} \text{ - mutual inductance of } S_1 \text{ w.r.t } S_2]$$

Magnetic field due to current I_2 in S_2, $B_2 = \mu_0 n_2 I_2$

$\therefore$ Magnetic flux through S_1, $\phi_1 = B_2 A_1 N_1$

$$= \mu_0 n_2 I_2 \times \pi r_1^2 \times n_1 l$$

$$= \mu_0 n_1 n_2 \pi r_1^2 l \cdot I_2 \quad \text{--- (2)}$$

From (1) & (2)

$$\boxed{M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l} \quad \text{--- (3)}$$

Case II (Reverse Case)

When current I_1 is passed through solenoid S_1, it sets up magnetic flux through S_2

$$\phi_2 = M_{21} I_1 \quad \text{--- (4)} \quad [M_{21} \text{ - mutual inductance of } S_2 \text{ w.r.t. } S_1]$$

Now, the flux due to I_1 in S_1 is considered to be confined solely inside S_1 (as the solenoids are very long)

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$$\therefore \quad \phi_2 = B_1 A_1 N_2$$
$$= \mu_0 n_1 I_1 \times \pi r_1^2 \times n_2 l$$
$$= \mu_0 n_1 n_2 \pi r_1^2 l I_1 \quad \text{—— (5)}$$

From (4) & (5)

$$\boxed{M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l} \quad \text{—— (6)}$$

Now, from (3) & (6), $\boxed{M_{12} = M_{21} = M = \mu_0 n_1 n_2 \pi r_1^2 l}$

<u>Note</u>

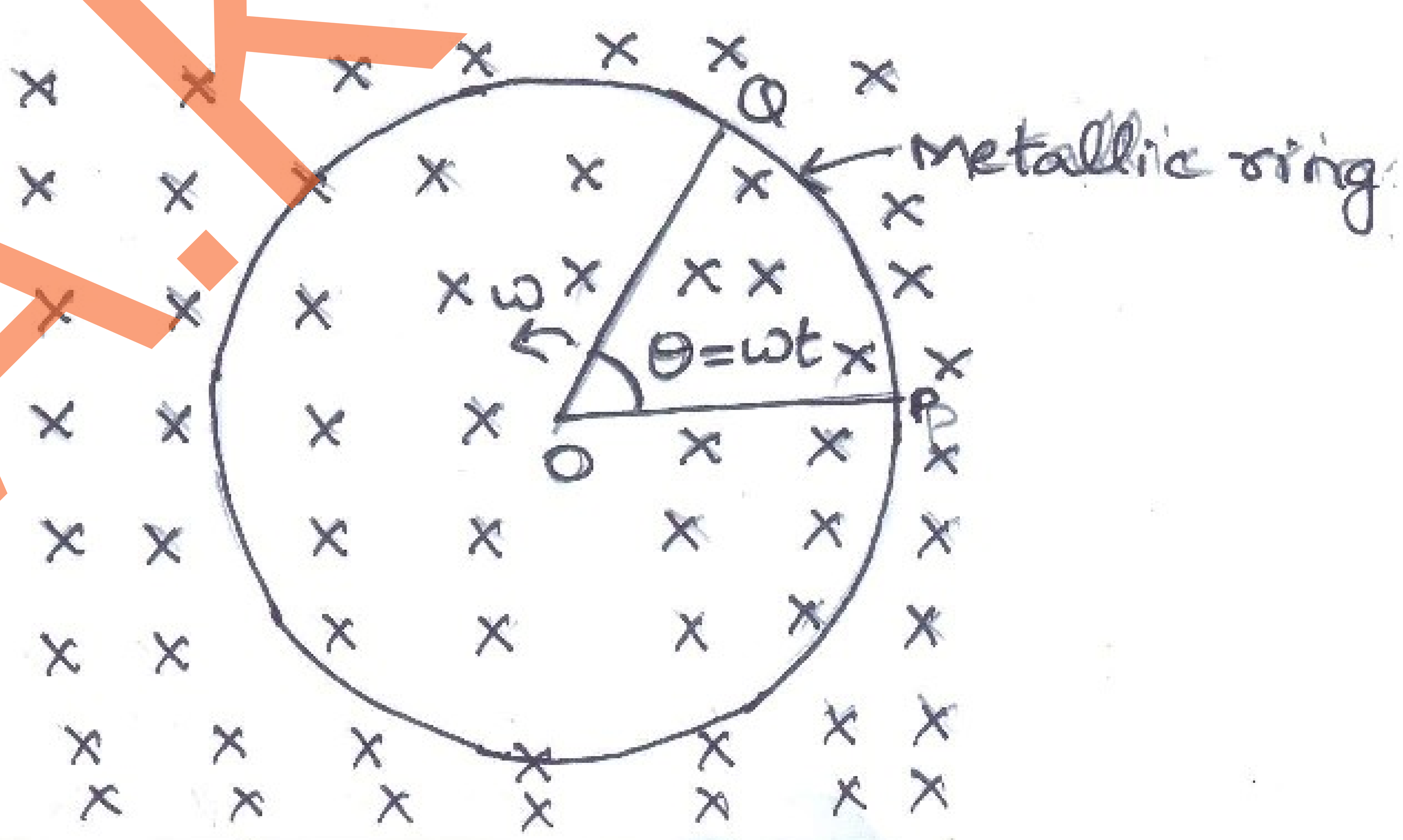
- In case of a medium of relative permeability μ_r

$$M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l$$

- $M = \mu_0 n_1 n_2 \pi r_1^2 l = \mu_0 \times \frac{N_1}{l} \times \frac{N_2}{l} \times A \times l = \frac{\mu_0 N_1 N_2 A}{l}$

- Mutual inductance of a pair of solenoids depends
  (i) separation of solenoids/coils
  (ii) relative orientation of the solenoids

<u>Induced EMF in a rotating rod</u>



<u>Method I</u>

- As the rod is rotated, free electrons in the rod move towards the outer end (due to Lorentz force) and get distributed over the ring.

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- The resulting separation of charges produces an emf across the ends of the rod.
- At a certain value of emf, there is no more flow of electrons & a steady state is reached.

The magnitude of emf generated across length 'dr' of rod as it moves at right angles to magnetic field is

$$dE = Bv\,dr$$

Integrating both sides

$$E = \int_0^R Bv\,dr$$

$$= \int_0^R B(\omega r)\,dr$$

$$= B\omega \int_0^R r\,dr$$

$$= B\omega \left[\frac{r^2}{2}\right]_0^R$$

$$\boxed{E = \frac{1}{2} B\omega R^2}$$

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Method II

- Consider a closed loop OPQ in which points 'O' and 'P' are connected with a resistor & OQ is rotating rod.
- Potential difference across resistor is equal to induced emf

$\therefore$ $E = B \times$ (rate of change of area of loop)

$$\left[\begin{aligned} E &= \frac{d\phi}{dt} \\ &= \frac{d}{dt} BA \\ &= B\frac{dA}{dt} \end{aligned}\right]$$

Area of sector OPQ $= \pi R^2 \times \dfrac{\theta}{2\pi}$

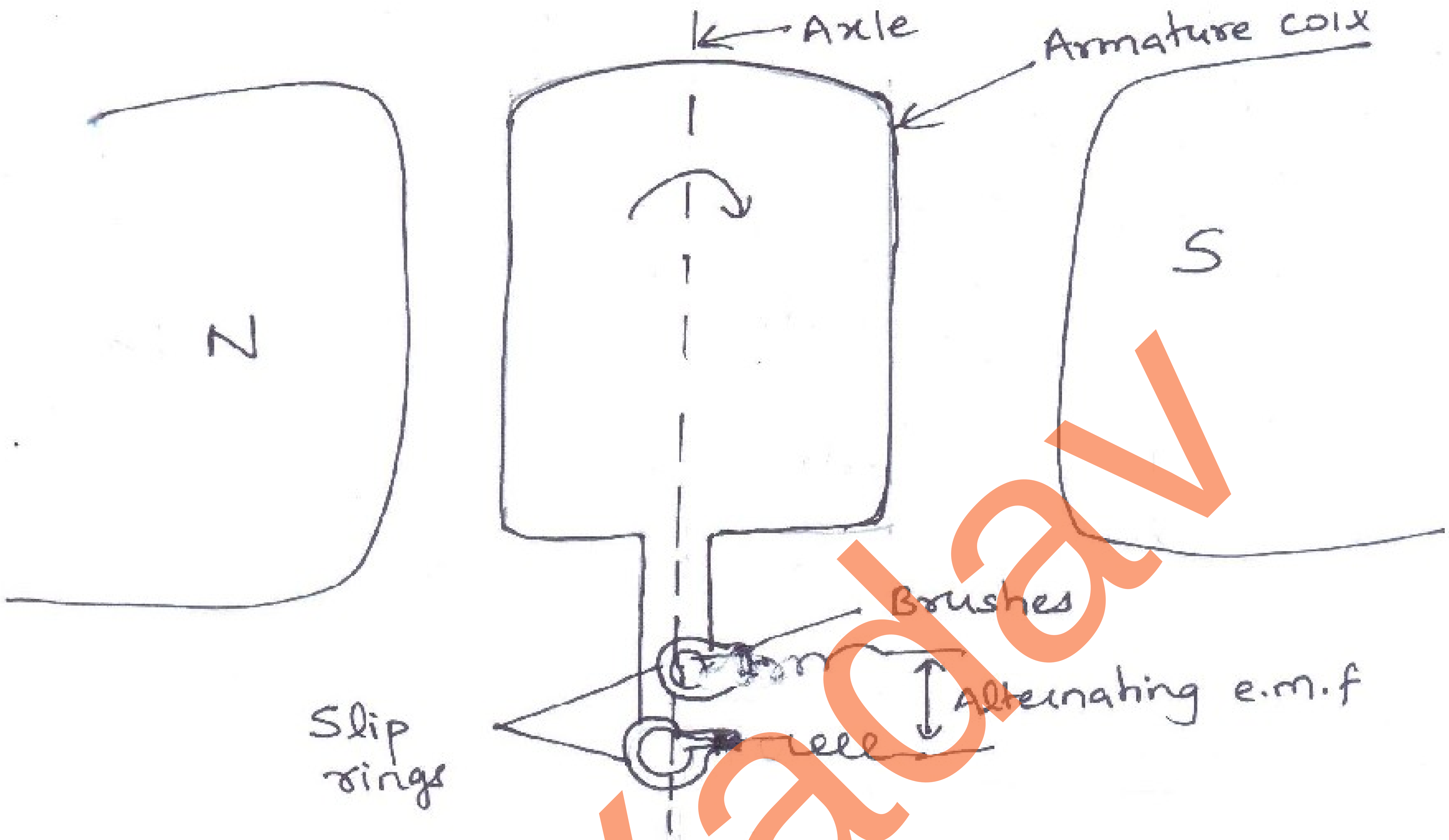
$= \dfrac{1}{2} R^2 \theta$ [R - radius of circle]

$$\therefore\ E = B\frac{d}{dt}\left(\frac{1}{2}R^2\theta\right) = \frac{1}{2} BR^2 \frac{d\theta}{dt}$$

$$\boxed{E = \frac{1}{2} BR^2 \omega}$$

AC Generator (Converts mechanical to electrical energy)

Principle: It is based on concept of electromagnetic induction.
"Whenever magnetic flux linked with a coil changes, an e.m.f is induced in the coil."



Construction

1. Armature – Rectangular coil ABCD having large no. of turns of copper wire wound over soft iron core.
2. Slip rings – provide movable contact
3. Brushes – pass current from armature to slip rings.
4. Field magnet – Strong electromagnet with concave poles.

Working

Direction of flow of current

- Initially at $t = 0$, armature coil PQRS is vertical with PQ down & RS up.
- During the motion of PQRS from $t = 0$ & $t = T/2$, PQ moves up & RS moves down.

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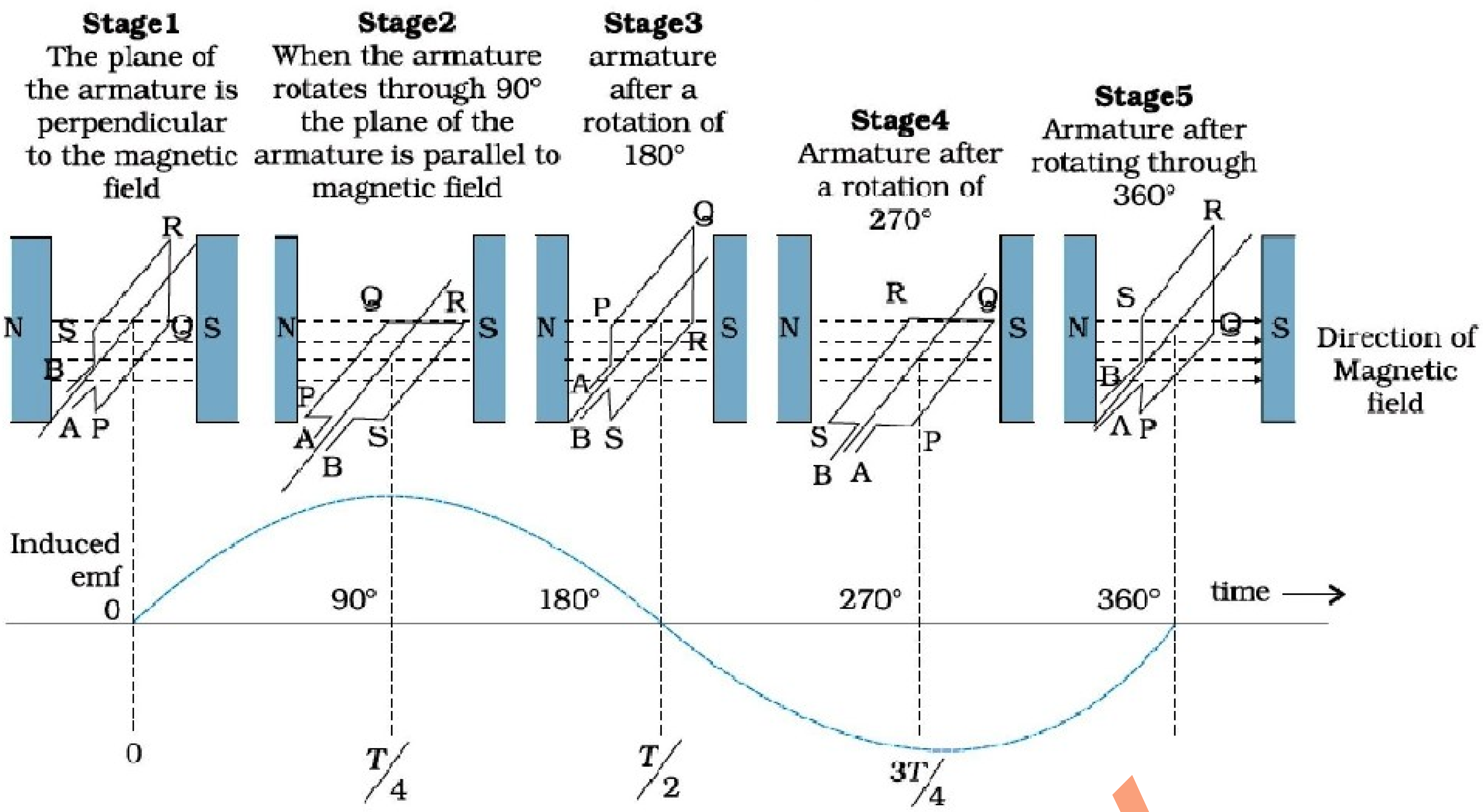

- By Fleming's right hand rule, direction of current in armature will be along PQRS.
- During the motion of armature PQRS betn $t = T/2$ to $t = T$, PQ moves down & RS moves up.
- By Fleming's right hand rule, direction of current in armature will be along SRQP.

<u>Magnitude of induced e.m.f</u>

Let N - no. of turns in the coil
B - strength of magnetic field
A - area enclosed by each turn of coil
θ - angle betn magnetic field & area vector
ω - constant angular speed of coil

Magnetic flux linked with the coil at any time 't' is

$$\phi = BA\cos\theta = BA\cos\omega t$$

The emf induced for rotating coil of 'N' turns $\varepsilon = -N\frac{d\phi}{dt}$

$$\varepsilon = -N\frac{d}{dt}(BA\cos\omega t)$$

$$= -NBA\frac{d}{dt}\cos\omega t$$

$$= -NBA(-\sin\omega t)\omega$$

$$\varepsilon = NBA\omega\sin\omega t$$

$$\boxed{\varepsilon = \varepsilon_0 \sin\omega t}$$

where $\boxed{\varepsilon_0 = NBA\omega}$ max. value of emf

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Magnetic flux linked with the coil at any time 't' is

$$\phi = BA\cos\theta$$
$$= BA\cos\omega t$$

The emf induced for rotating coil of 'N' turns

$$\varepsilon = -N\cancel{\frac{d\phi}{dt}} \quad -N\frac{d\phi}{dt}$$

$$= -N\frac{d}{dt}(BA\cos\omega t)$$

$$= -NBA\frac{d}{dt}\cos\omega t$$

$$= -NBA(-\sin\omega t)\omega$$

$$\varepsilon = NBA\omega\sin\omega t$$

$$\boxed{\varepsilon = \varepsilon_0\sin\omega t}$$

where $\boxed{\varepsilon_0 = NBA\omega}$ max. value of emf

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Alternating Current

So, far we have studied d.c. circuits but most of the electric power generated is in the form of a.c. because:

(i) alternating voltages can be easily & efficiently converted from one value to other by using transformers.

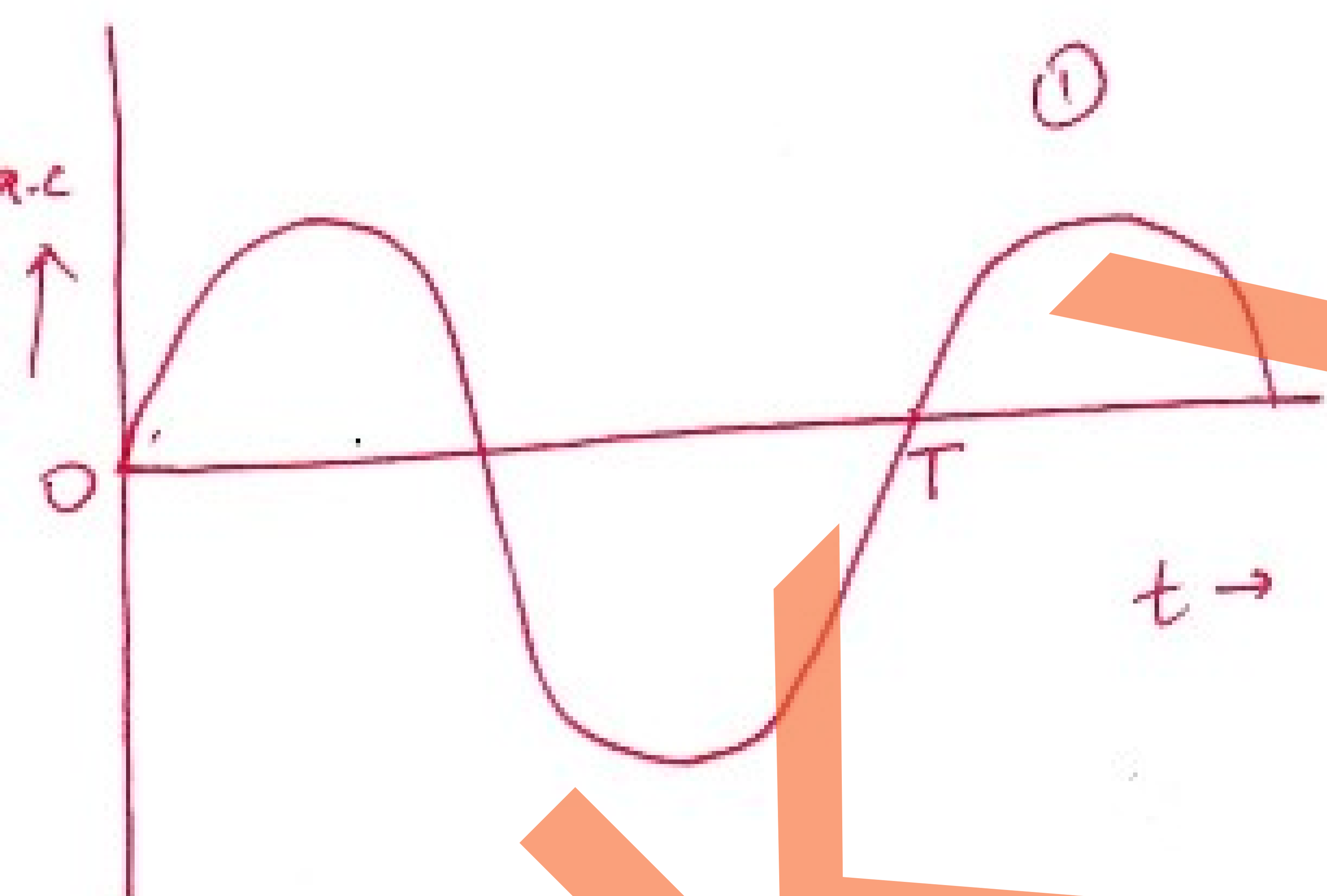
(ii) the a.c. energy can be transmitted & distributed over long distances without much loss of energy.

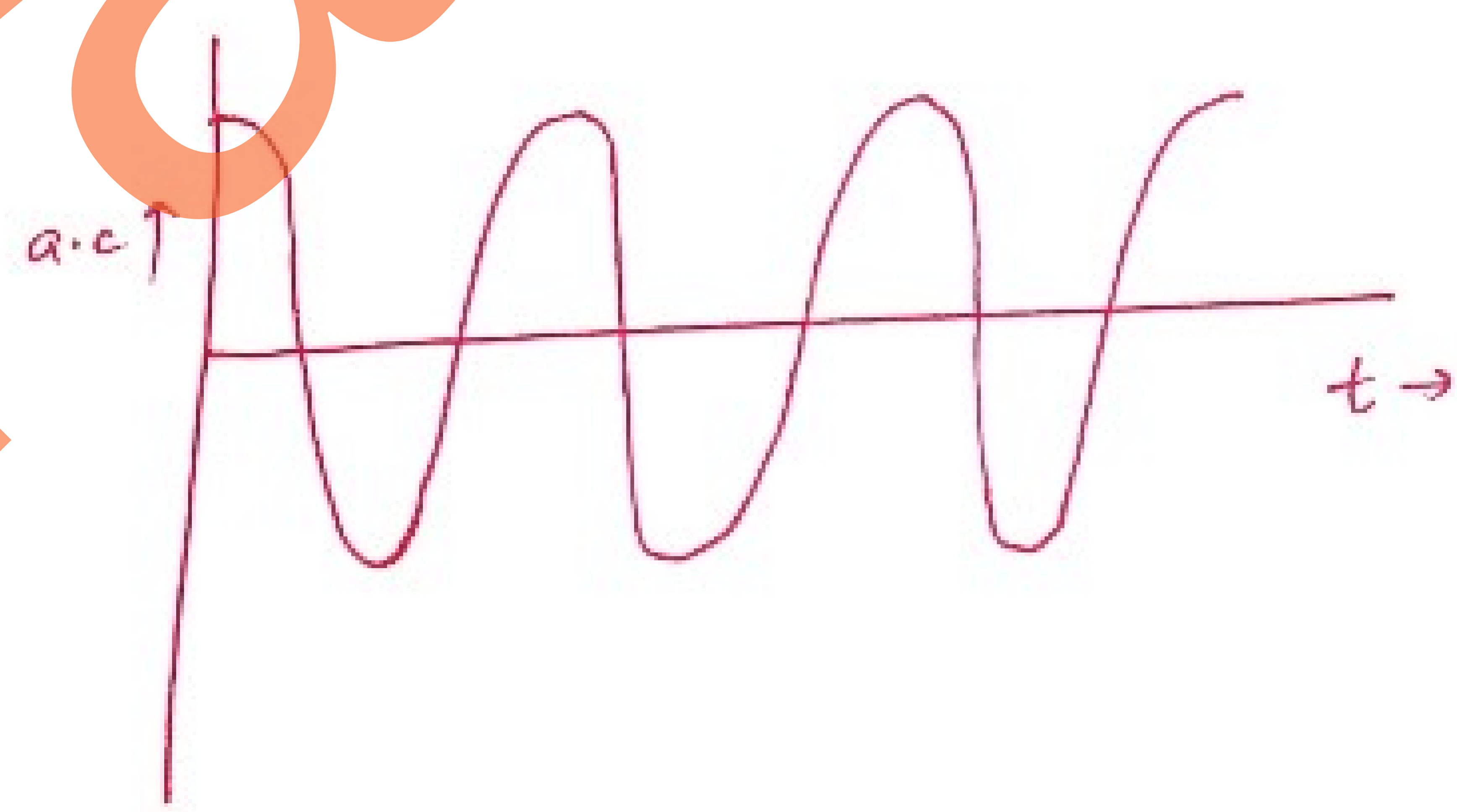
Alternating current

The magnitude of a.c. changes continuously with time & its direction is reversed periodically.

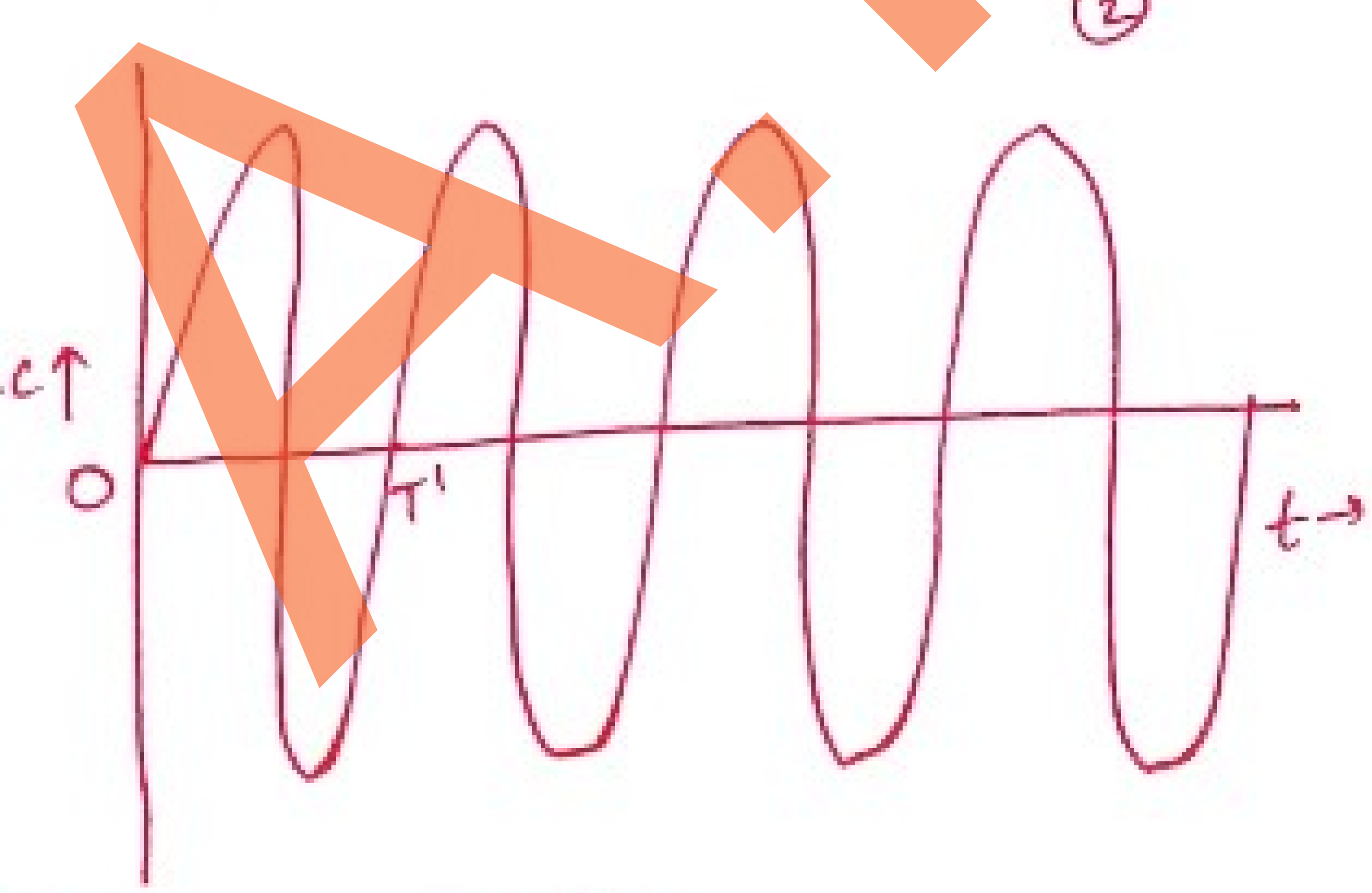
$$I = I_0 \sin\omega t \quad \text{or} \quad I = I_0 \cos\omega t$$

①





②



from ① & ②

$v' = 2v$

$T = 2T'$

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Mean value or average value of a.c.

The mean value of a.c. over any half cycle is defined as that value of steady current which would send the same amount of charge through a circuit in the time of half cycle as is sent by the a.c. through the same circuit, in the same time.

$$I = I_0 \sin\omega t$$

Also, $$I = \frac{dq}{dt}$$

$$dq = I\,dt$$

Let q be the total charge sent by a.c. in the first half cycle

$$q = \int_0^{T/2} I\,dt \quad \text{--- (1)}$$

$$= \int_0^{T/2} I_0 \sin\omega t\,dt$$

$$= I_0 \left[-\frac{\cos\omega t}{\omega} \right]_0^{T/2}$$

$$= \frac{-I_0}{\omega} \left[\cos\omega\frac{T}{2} - \cos 0^\circ \right]$$

$$= \frac{-I_0}{\omega} \left[\cos\pi - \cos 0^\circ \right]$$

$$= \frac{-I_0}{\omega} \left[-1 - 1 \right]$$

$$q = \frac{2I_0}{\omega} \quad \text{--- (2)}$$

If I_m represents the mean value of a.c. over 1st h.cy then

$$q = I_m \times \frac{T}{2} \quad \text{--- (3)}$$

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from (2) & (3)

$$I_m \times \frac{T}{2} = \frac{2 I_0 \times T}{2\pi}$$

$$\boxed{I_m = \frac{2}{\pi} I_0 = 0.637\, I_0}$$

Hence, the mean value of a.c. over +ve half cycle is 0.637 times the peak value of a.c.

Similarly, for −ve half cycle $I_m = -0.637 I_0$ [Integrate eqⁿ (1) from $T/2$ to T]

So, the mean value of a.c. over one complete cycle is

$$0.637 I_0 - 0.637 I_0 = 0$$

[Can be derived directly by integrating eqⁿ (1) from 0 to T]

<u>Mean value of alternating e.m.f</u>

The mean value of alternating e.m.f over a half cycle is that value of constant e.m.f which would send the same amount of charge through a circuit in the time of half cycle as is sent by alternating e.m.f through the same circuit in the same time.

$$E = E_0 \sin\omega t$$

$$I = \frac{E}{R} = \frac{E_0 \sin\omega t}{R}$$

Now, $I = \frac{dq}{dt}$

$$dq = I\,dt$$

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$$dq = \frac{E_0}{R} \sin \omega t \, dt$$

$$q = \int_0^{T/2} \frac{E_0}{R} \sin\omega t \, dt$$

$$= \frac{E_0}{R} \left[-\frac{\cos\omega t}{\omega} \right]_0^{T/2}$$

$$= \frac{-E_0}{\omega R} \left[\cos \omega \frac{T}{2} - \cos 0^\circ \right]$$

$$= \frac{-E_0}{\omega R} [-2]$$

$$q = \frac{2E_0}{\omega R} \quad —— (1)$$

If E_m is mean value of alternating e.m.f over the first half cycle, then

$$q = \frac{E_m}{R} \frac{T}{2} \quad —— (2)$$

from (1) & (2)

$$\boxed{E_m = \frac{2}{\pi} E_0 = 0.637 E_0}$$

Hence, mean value of alternating e.m.f over positive half cycle is 0.637 times the peak value of alt. e.m.f.

Similarly, over −ve half cycle, $E_m = -0.637 I_0$ $[T/2 \rightarrow T]$

" one complete " , $E_m = 0$ $[0 \rightarrow T]$

* Ordinary d.c. ammeter & voltmeter can't measure a.c or alt. voltage (they show zero reading) because average value of alt current/voltage over a full cycle is zero.

R.M.S. value of a.c. [Effective or virtual value of a.c]

The r.m.s value of a.c. is defined as that value of steady current, which would generate the same amount of heat in a given resistance in a given time, as is done by the a.c., when passed through the same resistance for the same time.

An alternating current is

$$I = I_0 \sin \omega t$$

Let this current flows through resistance R. In small time 'dt', the amount of heat produced in resistance R is

$$dH = I^2 R\, dt$$

In one complete cycle, the total amount of heat produced in resistance R is

$$H = \int_0^T I^2 R\, dt$$

$$= \int_0^T I_0^2 \sin^2 \omega t\, R\, dt$$

$$= I_0^2 R \int_0^T \sin^2 \omega t\, dt$$

$$= I_0^2 R \int_0^T \frac{1 - \cos 2\omega t}{2}\, dt$$

$$= \frac{I_0^2 R}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t\, dt\right]$$

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$$= \frac{I_0^2 R}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{I_0^2 R}{2}\left[T - 0 - \left(\frac{\sin 2\omega T}{2\omega} - \sin 0^\circ\right)\right]$$

$$= \frac{I_0^2 R}{2}\left[T - \frac{\sin 2\times 2\pi}{2\omega}\right] \qquad [\because \omega T = 2\pi]$$

$$H = \frac{I_0^2 RT}{2} \quad \text{—— (1)} \qquad [\because \sin 4\pi = 0]$$

If $I_{r.m.s}$ is the r.m.s value of a.c., then the amount of heat produced in the same resistance, in the same time T would be

$$H = I_{r.m.s}^2 RT \quad \text{—— (2)}$$

from (1) & (2)

$$I_{rms}^2 RT = \frac{I_0^2 RT}{2}$$

$$\boxed{I_{r.ms} = \frac{I_0}{\sqrt{2}} = 0.707\, I_0}$$

Hence, the r.m.s value of a.c. is 0.707 times the peak value of a.c.

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R.M.S. value of alternating E.M.F

The r.m.s value of alternating e.m.f is defined as that value of steady voltage; which would generate the same amount of heat in a given resistance in a given time, as is done by the alternating e.m.f, when applied to the same resistance for the same time.

Let the alternating e.m.f is $E = E_0 \sin\omega t$

Small amount of heat produced when this alternating e.m.f is applied to a resistance R for a small time 'dt' is

$$dH = \frac{E^2}{R} dt$$

$$= \frac{E_0^2}{R} \sin^2\omega t \, dt$$

The total amount of heat produced in resistence R in one complete cycle is

$$H = \int_0^T \frac{E_0^2}{R} \sin^2\omega t \, dt$$

$$= \frac{E_0^2}{R} \int_0^T \frac{1-\cos 2\omega t}{2} dt$$

$$= \frac{E_0^2}{2R} \left[\int_0^T dt - \int_0^T \cos 2\omega t \, dt \right]$$

$$= \frac{E_0^2}{2R} \left[[t]_0^T - \left[\frac{\sin 2\omega t}{2\omega} \right]_0^T \right]$$

$$= \frac{E_0^2}{2R} \left[T - 0 - \left(\frac{\sin 2\omega T}{2\omega} - \sin 0^\circ \right) \right]$$

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$$= \frac{E_0^2}{2R}\left[T - \frac{\sin 2\times 2\pi}{2\omega}\right]$$

$$= \frac{E_0^2}{2R}\left[T - 0\right] \qquad [\because \sin 4\pi = 0]$$

$$H = \frac{E_0^2 T}{2R} \quad \text{—— (1)}$$

If E_{rms} is the rms value of alternating e.m.f, then the amount of heat produced in the same resistance R in the same time T is

$$H = \frac{E_{rms}^2 T}{R} \quad \text{—— (2)}$$

$$\frac{E_{rms}^2 T}{R} = \frac{E_0^2}{2R}$$

$$\boxed{E_{rms} = \frac{E_0}{\sqrt{2}}}$$

$$\boxed{E_{rms} = 0.707\, E_0}$$

Hence, r.m.s. value of alternating e.m.f is 0.707 times the peak value of alternating e.m.f

* 1. A.C/A.V. are measured by a.c. ammeter/voltmeter resp.

2. These instruments are called hot wire instruments & they measure only virtual values of a.c/a.v.

3. Example:

220 V a.c. means $E_{rms} = 220V$

1A a.c " $I_{rms} = 1A$

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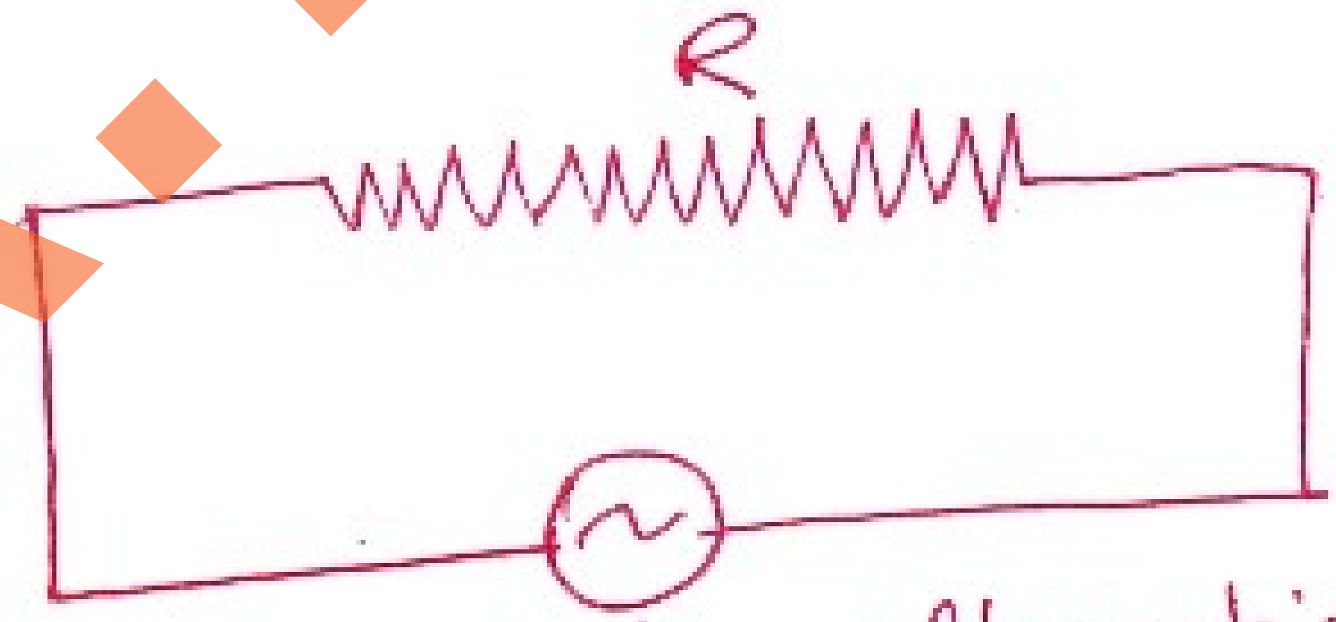
Phasor Diagram

- The analysis of an a.c. circuit is facilitated by the use of phasor diagram.
- <u>Phasor</u> - A rotating vector that represents a quantity varying sinusoidally with time is called phasor.
  - It is imagined to rotate with angular velocity equal to angular velocity of that quantity.
- In phasor diagram, peak values of a.c (I_0) & a.v (E_0) are represented by arrows called phasors.
- They are inclined to horizontal axis at $\angle \omega t$ & rotate in anti-clockwise direction.
- The length of arrow represents the max. values of quantity i.e. I_0 & E_0.
- The projection of arrow of any axis represents the instantaneous value of the quantity.

  sine form - projection is taken on vertical axis
  cosine form - " " " " horizontal "
- The phase difference betn the 2 alternating quantities is represented by the angle betn them.

A.C. circuit containing resistance only



Let a source of alternating e.m.f is connected to a pure resistance R.
The alternating e.m.f supplied is

$$E = E_0 \sin\omega t \quad \text{--- (1)}$$

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Let I be the current in the circuit at any instant 't'.

$$E = IR$$

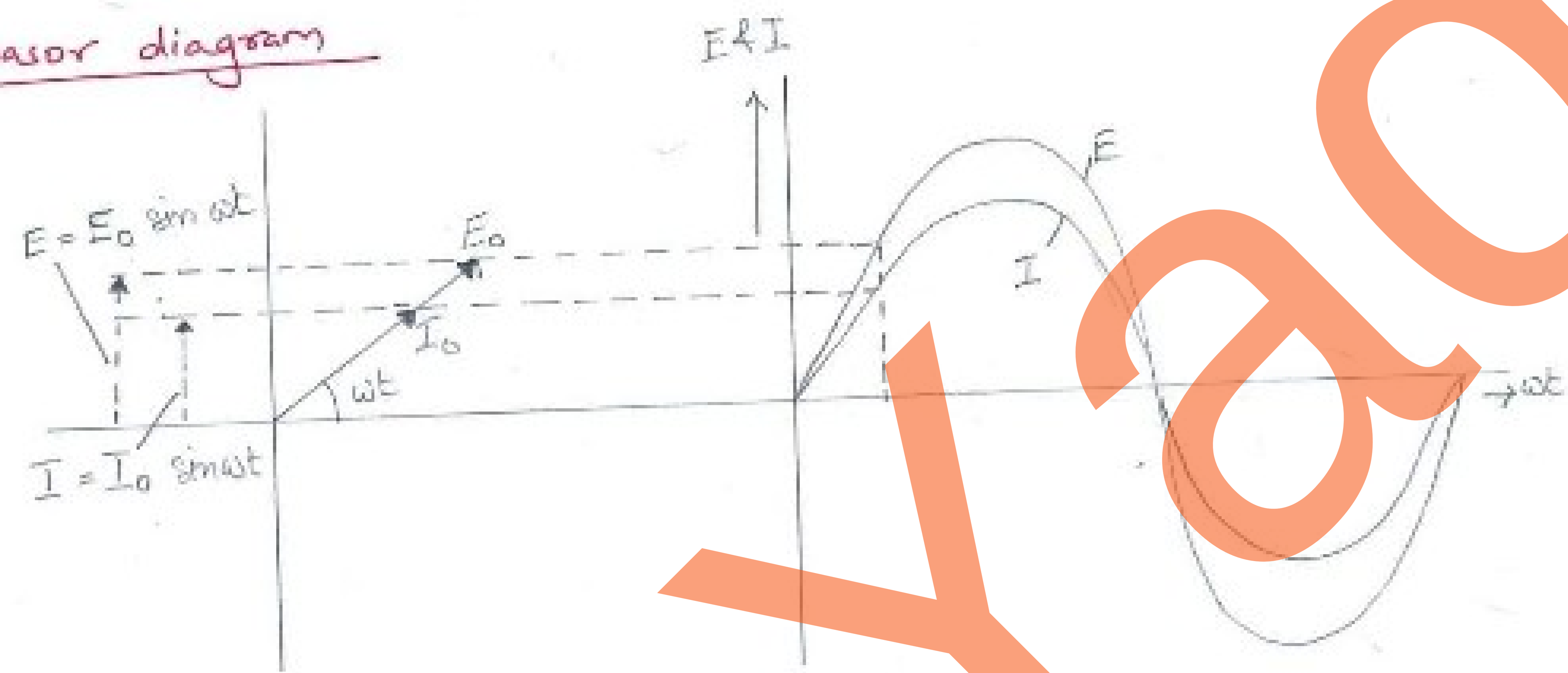
$$\boxed{I = \frac{E}{R} = \frac{E_0 \sin\omega t}{R} = I_0 \sin\omega t} \quad (2)$$

where $I_0 = \dfrac{E_0}{R}$

Now, $I_0 = \dfrac{E_0}{R}$ & $I = \dfrac{V}{R}$ so we find that R is same in a.c or d.c.

Also, from (1) & (2) we find that <u>E & I are in phase</u>

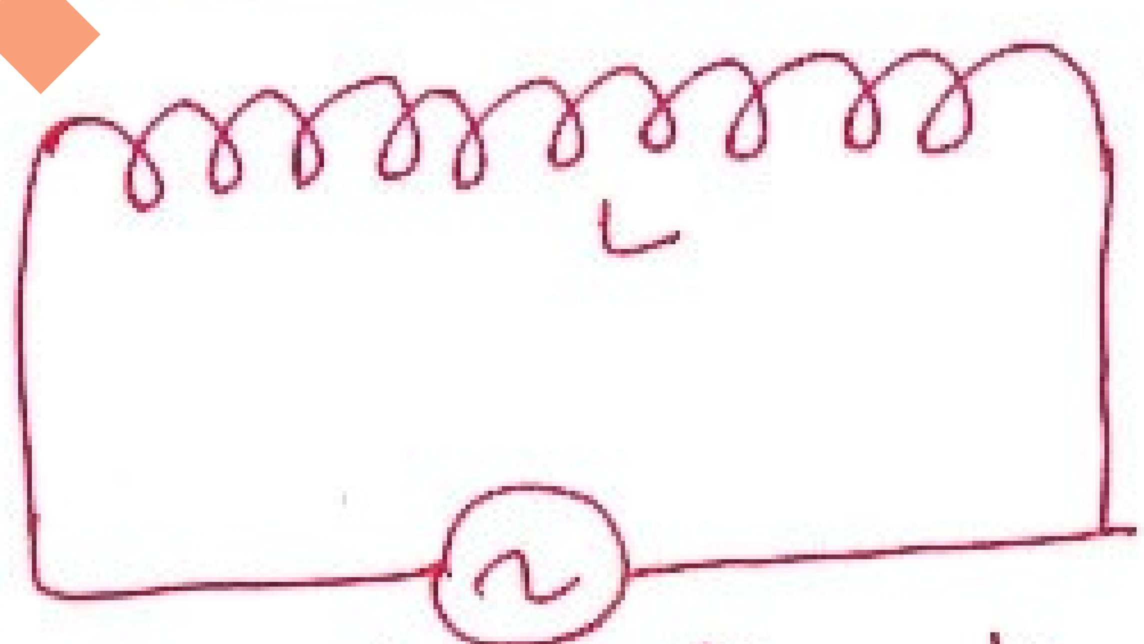
<u>Phasor diagram</u>



→ I & V in same phase so phasors (I_0 & E_0) are in same direction making angle ωt with X-axis

→ angle betn a.c & a.v – zero.

<u>A.C. circuit containing Inductor only</u>



Let a source of alternating e.m.f is connected to a circuit containing a pure inductance 'L'.

Let the alternating e.m.f is

$$E = E_0 \sin\omega t \quad (1)$$

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If $\frac{dI}{dt}$ is the rate of change of current through L then the e.m.f induced in L is

$$e = -L\frac{dI}{dt}$$

To maintain the flow of current in the circuit applied voltage must be equal & opposite to the induced voltage

$$E = -\left(-L\frac{dI}{dt}\right)$$

$$E_0 \sin\omega t = L\frac{dI}{dt}$$

$$dI = \frac{E_0}{L}\sin\omega t\, dt$$

$$I = \frac{E_0}{L}\int \sin\omega t\, dt$$

$$= \frac{E_0}{L}\left(-\frac{\cos\omega t}{\omega}\right)$$

$$= -\frac{E_0}{\omega L}\cos\omega t$$

$$= \frac{-E_0}{\omega L}\sin\left(\frac{\pi}{2}-\omega t\right)$$

$$= \frac{E_0}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right) \quad (2)$$

Current will be max. $(I = I_0)$ if $\sin(\omega t - \pi/2) = 1$

so, $$I_0 = \frac{E_0}{\omega L} \quad (3)$$

$$\therefore I = I_0 \sin(\omega t - \pi/2) \quad (4)$$

from (1) & (4) we find that I lags behind E by a phase angle of 90°

Comparing eqⁿ ① with ohm's law we find that ωL represents effective resistance offered by L called as inductive reactance (X_L)

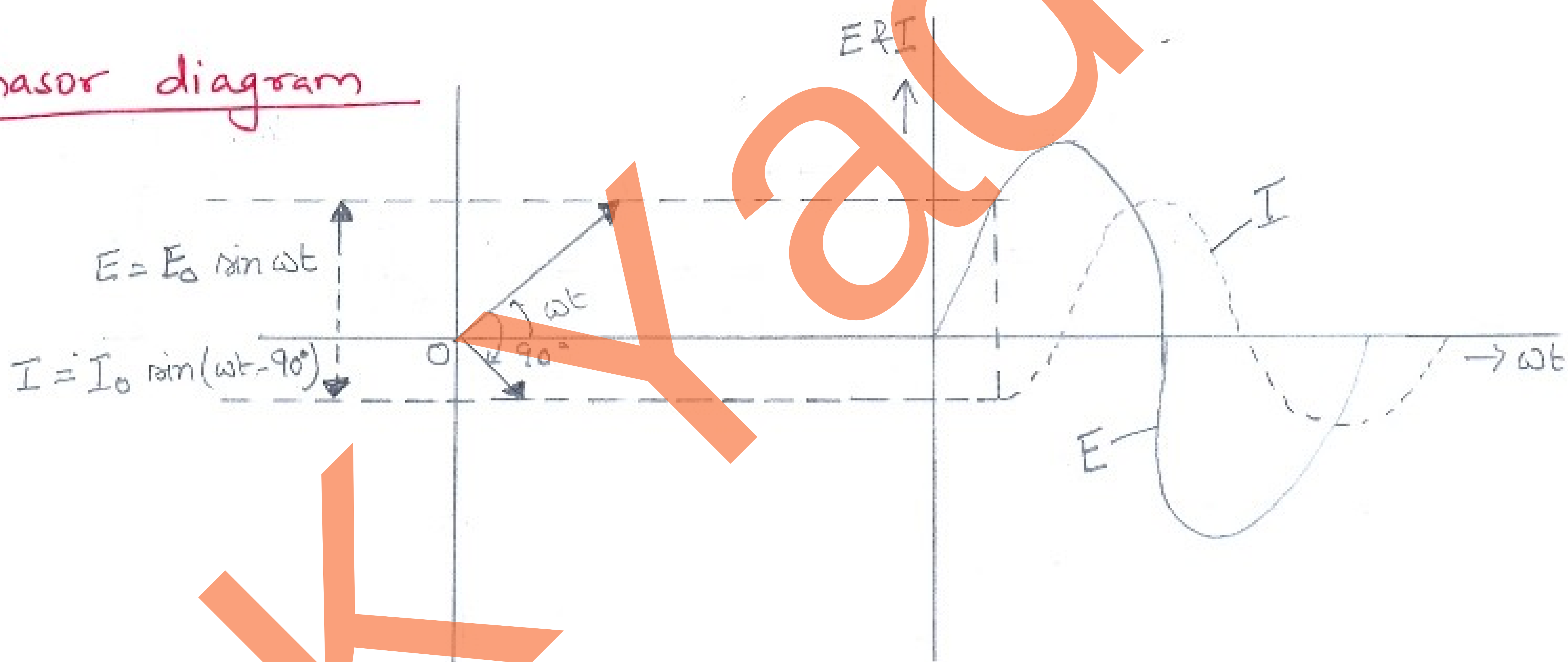
$$\boxed{X_L = \omega L = 2\pi \nu L}$$

* In d.c. circuit $\nu = 0$ so $X_L = 0$
i.e. A pure inductance offers zero resistance to d.c. & it can't reduce dc.

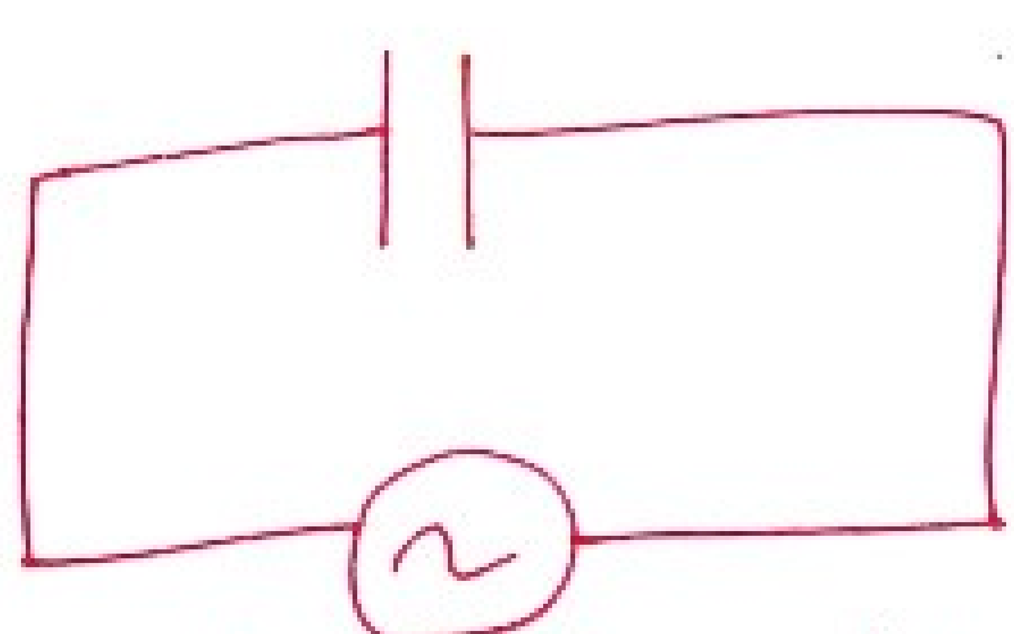
Unit of X_L

$$X_L = \omega L = \frac{1}{sec}\ \text{henry} = \frac{1}{sec}\ \frac{volt}{A\,s^{-1}} = \text{ohm}$$

Phasor diagram



A.C. circuit containing capacitance only



Let a source of alternating e.m.f be connected to a capacitor only.
The alternating e.m.f is $E = E_0 \sin \omega t$ —①

A.K Yadav

[The current flowing in the circuit transfers charge to the plates of capacitor.
This produces a potential difference betⁿ the plates.
The capacitor is alternately charged & discharged as the current reverses after every half cycle.]

At any instant, let q be the charge on capacitor, so p.d. across the plates is

$$E\ (V) = \frac{q}{C}$$

$$E_0 \sin\omega t = \frac{q}{C}$$

$$q = CE_0 \sin\omega t$$

$$I = \frac{dq}{dt}$$

$$= CE_0 \frac{d}{dt}\sin\omega t$$

$$= CE_0 \cos\omega t \times \omega$$

$$= \frac{E_0}{1/\omega C}\sin\left(\omega t + \frac{\pi}{2}\right)$$

When $\sin\left(\omega t + \frac{\pi}{2}\right) = 1$, $I = I_0$

$$\therefore\ I_0 = \frac{E_0}{1/\omega C} \quad (2)$$

So, $I = I_0 \sin\left(\omega t + \pi/2\right)$ — (3)

Comparing (1) & (3) we find that in an a.c. circuit containing C only, I leads E by 90°

Comparing eqⁿ (2) with ohm's law eqⁿ we find that $1/\omega C$ represents effective resistance offered by capacitor called as capacitive reactance (X_C)

$$\boxed{X_C = \frac{1}{\omega C} = \frac{1}{2\pi \nu C}}$$

A.K. Yadav

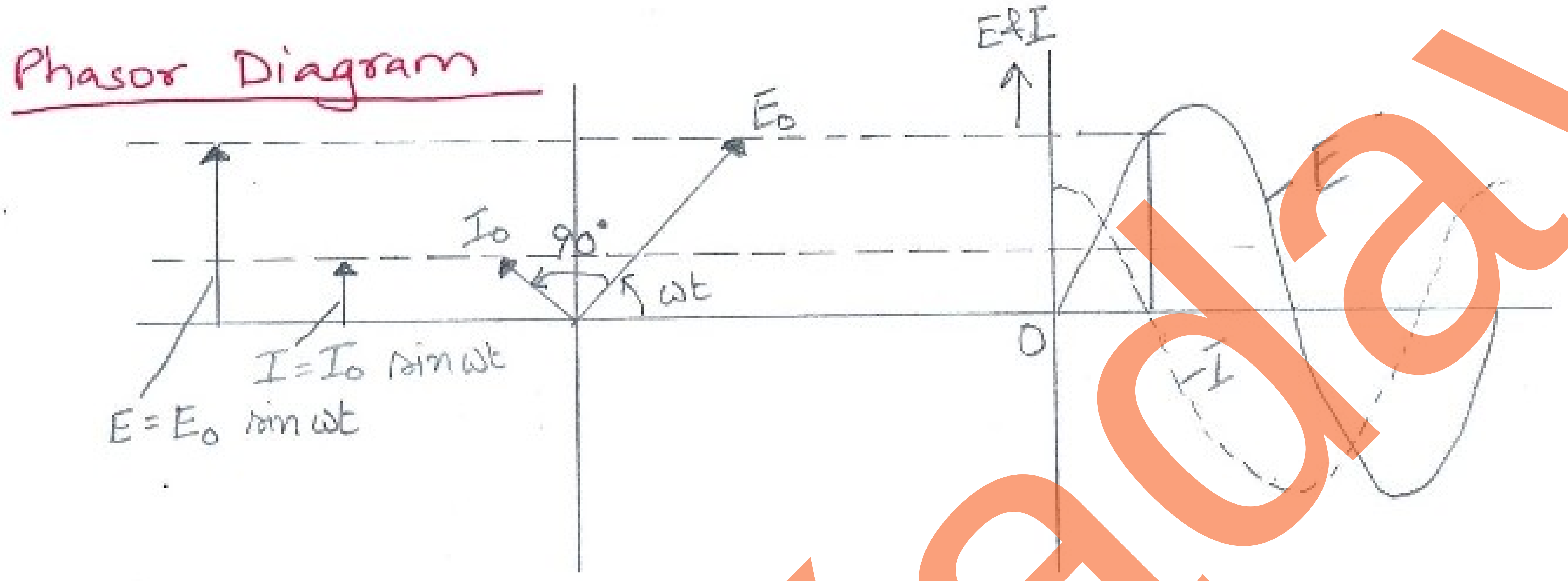
In a d.c. circuit $\nu = 0$ so $X_c = \infty$

i.e. a condenser will block d.c.

Unit of X_c

$$X_c = \frac{1}{\omega C} = \frac{sec}{farad} = \frac{sec}{CV^{-1}} = \frac{Vs}{As} = ohm$$

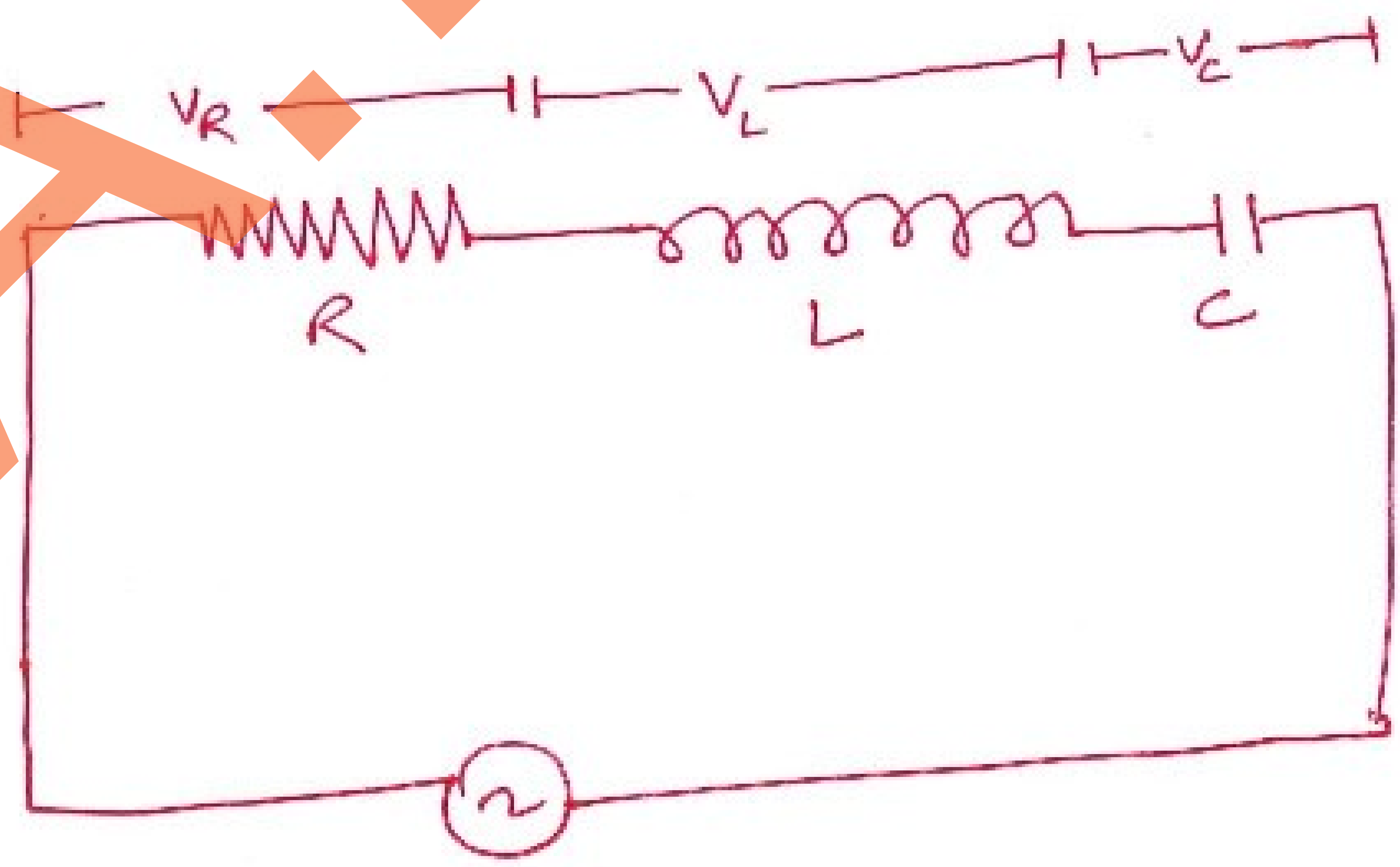
Phasor Diagram



LCR circuit

Let a pure resistance R, a pure inductance L & an ideal condenser of capacity C be connected in series to a source of alternating e.m.f

$$E = E_0 \sin \omega t$$



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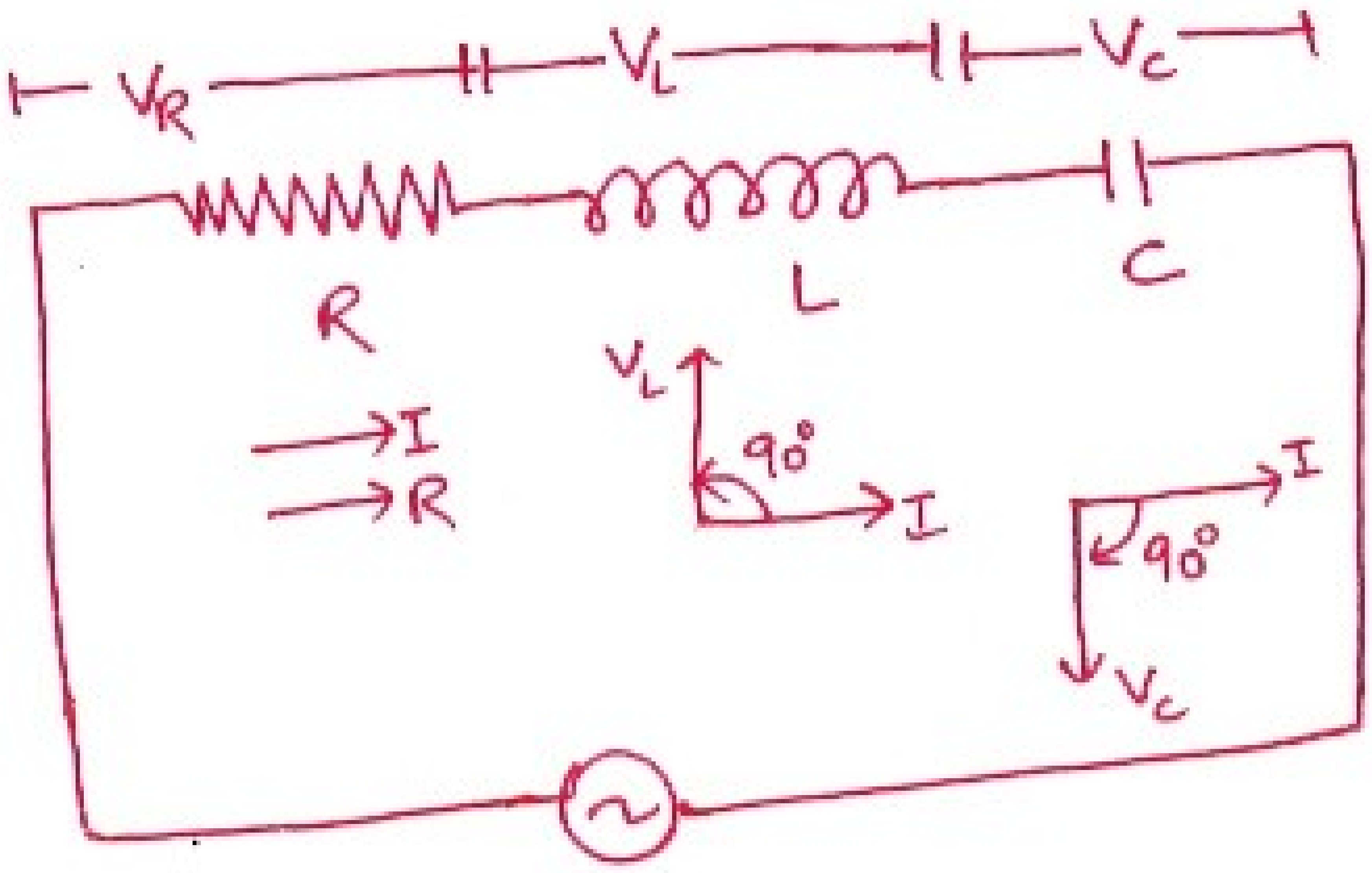
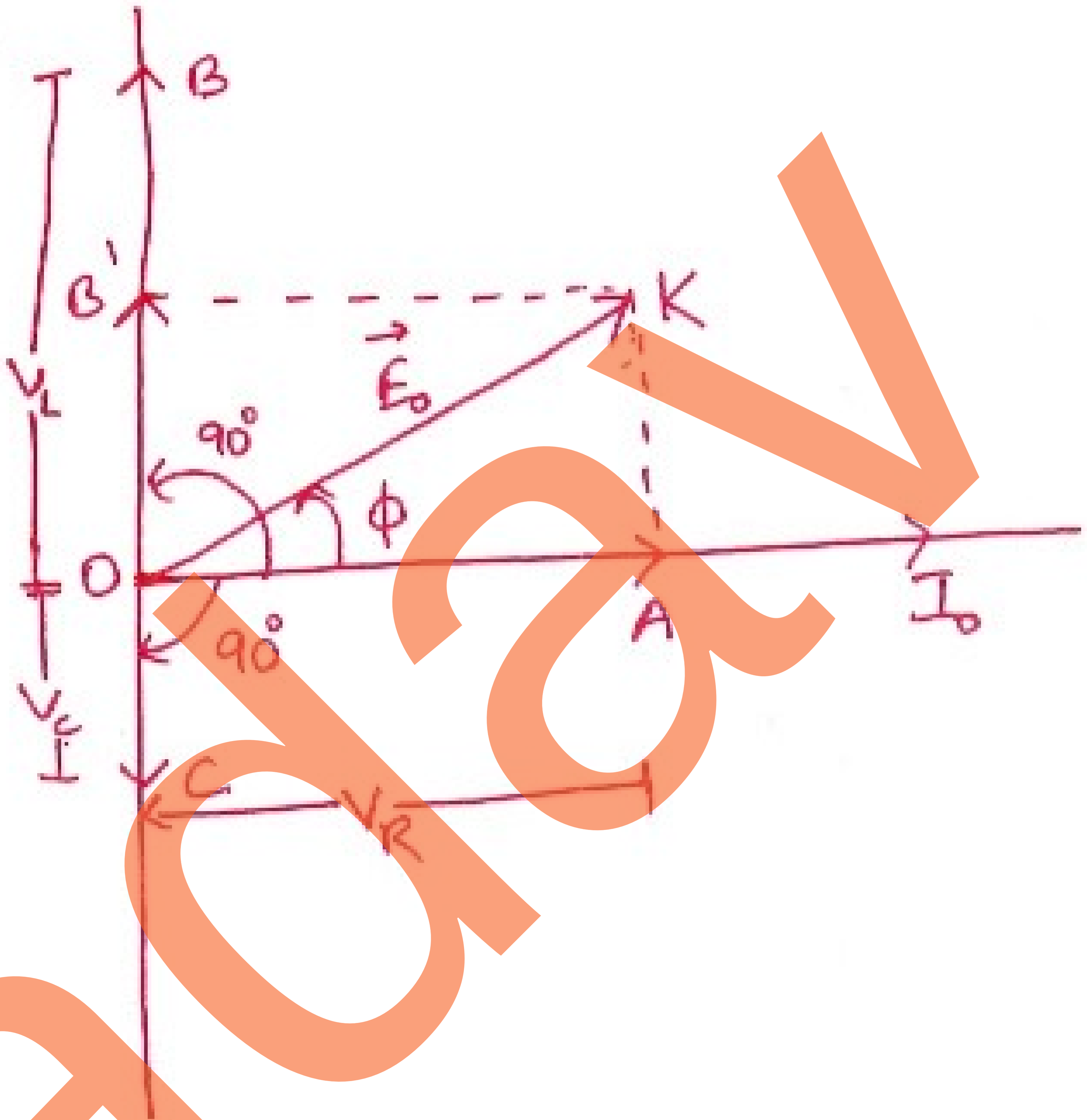
Phasor Treatment





Max. voltage across R, $\vec{V_R} = \vec{I_0} R$ $(\vec{OA})$

" " " L, $\vec{V_L} = \vec{I_0} X_L$ $(\vec{OB})$

" " " C, $\vec{V_C} = \vec{I_0} X_C$ $(\vec{OC})$

As voltage across L & C have phase difference of $180°$, the net reactive voltage is $(\vec{V_L} - \vec{V_C})$ (assuming $\vec{V_L} > \vec{V_C}$) represented by $\vec{OB'}$

The vector sum of V_L, V_R & V_C is phasor $\vec{E_0}$ $(\vec{OK})$ making an angle ϕ with $\vec{I_0}$.

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$$OK = \sqrt{OA^2 + OB'^2}$$

$$E_0 = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2}$$

$$= I_0 \sqrt{R^2 + (X_L - X_C)^2}$$

$$E_0 = I_0 Z$$

where $\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}}$

impedance of the circuit

$$\tan\phi = \frac{AK}{OA} = \frac{OB'}{OA} = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

$$\boxed{\tan\phi = \frac{X_L - X_C}{R}}$$

(i) $X_L = X_C$, $\tan\phi = 0$

$\therefore \phi = 0°$

V & I in same phase. circuit - non-inductive

(ii) $X_L > X_C$, $\tan\phi = +ve$, $\phi = +ve$

V leads I by ϕ, circuit - inductance dominated

(iii) $X_L < X_C$, $\tan\phi = -ve$, $\phi = -ve$

V lags I by ϕ, circuit - capacitance dominated

A.K. Yadav

Resonance

1. The phenomenon of resonance is common among systems that have a tendency to oscillate at a particular frequency, called as natural frequency of oscillation of the system.
2. If such a system is driven by an energy source whose frequency is equal to the natural frequency of the system, the amplitude of oscillations becomes large and resonance is said to occur.

Example: a child on a swing

(i) A child on a swing has a natural frequency of swinging back & forth.

(ii) If another child pushes the swing at regular intervals & the frequency of the pushes is almost the same as the frequency of swinging, the amplitude of swing becomes very large.

(iii) We conclude resonance has occured.

Resonance in LCR circuit

In a LCR circuit with voltage V_m & frequency ω, the current is

$$I_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

At a particular frequency ω_0, $X_C = X_L$, Z - min.

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

ω_0 - resonant frequency

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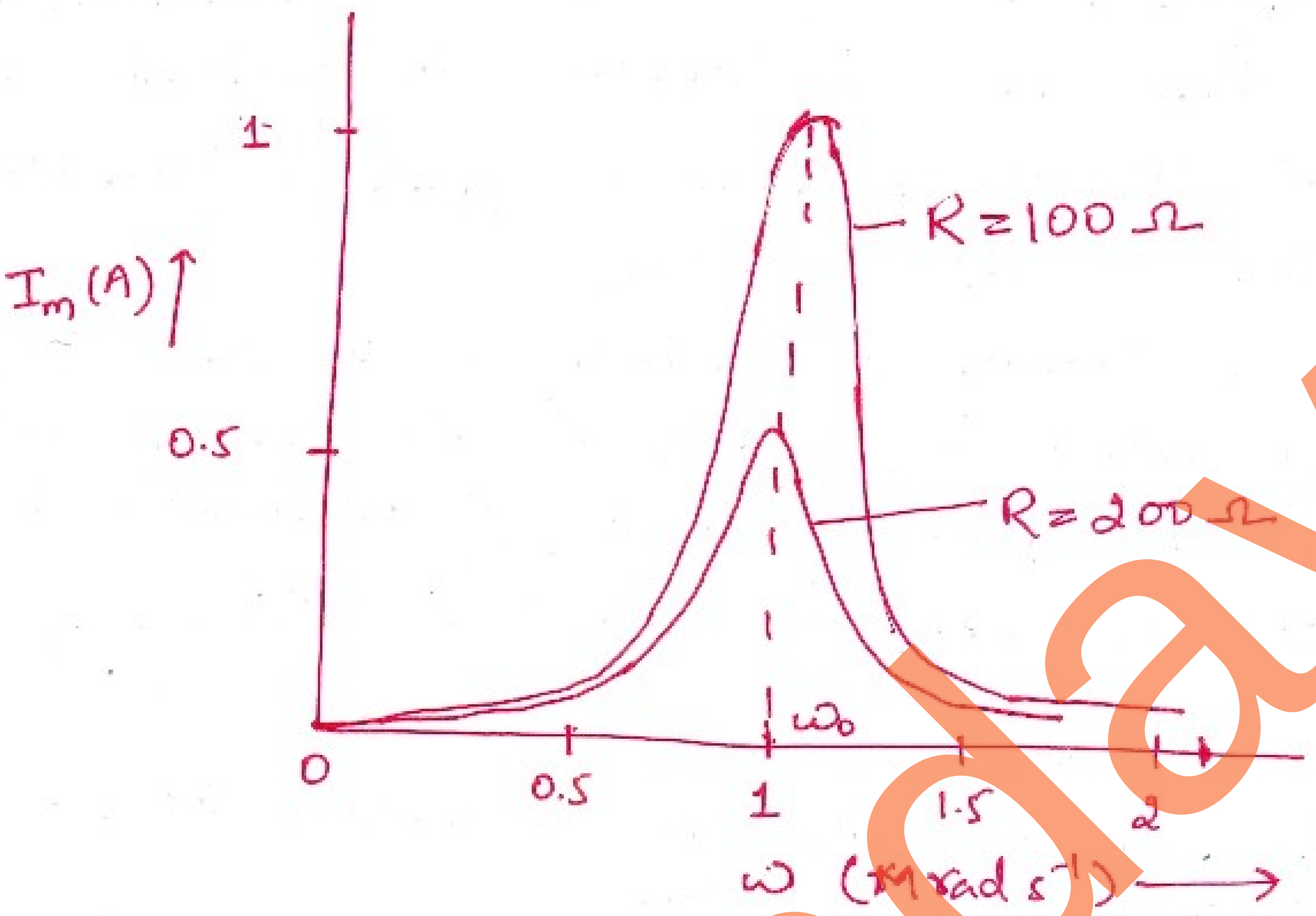
At resonant frequency, $I_m = \frac{V_m}{R}$ (max.)



Here is the variation of I_m with ω in a LCR circuit with $L = 1\,mH$, $C = 1\,nF$ & $V_m = 100\,V$

Here, $\omega_0 = \frac{1}{\sqrt{LC}} = 1 \times 10^6$ rad s^{-1}

from graph I_m (max) at ω_0 $\left[I_m = \frac{V_m}{R} = \frac{100}{100} = 1,\ \omega_0 = 1 \right]$

A.K. Yadav

Applications of electrical resonance

① Tuning mechanism of a radio or TV

→ The antenna of a radio accepts signals from many broadcasting stations.

→ To hear one particular radio station, we tune the radio.

→ In tuning, we vary the capacitance of a capacitor in the tuning circuit such that the resonant frequency of circuit becomes nearly equal to the frequency of radio signal received.

→ Hence resonance occurs & the amplitude of current with the frequency of signal from desired station becomes max. & it is received in our radio.

(2) Metal detector

→ When we walk through a metal detector, actually we are walking through a coil of many turns.

→ The coil is connected to a capacitor tuned so that the circuit is in resonance.

→ When you walk through with metal in your pocket, the impedance of circuit changes, which results in significant change in current in circuit.

→ This change in current is detected & alarm gets activated.

Q-Factor of Resonance circuit (Sharpness of resonance)

It is defined as ratio of voltage drop across inductor or capacitor to the applied voltage.

$$Q = \frac{\text{voltage across L or C}}{\text{applied voltage}}$$

$$= \frac{I X_L}{IR} = \frac{\omega_0 L}{R}$$

$$= \frac{1}{\sqrt{LC}} \frac{L}{R}$$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

→ R increases Q decreases.

→ The electronic circuits with high Q values would respond to very narrow range of frequencies & vice-versa.

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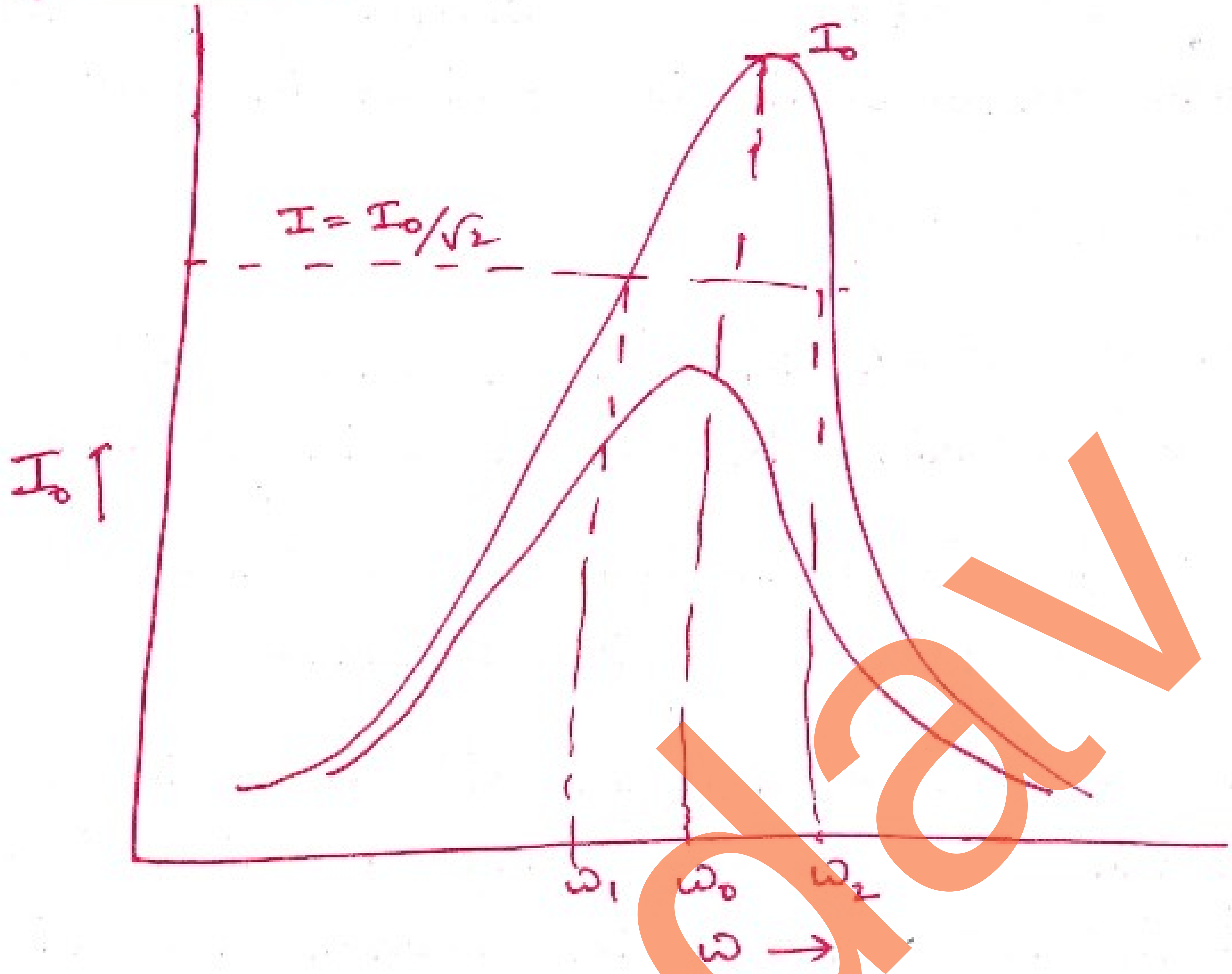
Another definition of Q Factor



→ Suppose we choose a value of ω for which the current amplitude is $\frac{1}{\sqrt{2}}$ times max. value.

→ From fig. we find there are 2 such values of ω symmetrical about ω_0 — ω_1 & ω_2

$$\omega_1 = \omega_0 - \Delta\omega$$
$$\omega_2 = \omega_0 + \Delta\omega$$

The diff. bet^n $(\omega_2 - \omega_1) = 2\Delta\omega$ is called <u>band-width</u> of the circuit.

→ The quantity $\frac{\omega_0}{2\Delta\omega}$ is called <u>sharpness of resonance</u>.

→ Large band-width → I_0 less → circuit close to resonance for large range of frequencies
↓
Tuning not good.

→ So, we require <u>smaller bandwidth</u> & <u>narrower resonance</u>.

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Energy stored in an inductor

Let an a.c. is applied to an inductor of inductance L. If I is the current at any instant t, then the magnitude of induced e.m.f is

$$e = L\frac{dI}{dt}$$

* The self induced e.m.f is also called back e.m.f as it opposes any change in the current in the circuit.

Now, the rate of doing work is

$$\frac{dW}{dt} = eI$$

$$\frac{dW}{dt} = LI\frac{dI}{dt}$$

$$dW = LI\,dI$$

Total work done is

$$W = \int_0^I LI\,dI = \frac{1}{2}LI^2$$

So, energy required to build up current in an inductor = Energy stored in inductor

$$\boxed{U_B = W = \frac{1}{2}LI^2}$$

LC oscillations

- A capacitor & an inductor can store electrical and magnetic energy respectively.
- When an initially charged capacitor is connected to an inductor, the charge on the capacitor & the current in the circuit exhibit the phenomenon of electrical oscillations called LC oscillations (similar to mechanical oscillations)

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Power in AC circuit

for an LCR circuit

$$E = E_0 \sin\omega t$$

$$\& I = I_0 \sin(\omega t + \phi)$$

where $I_0 = \dfrac{E_0}{Z}$, $\phi = \tan^{-1}\left(\dfrac{X_C - X_L}{R}\right)$

$\therefore$ The instantaneous power supplied by source is

$$p = VI$$
$$= E_0 \sin\omega t \times I_0 \sin(\omega t + \phi)$$
$$= \frac{E_0 I_0}{2}\left[\cos\phi - \cos(2\omega t + \phi)\right]$$

The average power over a cycle is

$$P = \frac{E_0 I_0}{2}\cos\phi$$

[$\because$ In $\cos(2\omega t + \phi)$ average power is 0 as the half of cos cancels $-$ve half of cos]

$$= \frac{E_0}{\sqrt{2}}\frac{I_0}{\sqrt{2}}\cos\phi$$

$$\boxed{P = VI\cos\phi} \quad \text{or} \quad \boxed{P = I^2 Z\cos\phi}$$

$\cos\phi$ – power factor

Case I → Resistive circuit , $\phi = 0$, $\cos\phi = 1$ max. power dissipation

Case II → Purely inductive or capacitive circuit

phase diff. betn V & I $-\frac{\pi}{2}$, $\cos\phi = 0$

no power dissipated even though current is flowing in the circuit. This current is called wattless current.

Case III Power dissipated at resonance in LCR circuit

$X_L = X_C$, $\phi = 0$, $\cos\phi = 0$, P – max

max. power dissipated through R

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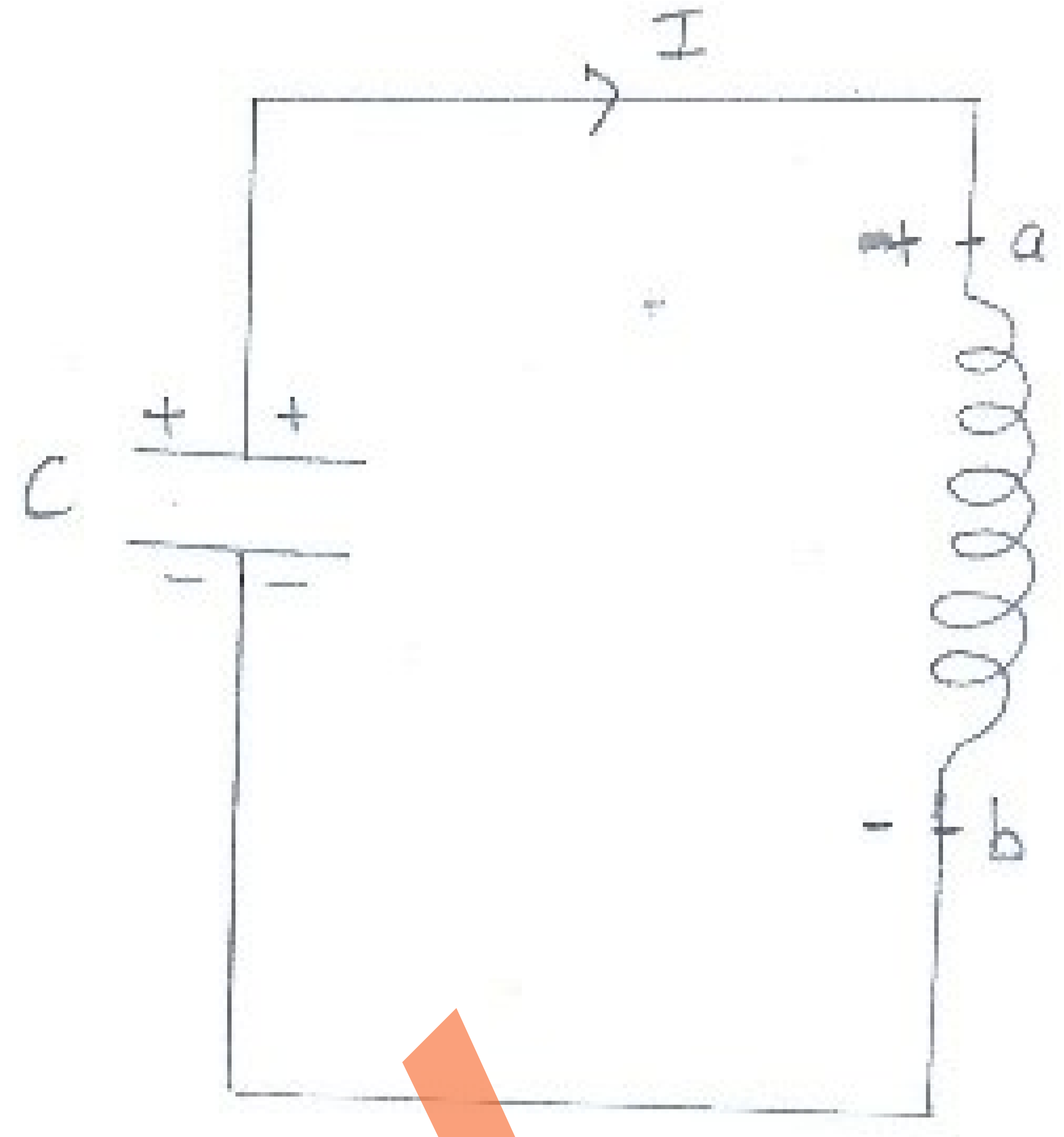
Let a capacitor be charged q_0 & connected to an inductor.



The charge on the capacitor starts decreasing, giving rise to current in the circuit.

Let q & I be the charge & current in the circuit at time 't'

As $\frac{dI}{dt}$ is positive, the induced e.m.f in L will have polarity as shown i.e. $V_b < V_a$

Acc. to Kirchoff's loop rule

$$\frac{q}{C} - L\frac{dI}{dt} = 0$$

As q decreases, I increases so $I = -\frac{dq}{dt}$

$$\therefore \quad \frac{q}{C} + L\frac{d^2q}{dt^2} = 0$$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

Eqn for S.H.M is $\frac{d^2x}{dt^2} + \omega^2 x = 0$

so, $\boxed{\omega = \frac{1}{\sqrt{LC}}}$ [Natural frequency with which the charge oscillates]

The variation of charge with time is

$$q = q_0 \cos\omega t$$

$$I = -\frac{dq}{dt}$$

$$\boxed{I = -\frac{d}{dt}(q_0 \cos\omega t) = q_0\omega \sin\omega t = I_0 \sin\omega t}$$

where $I_0 = q_0\omega$

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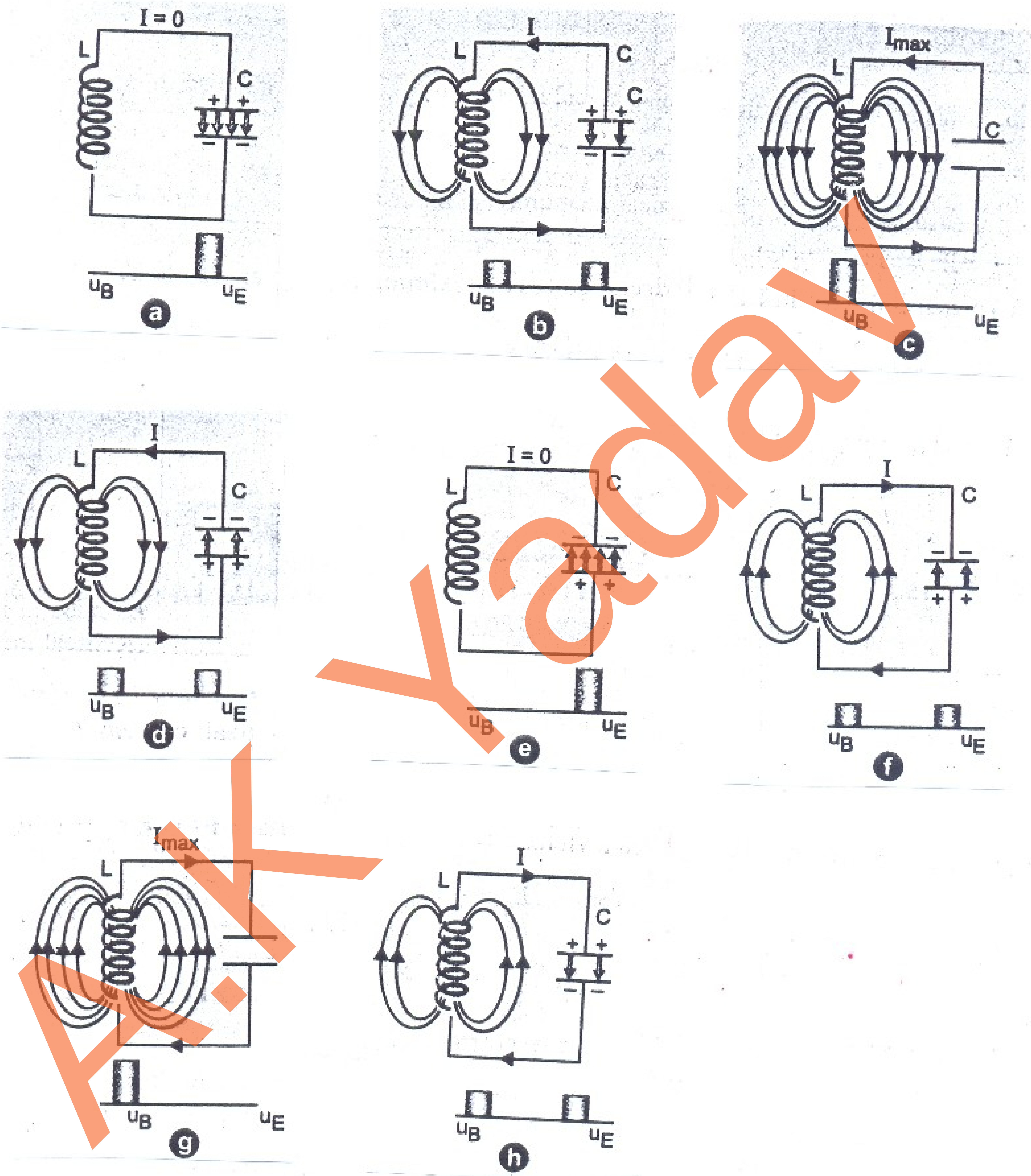

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Limitations of LC oscillations

① Every inductor has some resistance which introduces a damping effect on charge & current in the circuit. So, the oscillations finally die away.

② Even if resistance is zero, the total energy of system won't be constant. It is radiated away from the system in the form of e-m waves.

Advantages of A.C over D.C

(i) A.C. can be transmitted over long distances using step up transformers.

(ii) Can be easily converted to d.c.

(iii) A.C. is easier & cheaper to generate than d.c.

(iv) A.C. voltages can be easily varied using transformers

Drawbacks

(i) It is more dangerous to work with a.c. at high voltages. If insulation is faulty, one gets severe shock.

(ii) Shock of a.c. is attractive, d.c. is repulsive

(iii) Phenomenon like electroplating & electrorefining can't be done with a.c.

(iv) A.C. is transmitted more from the surface of conductor than from inside. So, several fine insulated wires are required for the transmission of a.c.

A.K. Yadav

Q. Show that in the free oscillations of an LC circuit the sum of energies stored in the capacitor and the inductor is constant in time.

Ans

Consider an LC circuit having oscillation with frequency $\omega = \frac{1}{\sqrt{LC}}$

At an instant t, $q = q_0 \cos\omega t$

$I = -q_0 \omega \sin\omega t$

Energy stored in the capacitor is

$$U_E = \frac{1}{2} C V^2 = \frac{1}{2} \frac{q^2}{C} = \frac{q_0^2}{2C} \cos^2\omega t$$

Energy stored in the inductor is

$$U_M = \frac{1}{2} L I^2 = \frac{1}{2} L q_0^2 \omega^2 \sin^2\omega t = \frac{q_0^2}{2C} \sin^2\omega t$$

$$U_E + U_M = \frac{q_0^2}{2C} (\cos^2\omega t + \sin^2\omega t)$$

$$= \frac{q_0^2}{2C}$$

$\therefore$ Sum is constant in time.

Transformer

It is an electrical device which is used for changing the a.c. voltages

step up transformer – increases the a.c. voltage

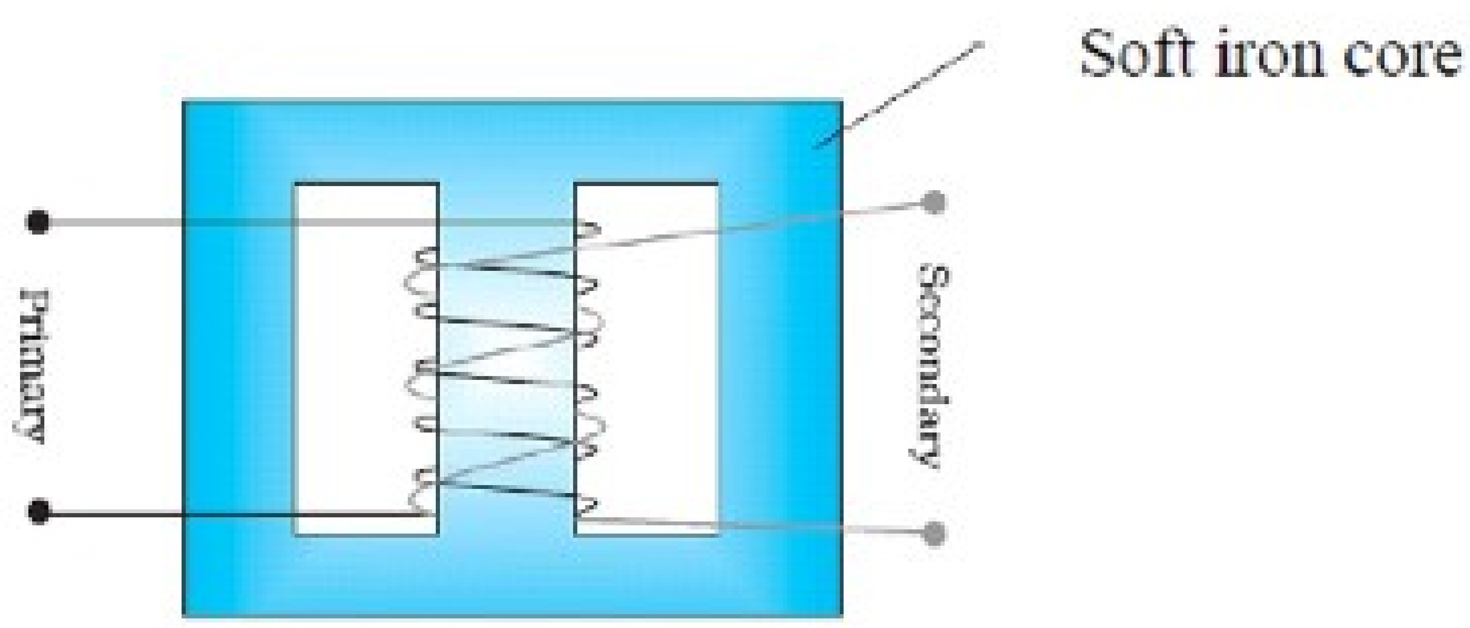
" down " – decreases " " "

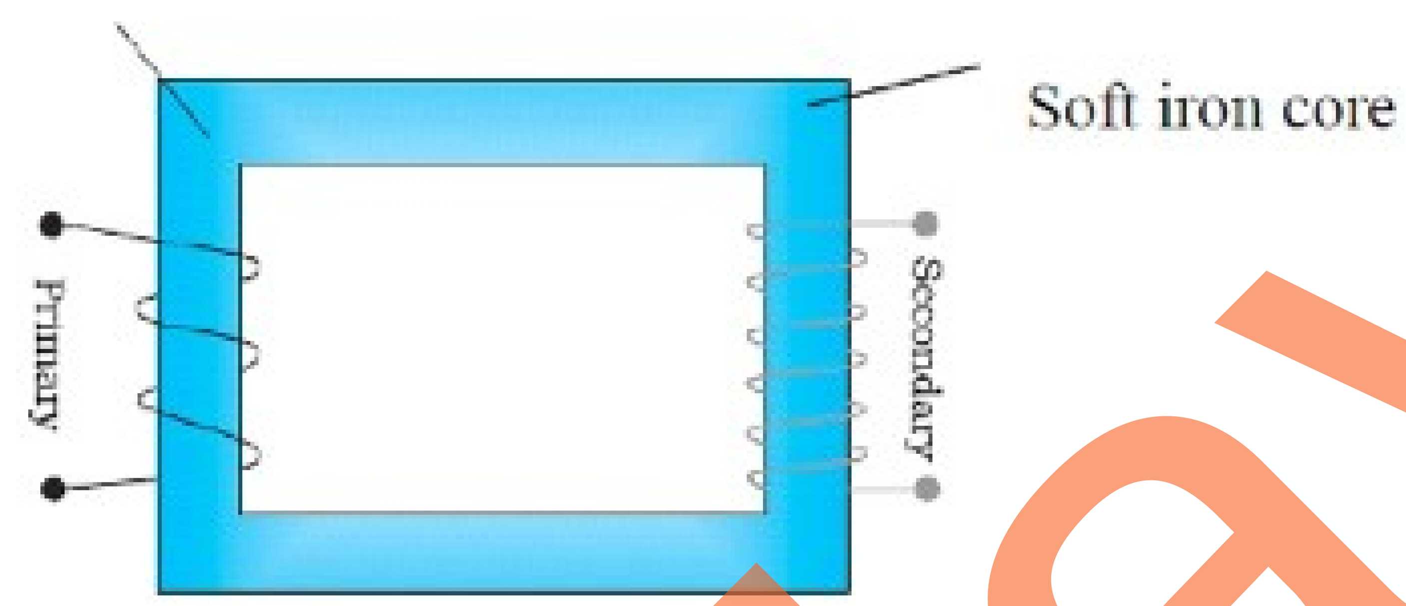
A. K. Yadav

Principle

It is based on the principle of mutual induction.

Construction





→ It consists of 2 sets of coils, insulated from each other, & wound on a soft iron core.

→ One coil is primary (input coil) having N_p turns.
other " " secondary (output ") " N_s " .

Assumptions

(i) the primary resistance & current are small
(ii) secondary current is small.
(iii) same magnetic flux links both primary & secondary

Theory & working

When an alternating e.m.f (voltage) is applied to the primary, the resulting current produces an alternating magnetic flux which links the secondary and induces an emf in it.

If ϕ is the flux in each turn of core when v_p is applied to primary then emf induced in secondary is

$$\mathcal{E}_s = -N_s \frac{d\phi}{dt}$$

A.K. Yadav

The emf induced in primary is

$$E_p = -N_p \frac{d\phi}{dt}$$

Now, $E_p = V_p$ & $E_s = V_s$, so

$$V_s = -N_s \frac{d\phi}{dt} \quad \& \quad V_p = -N_p \frac{d\phi}{dt}$$

$$\therefore \boxed{\frac{V_s}{V_p} = \frac{N_s}{N_p}}$$

$\frac{N_s}{N_p}$ - transformation ratio

If the transformer is assumed to be 100% efficient then

power input = power output

$$I_p V_p = I_s V_s$$

$$\therefore \boxed{\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p}}$$

(a) Step up transformer

for a step-up transformer $N_s > N_p$, so voltage is stepped up $V_s > V_p$ but less current $I_s < I_p$

eg: $N_p = 100$ turns, $N_s = 200$ turns, $V_p = 220V$, $I_p = 10A$

So, $\frac{N_s}{N_p} = 2$, $V_s = 440V$, $I_s = 5A$

(b) Step down transformer

$$\boxed{N_s < N_p \; ; \; V_s < V_p \; ; \; I_s > I_p}$$

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Energy losses in a transformer

① Copper loss

It is the energy loss in the form of heat in the copper coils of the transformer.

Solⁿ → minimised by using thick wires

② Iron loss

It is the energy loss in the form of heat in the iron core in the transformer due to formation of eddy currents in the core.

Solⁿ → laminated cores.

③ Hysteresis loss

It is the loss of energy due to repeated magnetisation & demagnetisation of the iron core when a.c is fed to it.

Solⁿ → using a magnetic material having low hysteresis loss.

④ Flux leakage

Despite best insulation, some flux leaks that is not all the flux due to primary passes through the secondary (It (leakage) can be due to poor design of core or due to air gaps in the core)

Solⁿ → Winding the primary & secondary coils over one-another.

Q Explain the large scale transmission & distribution of electrical energy over long distances using transformers.

Ans (i) Voltage output of generator is stepped up so that I is reduced & I^2R loss is cut down.

(ii) It is then transmitted over long distances to the sub-station

(iii) There voltage is stepped down.

(iv) It is further stepped down at distributing sub-stations & utility poles before power supply of 240 V reaches our homes

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