Current Electricity

Current carriers

The charged particles whose flow in a definite direction constitutes the electric current are called current carriers.

Current carriers in :

(a) Solids — valence electrons

Valence electrons of the atoms do not remain attached to individual atoms but are free to move throughout the volume of conductor.

Under the effect of external electric field, the valence electrons move in a definite direction causing flow of electric current.

(b) Liquids — positively & negatively charged ions.

(c) Gases — positive ions & electrons

Normally, gases are insulators of electricity.

But they can be ionized by applying high p.d. at low pressure or by exposure to X-rays. So, the ionized gas contains +ve ions & electrons.

Electric current (I)

Rate of flow of electric charge

$$I = \frac{q}{t} = \frac{dq}{t} = \frac{ne}{t}$$

Unit - Ampere (A)

$$1\,A = \frac{1\,C}{1\,s}$$

* Current is a scalar quantity as it does not follows laws of vector addition. [Angle betn wires carrying current does not affect the total current in the circuit]

E.M.F.

→ The potential difference betⁿ the 2 poles of the cell in an open circuit (when no current is drawn from the cell) is the e.m.f. of the cell.

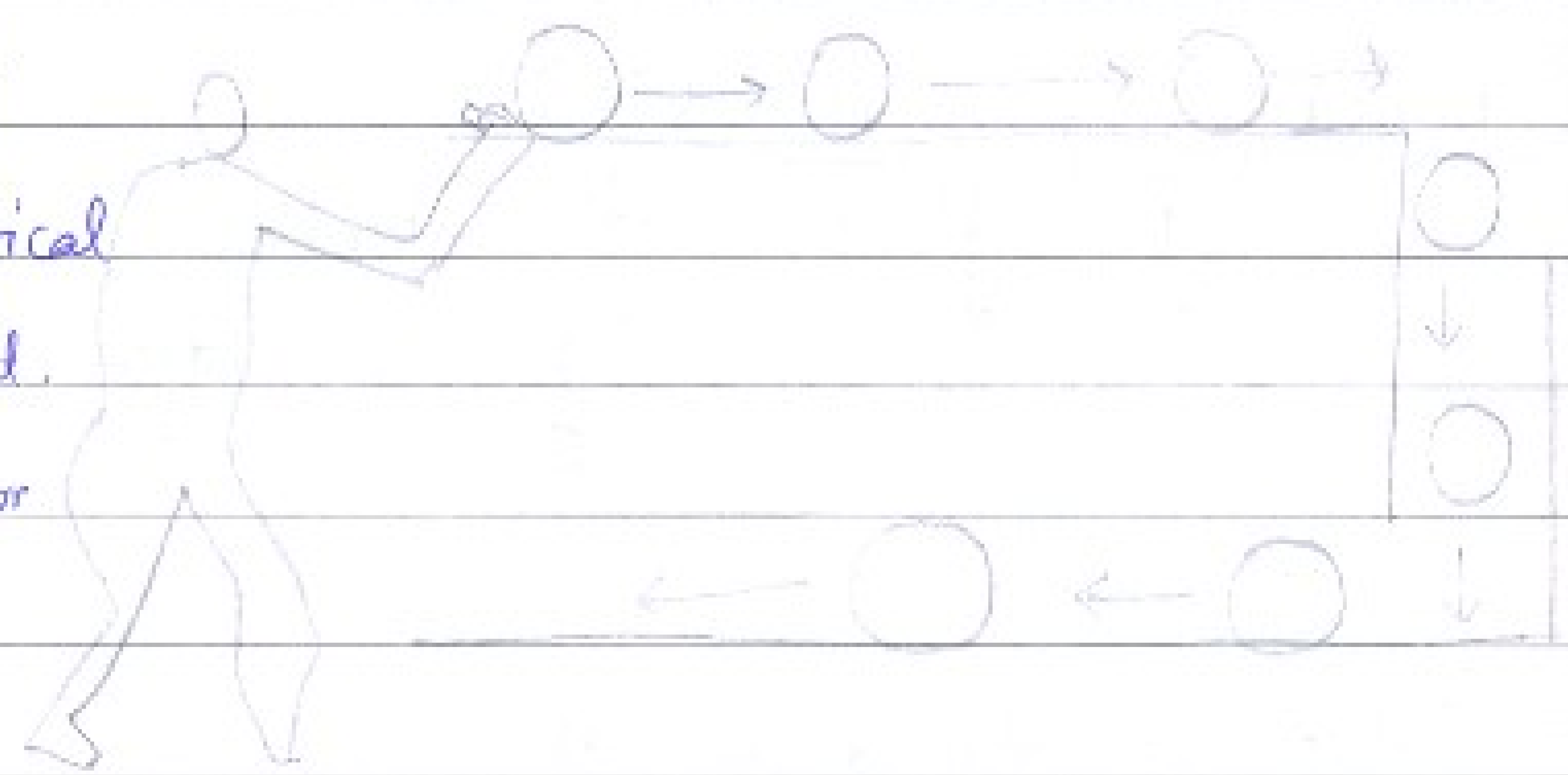
→ Due to its e.m.f., a cell drives the current carriers around the circuit, so e.m.f of a cell is also defined as the energy supplied by cell to drive a unit charge round the complete circuit.

S.I. unit – Volt (V) or JC^{-1}

The e.m.f. of a cell is called 1V, if 1J of work is performed by the cell to drive 1C of charge around the circuit.

* The word e.m.f. is a misnomer, as it doesn't represent force on current carriers. Instead it represents the work done per unit charge to drive the carriers of electricity.

Mechanical analogy of e.m.f



→ Iron balls dropped at steady rate through vertical column of viscous liquid.

→ Man lifts ball from floor & transfers to top.

→ Thus balls go through the closed path at a steady rate in the same manner as electric charges flow through a closed circuit at steady speed causing steady current.

→ So, man is the source of e.m.f

* Transferring the ball from bottom to top is at the cost of man's own internal chemical energy.

Drift velocity

→ Metal conductor has a large no. of free electrons whose no. density (no. of electrons / volume) is $10^{29}\ m^{-3}$.

→ These electrons move randomly within the body of conductor, like molecules of a gas.

→ The directions of motion of these free electrons are so randomly distributed that the average thermal velocity of electrons is zero.

If $\vec{u}_1, \vec{u}_2, \ldots \vec{u}_n$ are random thermal velocities of n free electrons in the metal conductor, then average thermal velocity of electrons is

$$\frac{\vec{u}_1 + \vec{u}_2 + \ldots + \vec{u}_n}{n} = 0$$

So, no net flow of electrons in one direction in conductor & hence no current.

→ When some p.d. is applied across the 2 ends of conductor, an electric field is set up inside the conductor

→ Due to this, the free electrons in the conductor experience a force in a direction opposite to that of electric field & are accelerated from −ve to +ve end of conductor.

→ On their way, the accelerated free electrons suffer frequent collisions against the atoms of the conductor & lose their gained kinetic energy.

→ After each collision, the free electrons are again accelerated due to electric field towards the positive end of the conductor & lose the gained K.E. in the next collision with the atoms of the conductor.

→ This process continues till the electrons reach the +ve end of conductor.

→ Thus under the effect of electric field, the free electrons accelerate and acquire a velocity component (in a direction opp. to electric field) in addition to their thermal velocities.

→ However the gain in velocity due to electric field takes place for a very short time because it is lost in the next collision with atoms of the conductor.

"The short time, for which a free electron accelerates before it undergoes a collision with the +ve ion/atom of the conductor is called relaxation time(τ)"

So, an electron having random thermal velocity $\vec{u}_1$ accelerates for a time τ_1 (before it suffers collision) then it will attain a velocity

$$\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$$

Similarly,

$$\vec{v}_2 = \vec{u}_2 + \vec{a}\tau_2$$
$$\vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3$$
$$\vec{v}_n = \vec{u}_n + \vec{a}\tau_n$$

Now, drift velocity (v_d) is defined as the average velocity with which free electrons in a conductor get drifted towards the positive end of the conductor under the influence of external electric field applied.

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \cdots + \vec{v}_n}{n}$$

$$= \frac{(u_1 + a\tau_1) + (u_2 + a\tau_2) + \cdots + (u_n + a\tau_n)}{n}$$

$$= \frac{\vec{u}_1 + \vec{u}_2 + \cdots + \vec{u}_n}{n} + \vec{a}\,\frac{\tau_1 + \tau_2 + \cdots + \tau_n}{n}$$

$$\vec{v}_d = 0 + \vec{a}\tau$$

where $\tau = \dfrac{\tau_1 + \tau_2 + \cdots + \tau_n}{n}$

↘ average relaxation time

$$\vec{v}_d = \vec{a}\tau \quad \text{———} \quad (1)$$

If V is the p.d applied across the ends of the conductor of length l, the magnitude of electric field set up is

$$E = \frac{\text{p.d.}}{\text{length}} = \frac{V}{l}$$

Also, $E = \dfrac{\vec{F}}{\vec{q}}$

So, force experienced by free electron, $\vec{F} = -e\vec{E}$

If m is the mass of electron, then the acceleration produced is

$$\vec{a} = \frac{-e\vec{E}}{m}$$

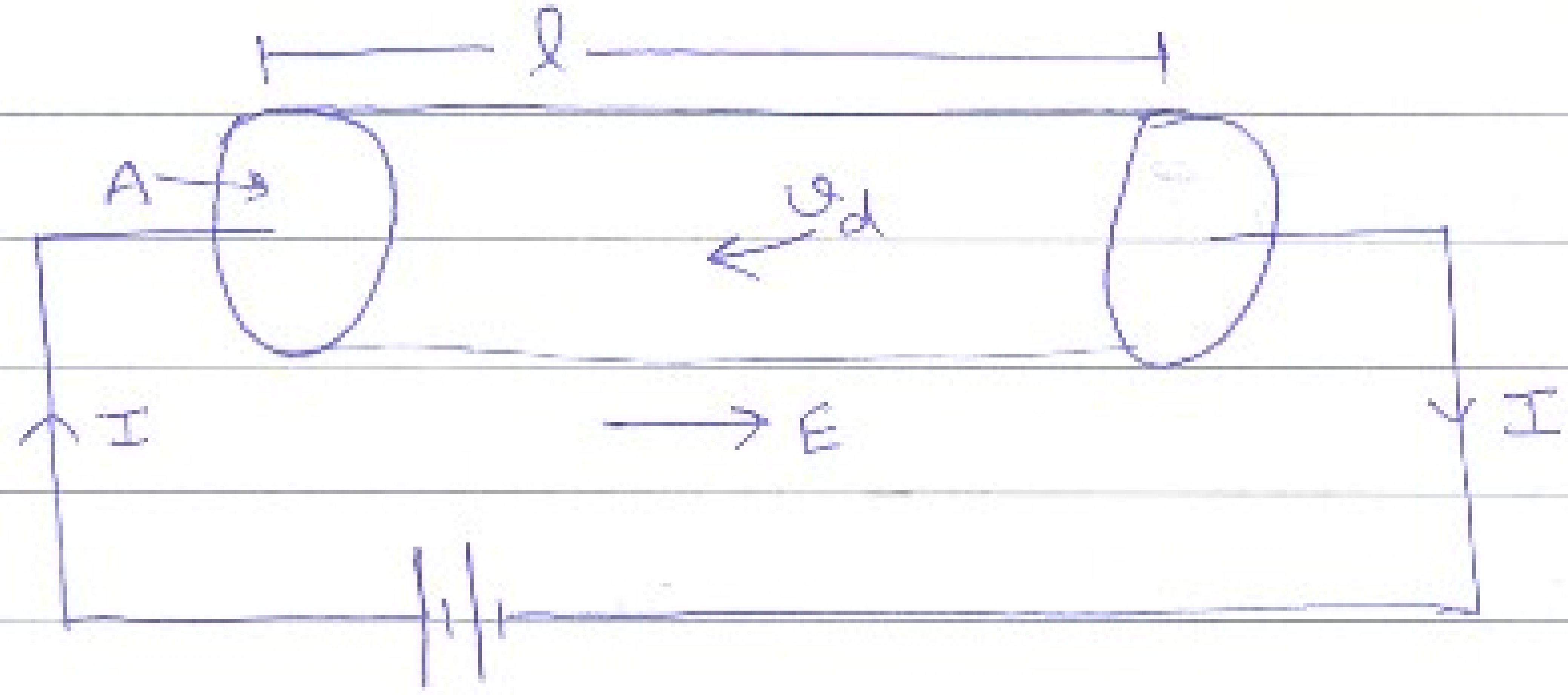
$$\therefore \quad \boxed{\vec{v}_d = \frac{-e\vec{E}}{m}\tau}$$

Mobility - It is defined as the magnitude of drift velocity of charge per unit electric field applied.

$$\boxed{\mu = \frac{v_d}{E} = \frac{qE\tau/m}{E} = \frac{q\tau}{m}}$$

Unit → $m^2 s^{-1} V^{-1}$

Relation betⁿ v_d & I



Consider a conductor of length 'l' & area 'A'.
∴ Volume of conductor $= Al$

Suppose that the conductor contains 'n' free electrons per unit volume.
So, no. of free electrons in the conductor $= nAl$

If e is the charge on an electron, then total charge on all free electrons in conductor

$$q = Alne$$

When a battery is connected across the 2 ends of the conductor, an electric field is set up.
If the electrons drift towards the +ve end of conductor with drift velocity v_d, then time taken by free electrons to cross the length of conductor is

$$t = \frac{l}{v_d}$$

Now, $I = \frac{q}{t} = \frac{Anle}{l} \times v_d$

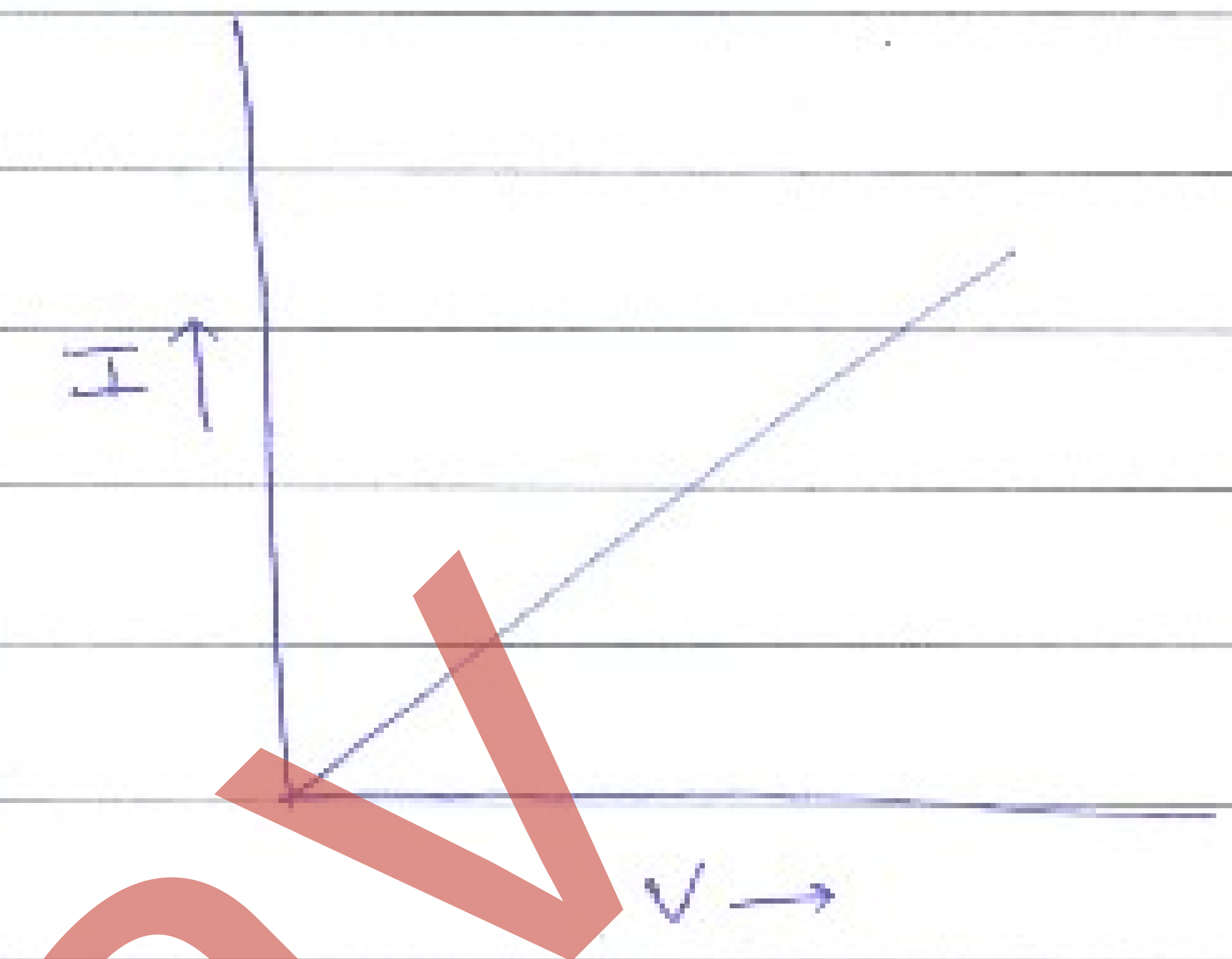
$$\boxed{I = Anev_d}$$

Ohm's law

The current flowing through a conductor is directly proportional to the potential difference across the ends of the conductor, provided that the physical conditions of conductor are kept constant.

$$V \propto I$$

$$\boxed{V = IR}$$



where R - Resistance

Deduction of Ohm's law

$$v_d = \frac{eE\tau}{m}$$

$$= \frac{eV\tau}{ml} \quad [E = V/l]$$

Also, $I = Ane v_d$

$$= Ane \times \frac{eV\tau}{ml}$$

$$I = \left(\frac{Ane^2\tau}{ml}\right) V$$

or $\boxed{\frac{V}{I} = \frac{ml}{Ane^2\tau} = \text{constant} = R}$

$$\boxed{V = IR}$$

Electrical resistance (R)

It is the ratio of potential difference across the ends of the conductor to the current flowing through it.

$$R = \frac{V}{I}$$

S.I. unit – ohm (Ω)

$$1\,\Omega = \frac{1\,V}{1\,A}$$

Dimensional formula

$$R = \frac{P.d.}{I} = \frac{W/q}{I} = \frac{ML^2T^{-2}/AT}{A} = [ML^2T^{-3}A^{-2}]$$

Cause of Resistance

P.d. applied across ends of conductor

↓

electric field is set up across the conductor

↓

free electrons gets accelerated

↓

As electrons move, they collide against the ions & atoms & thus their motion is opposed.

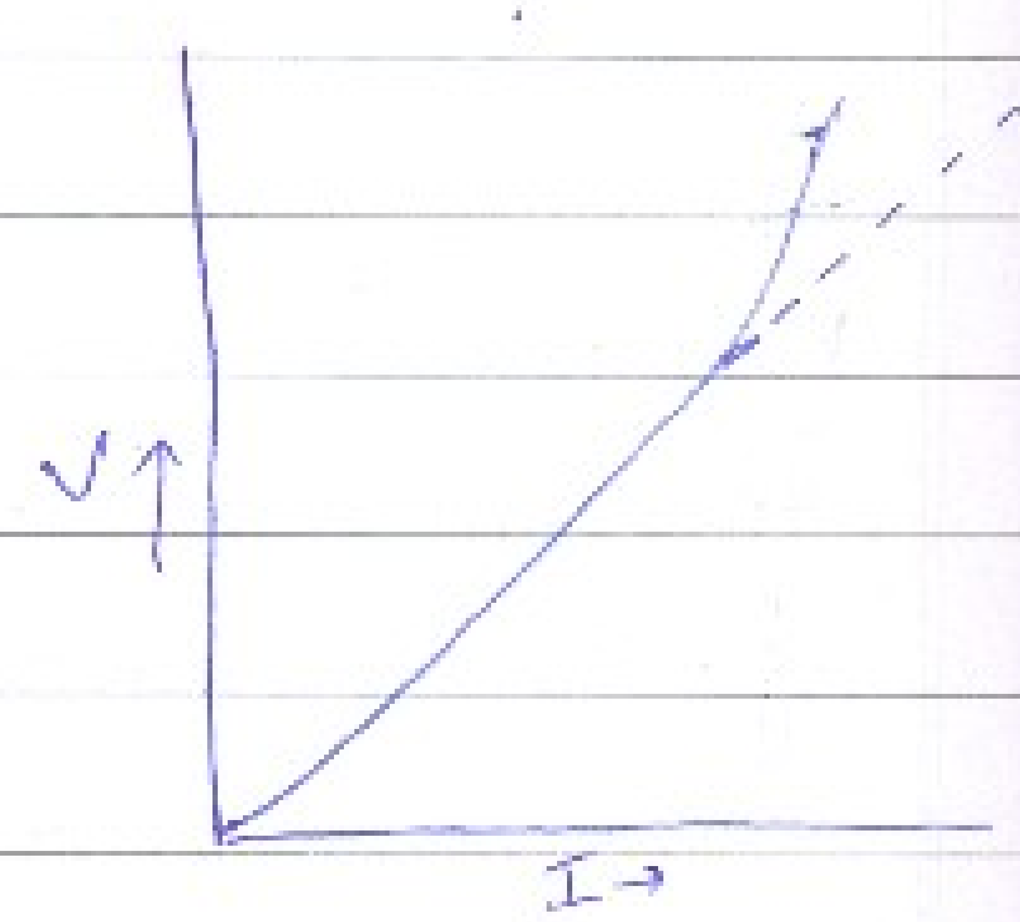
↓

This opposition offered by ions & atoms is termed as resistance of conductor.

* R depends upon tem., length, shape & nature of material of conductor.

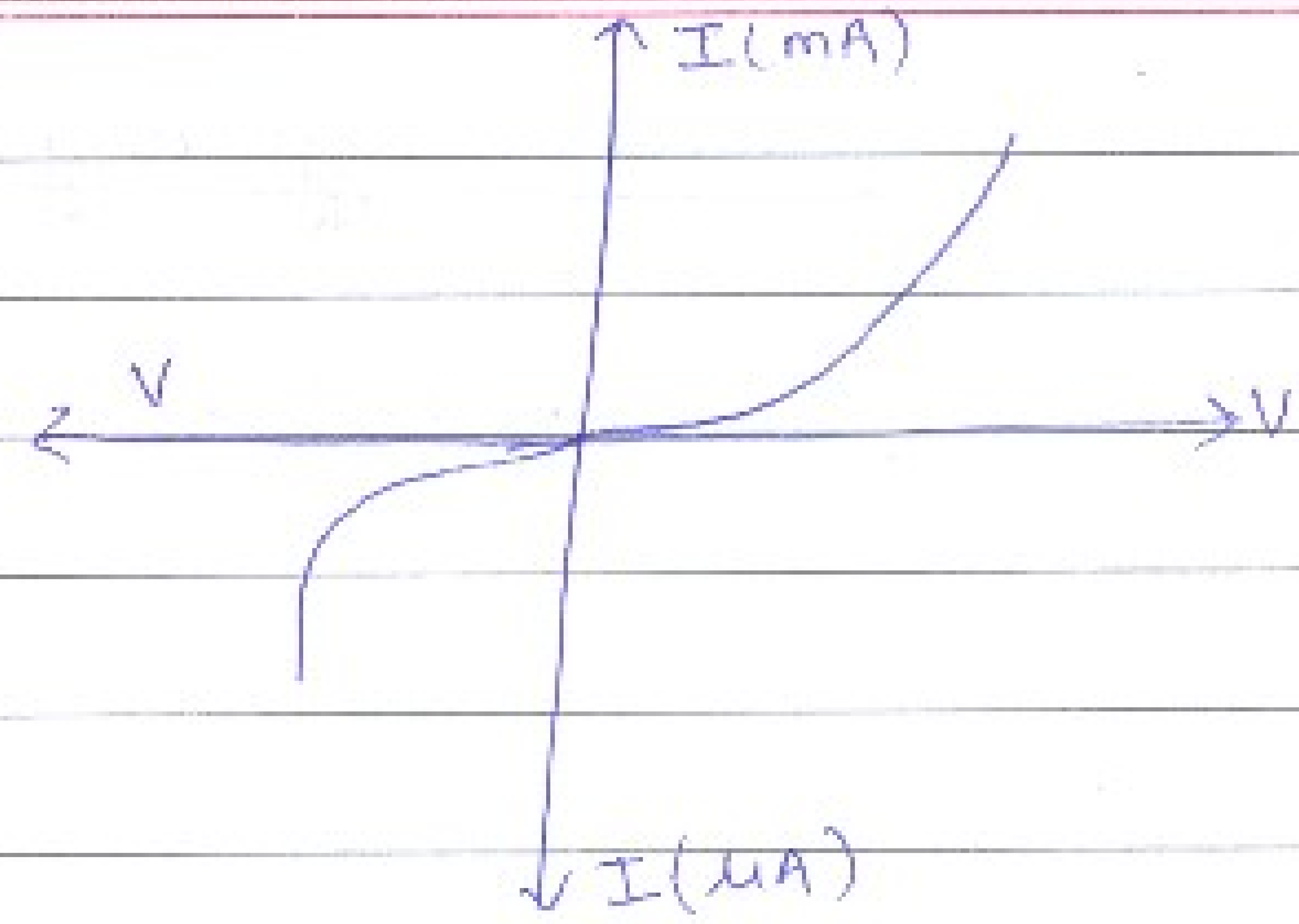
Limitations of Ohm's law

(1) Relation betn V & I is non-linear
[It happens due to heating effect of current]



(2) The relation betⁿ V & I depends on the sign of V.

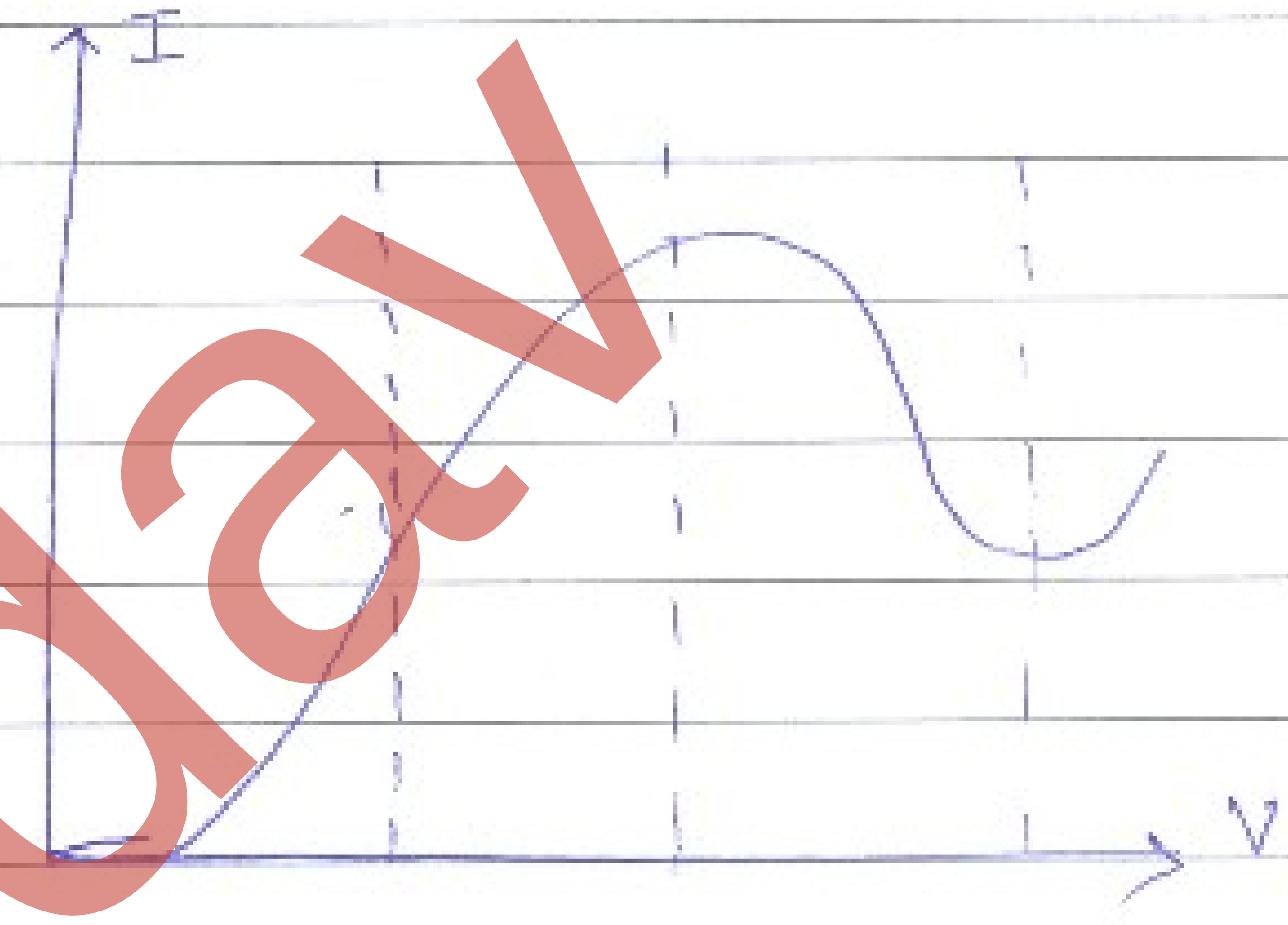
eg Junction diodes.



(3) The relation betⁿ V & I is not unique.

i.e. There is more than one value of V for the same current I.

eg GaAs (LED)



Resistivity (ρ)

$R \propto l$

$R \propto \frac{1}{A}$

$R \propto \frac{l}{A}$

$$\boxed{R = \rho \frac{l}{A}}$$

If $l = 1m$ & $A = 1m^2$, $\boxed{R = \rho}$

→ The resistivity of a substance is numerically equal to the resistance of a wire of length 1m and area $1m^2$.

Unit → Ωm

Dimension → $ML^3T^{-3}A^{-2}$

Factors affecting resistivity

$$R = \frac{\rho l}{A} \quad \text{--- (1)}$$

Also $R = \frac{V}{I} = \frac{ml}{Ane^2\tau} = \frac{m}{ne^2\tau}\frac{l}{A}$ --- (2)

from (1) & (2)

$$\boxed{\rho = \frac{m}{ne^2\tau}}$$

(1) $\rho \propto \frac{1}{n}$

n depends upon nature of material so, ρ depends upon nature of material

(2) $\rho \propto \frac{1}{\tau}$

τ depends upon tem. of conductor so, ρ (changes with) depends upon tem.

Increase in tem → τ decrease → ρ increase

Effect of tem. on resistance

$$R = \frac{m}{ne^2\tau}\frac{l}{A} \Rightarrow R \propto \frac{1}{\tau}$$

Tem of conductor increased → atoms/ions of conductor vibrate with greater amplitude & frequency about their mean position
↓
Due to increase in thermal energy, frequency of collision with atoms while drifting increases.
→ τ decreases but n doesn't decrease appreciably → R increases.

Variation of electrical resistivity with tem.

$$\rho = \frac{m}{ne^2\tau}$$

$$\boxed{\rho \propto \frac{1}{n\tau}}$$

(a) In most metals, n does not change with tem but τ decrease with increase in tem. (same explanation as in resistance)

So, ρ increases with increase in tem.

The tem. dependance of resistivity of a metal is given by the relation

$$\rho = \rho_0 [1 + \alpha (T - T_0)]$$

where ρ - resistivity at tem. T

ρ_0 - " " T_0

α - tem. coefficient of resistivity.

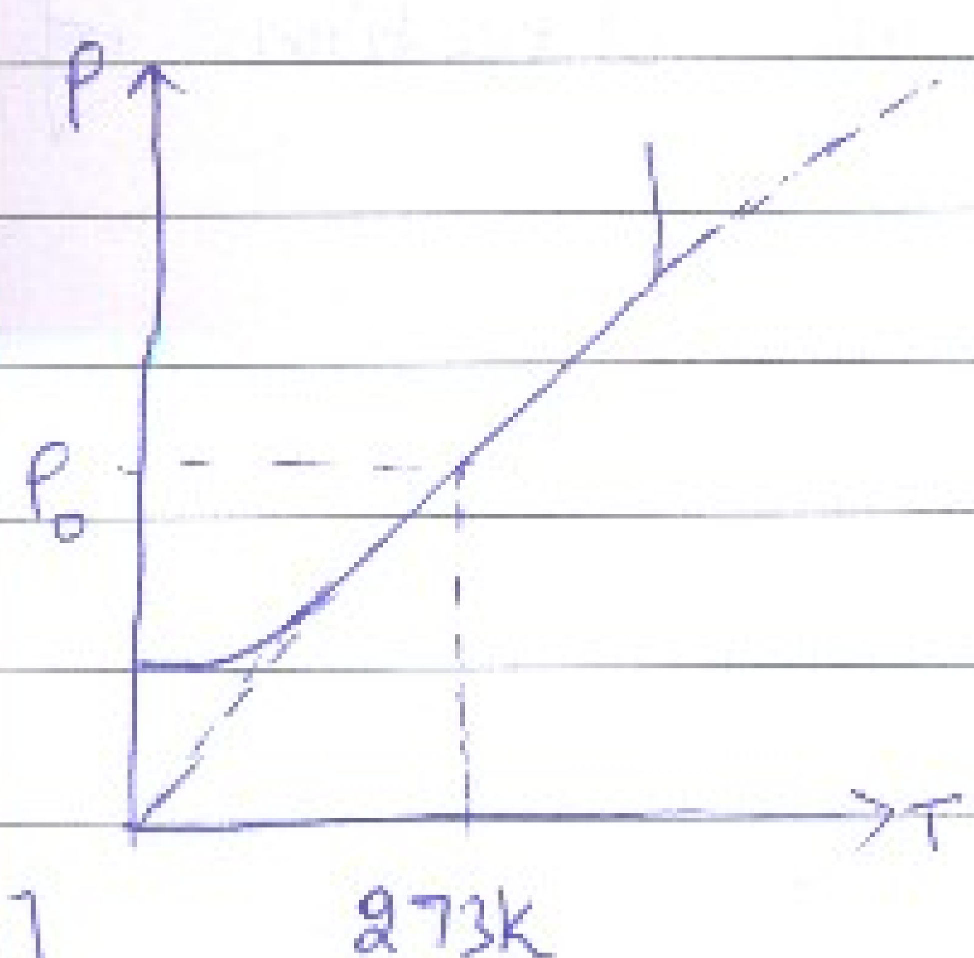
$$\boxed{\alpha = \frac{\rho - \rho_0}{\rho_0 (T - T_0)}}$$

* α metal conductor, α = +ve showing ρ increases with tem.

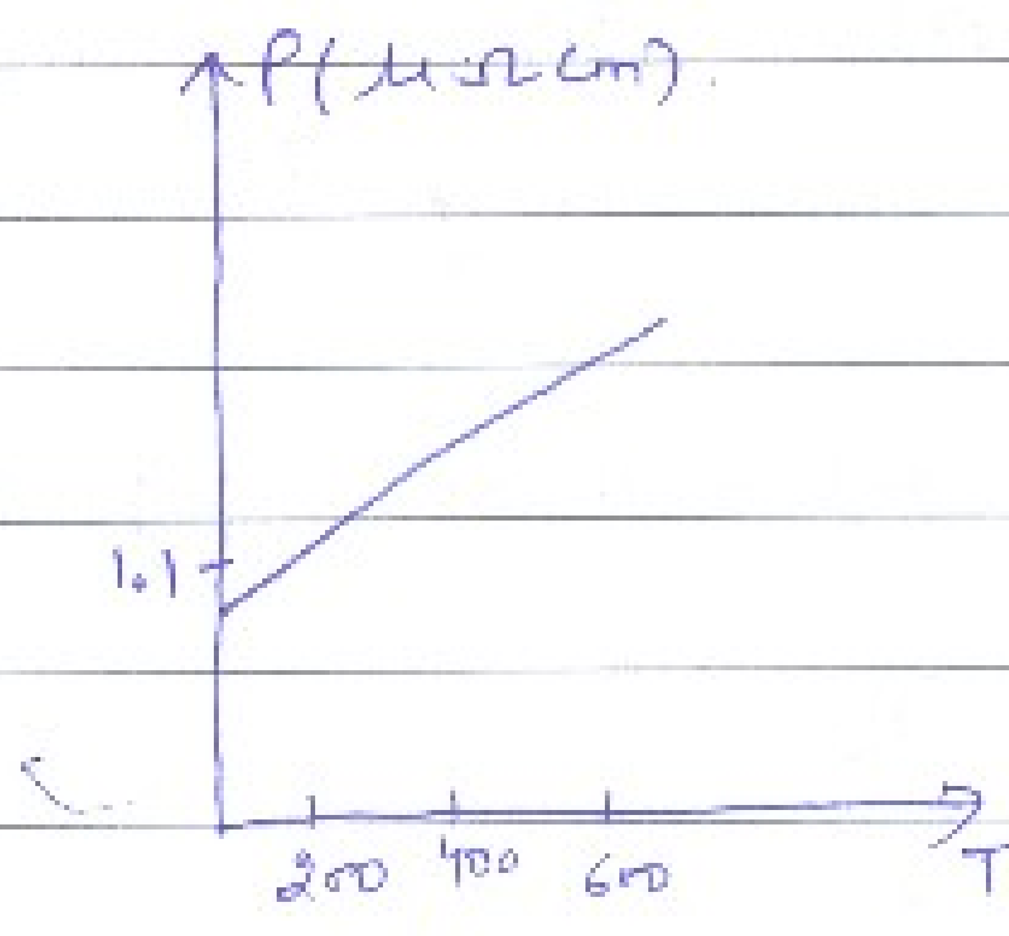
Semi-conductor = −ve " " decreases " "

Insulators = −ve " " " " "

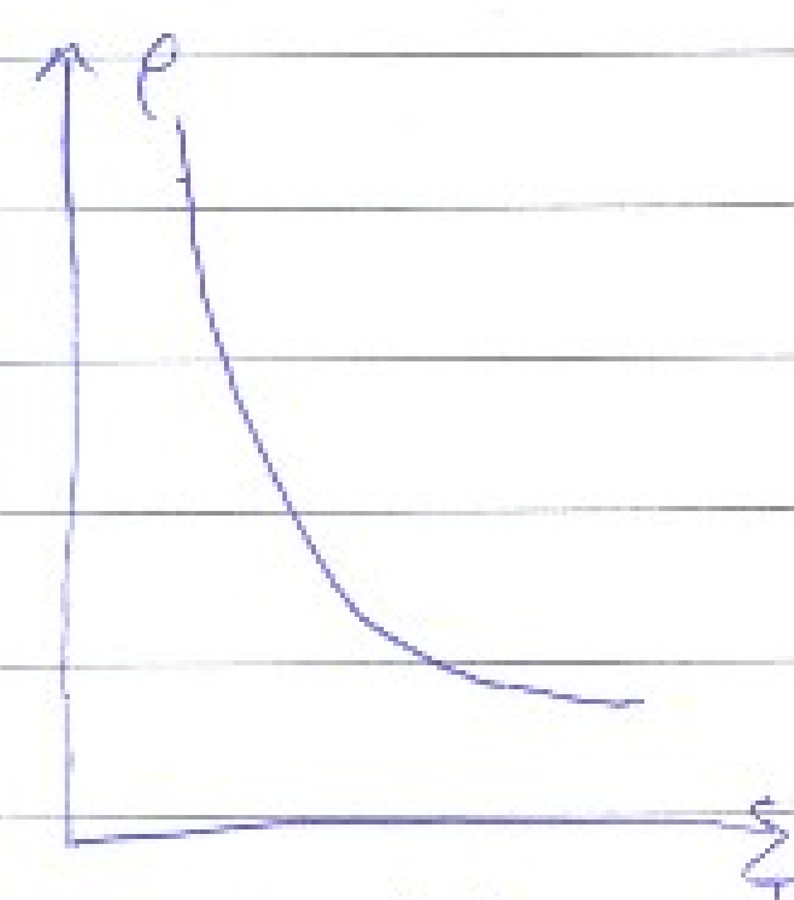
[In metals ρ increases with tem. over a tem range of 500k]



Metals



Alloys



Semiconductors & insulators

(b) For semi conductors, n increases with tem. [More than decrease in τ]

τ decreases " "

So, ρ decreases with increase in tem.

Q Why magnanin & constantan are used in making standard resistance coils?

Ans (a) small value of α (tem. coefficient of resistance)

(b) high resistivity

(c)

* Tem. coefficient of resistance (α)

$$\alpha = \frac{R_t - R_0}{R_0 \times t} = \frac{\text{increase in resistance}}{\text{original resistance} \times \text{rise in tem}}$$

Current density (J)

It is defined as the amount of current flowing per unit area of conductor provided that area is held perpendicular to current

$$J = \frac{I}{A}$$

$$= \frac{\cancel{A} n e v_d}{\cancel{A}}$$

$$\boxed{J = n e v_d}$$

Unit → Am^{-2}

* Vector quantity [Its direction is the direction of motion of +ve charge.]

Conductance (G) - inverse of resistance

$$G = \frac{1}{R}$$

Unit - mho or Siemen

Electrical conductivity (σ) - inverse of ρ

$$\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$$

unit - mho m^{-1} or Sm^{-1}

Relation betⁿ J, σ & E

$$I = Anev_d$$

$$= Ane\frac{eE\tau}{m}$$

$$I = \frac{Ane^2E\tau}{m}$$

$$\frac{I}{A} = \frac{ne^2E\tau}{m}$$

$$\boxed{J = \sigma E}$$

microscopic form of ohm's law

Relation betⁿ ρ & μ (mobility)

$$J = \sigma E = \frac{E}{\rho}$$

$$\frac{I}{A} = \frac{E}{\rho}$$

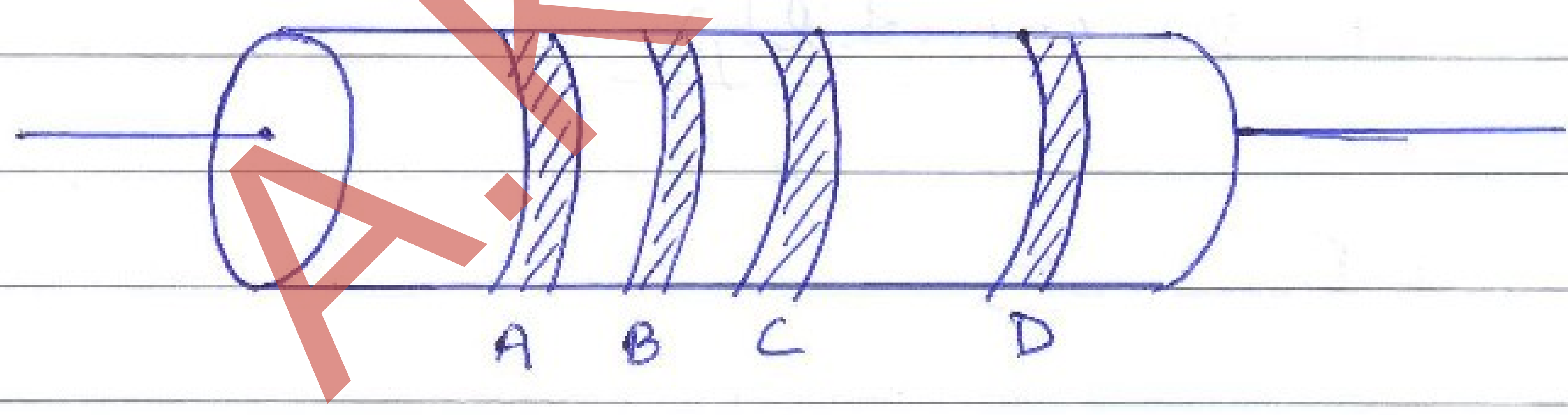
$$nev_d = \frac{E}{\rho}$$

$$ne\mu\cancel{E} = \frac{\cancel{E}}{\rho} \quad \because v_d = \mu E$$

$$\boxed{\rho = \frac{1}{ne\mu}}$$

Colour code for carbon resistors

Colour	Number	Multiplier	Tolerance	Memory Acronym
Black	0	10^0		B
Brown	1	10^1		B
Red	2	10^2		R
Orange	3	10^3		O
Yellow	4	10^4		Y
Green	5	10^5		Great
Blue	6	10^6		Britain
Violet	7	10^7		Very
Grey	8	10^8		Good
White	9	10^9		Wife
Gold		10^{-1}	$\pm 5\%$	Gold
Silver		10^{-2}	$\pm 10\%$	Silver
No colour			$\pm 20\%$	



Colour strip A – 1^{st} significant figure of resistance (in ohm)
B – 2^{nd} " " "
C – multiplier
D – tolerance

Q. For a given carbon resistor, let the first strip be yellow, second strip be red, third strip be orange and fourth be gold. What is its resistance?

Ans. $42 \times 10^3 \Omega \pm 5\%$

EMF, Internal resistance and terminal P.D. of a cell

EMF of a cell is the max. p.d. betⁿ 2 electrodes of the cell when no current is drawn from cell.

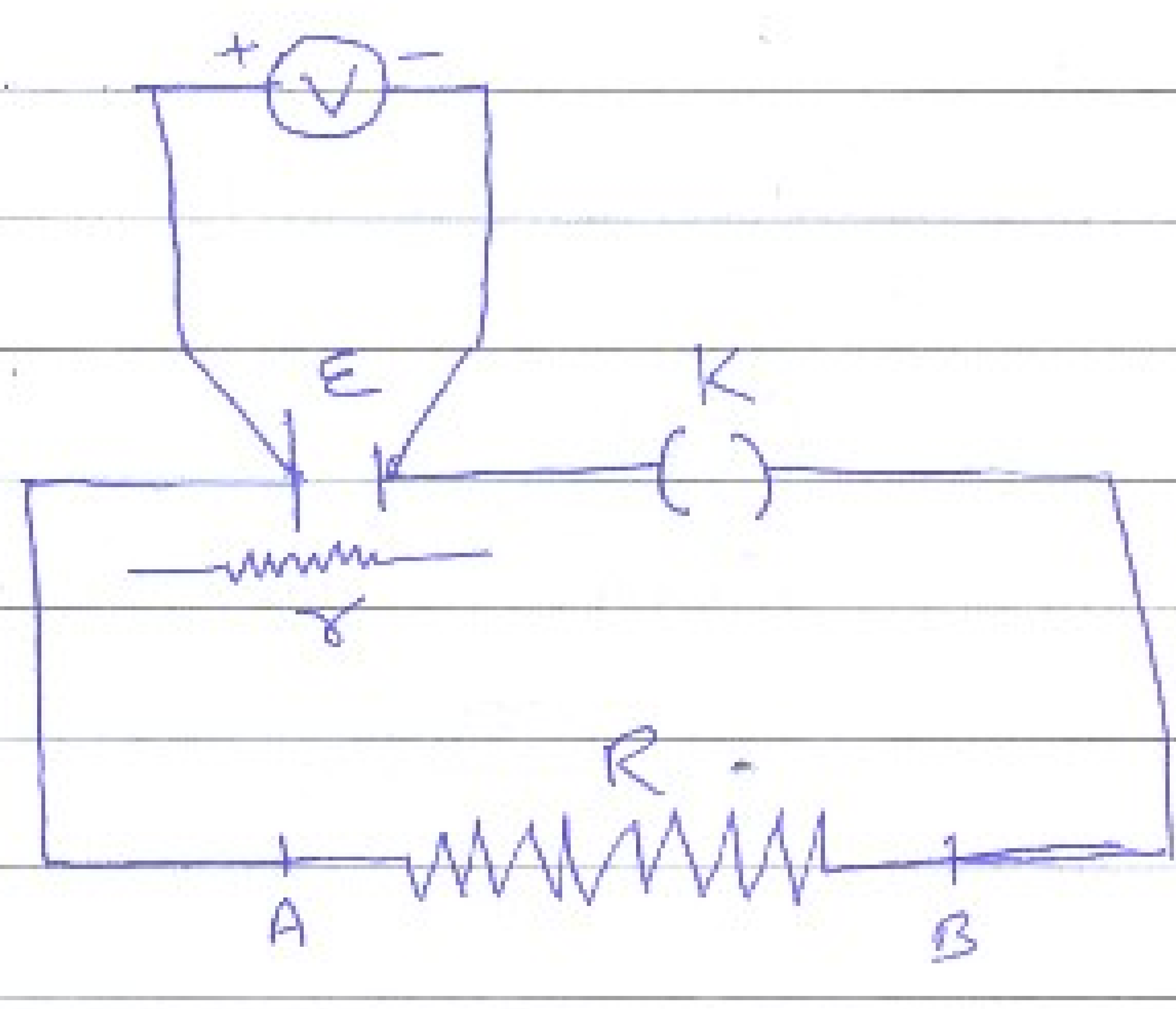
Internal resistance - resistance offered by electrodes & electrolyte when current flows through the cell.

Internal resistance depends on :
(a) distance betⁿ electrodes
(b) nature, conc. & tem. of electrolyte
(c) nature of electrodes
(d) area of electrodes.

Terminal potential difference (V)
It is defined as the potential difference betⁿ the 2 electrodes of a cell in a closed circuit.

→ When the electric cell is in a closed circuit, the current flows through the circuit.
→ Potential across the internal resistance of cell decreases
→ Terminal p.d. betⁿ the 2 electrodes of cell becomes less than emf of the cell.

Consider a cell of emf 'E' & internal resistance 'r' connected to external resistance 'R' & one way key 'K'
A high resistance voltmeter 'V' is connected across the 2 terminals of the cell.



- K not closed → circuit open → no current
- Reading of $V = \mathcal{E}$

K is closed, current flows through the circuit

Total resistance $= R + r$

current, $I = \dfrac{\mathcal{E}}{R+r}$

P.d. across $r = Ir$

$$\boxed{V = \mathcal{E} - Ir} \quad (1)$$

Now, terminal p.d. of a cell is equal to the p.d. across the external resistance R of the circuit, so

$$V = IR$$

$$I = \frac{V}{R}$$

from (1)

$$\boxed{r = \frac{\mathcal{E} - V}{V} \times R}$$

* The difference of emf & terminal voltage is called lost voltage as it is not indicated by voltmeter. It is equal to 'Ir'.

Grouping of Cells

Cells in Series

(a) When cells are of same e.m.f. & internal resistance

Total e.m.f. of cells = $E + E + \dots$ n times $= nE$

" internal resistance " $= r + r + \dots$ n times $= nr$

Total resistance of circuit $= R + nr$

$$I = \frac{\text{total e.m.f}}{\text{total resistance}} = \frac{nE}{R+nr}$$

If $R << nr$, $R + nr \approx nr$

$$I = \frac{nE}{nr} = \frac{E}{r} = \text{current due to single cell}$$

If $R >> nr$, $R + nr \approx R$

$I = \frac{nE}{R} =$ n times the current due to single cell.

"So, in order to have max. current, the cells should be connected in series, when the total internal resistances of the cells is negligible as compared to external resistance in the circuit."

(b) When cells are of different e.m.fs & internal resistances

Now, terminal p.d across first cell, $V_1 = E_1 - Ir_1$

& " " " 2^{nd} " , $V_2 = E_2 - Ir_2$

If V is the p.d. betn A & B

$$V = V_1 + V_2$$
$$= E_1 - Ir_1 + E_2 - Ir_2$$
$$V = (E_1 + E_2) - I(r_1 + r_2) \quad \text{—— (1)}$$

If E - effective emf & r - effective internal resistance of series combination of 2 cells then

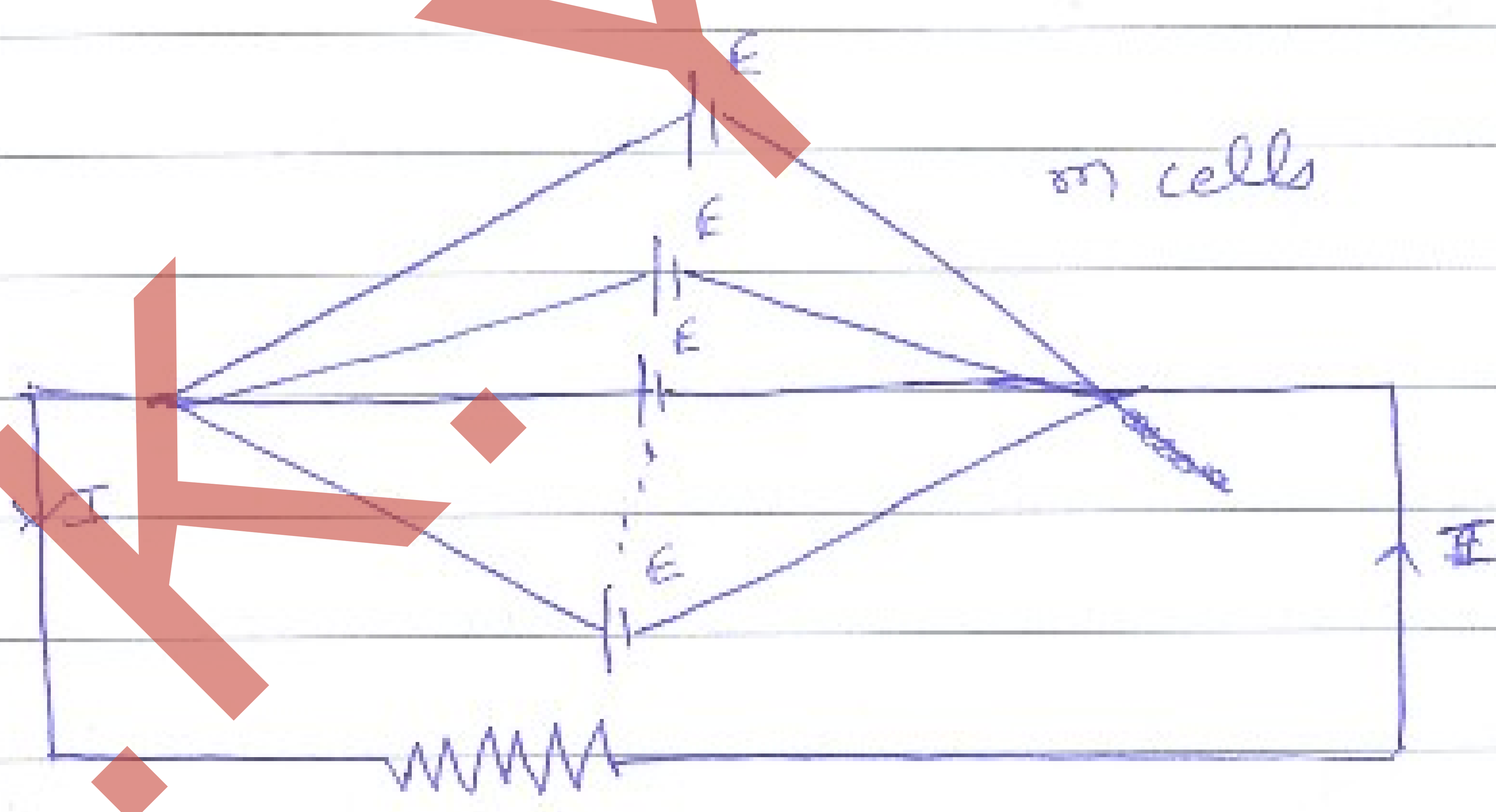
$$V = E - Ir \quad \text{—— (2)}$$

from (1) & (2)

$$E = E_1 + E_2$$
$$r = r_1 + r_2$$

Cells in parallel

(a) When cells are of same emf & internal resistance



Total e.m.f = E

Total internal resistance, $\frac{1}{r'} = \frac{1}{r} + \frac{1}{r} + \text{----} + m \text{ times}$

$$r' = \frac{r}{m}$$

Total current, $I = \frac{E}{R + r'}$

$$= \frac{E}{R + r/m}$$

If $R >> r/m \approx R$

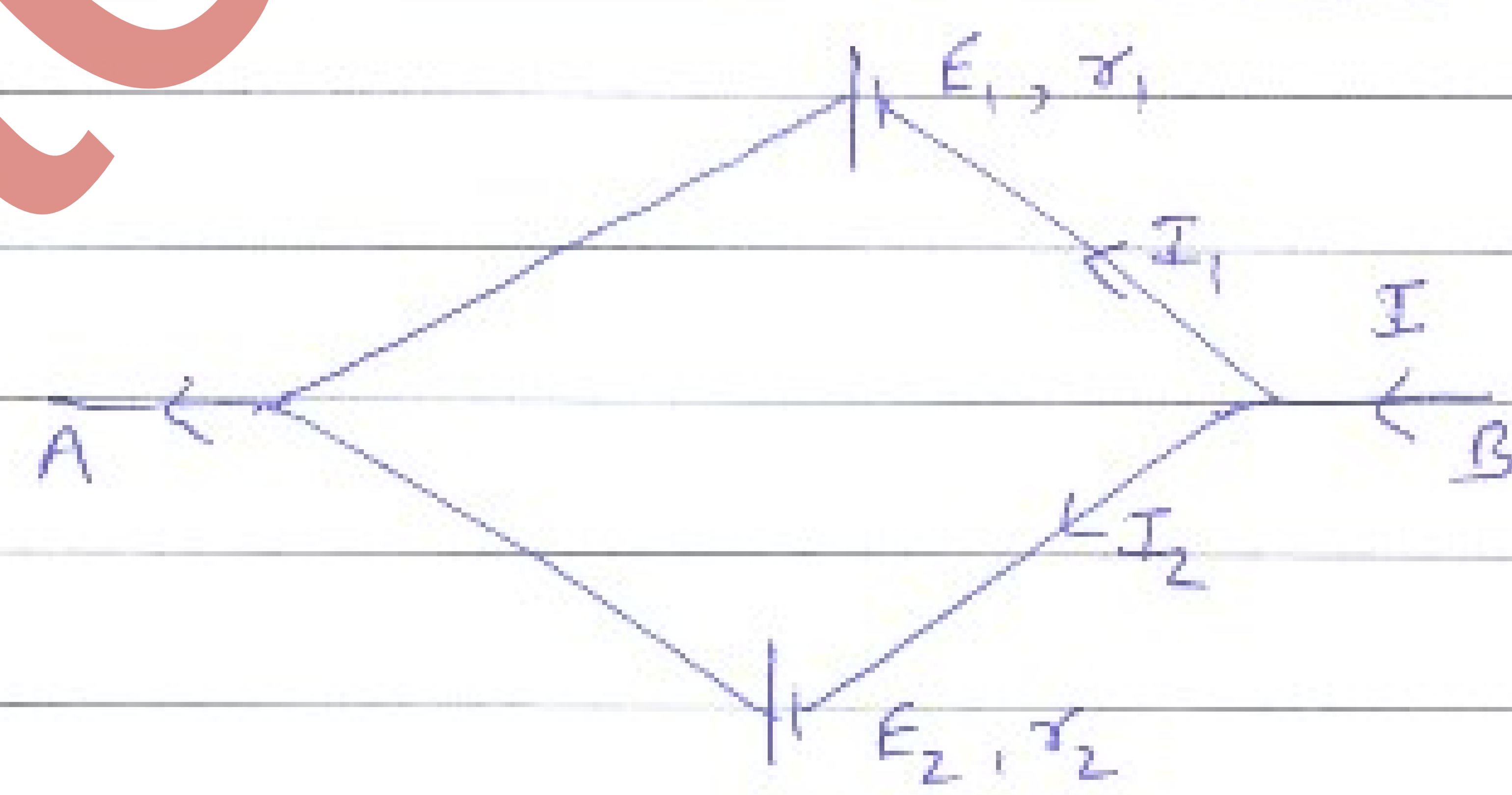
$$I = \frac{E}{R} = \text{current due to single cell}$$

If $R << r/m \approx r/m$

$$I = \frac{E}{r/m} = m\left(\frac{E}{r}\right) = \text{'m' times current due to single cell}$$

"In order to have max. current, the cells should be connected in parallel, when the external resistance in the circuit is negligible as compared to the total internal resistances of the cells."

(b) When cells are of different e.m.fs & internal resistances



$$I = I_1 + I_2$$

Now, $V = E_1 - I_1 r_1$

$$I_1 = \frac{E_1 - V}{r_1}$$

Also $V = E_2 - I_2 r_2$

$$I_2 = \frac{E_2 - V}{r_2}$$

$\therefore \quad I = I_1 + I_2$

$$= \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2}$$

$$= \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$V = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}\right) - I\left(\frac{r_1 r_2}{r_1 + r_2}\right) \longrightarrow (1)$$

If E – effective emf

& r – " resistance, so

$$V = E - Ir \longrightarrow (2)$$

from (1) & (2)

$$E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

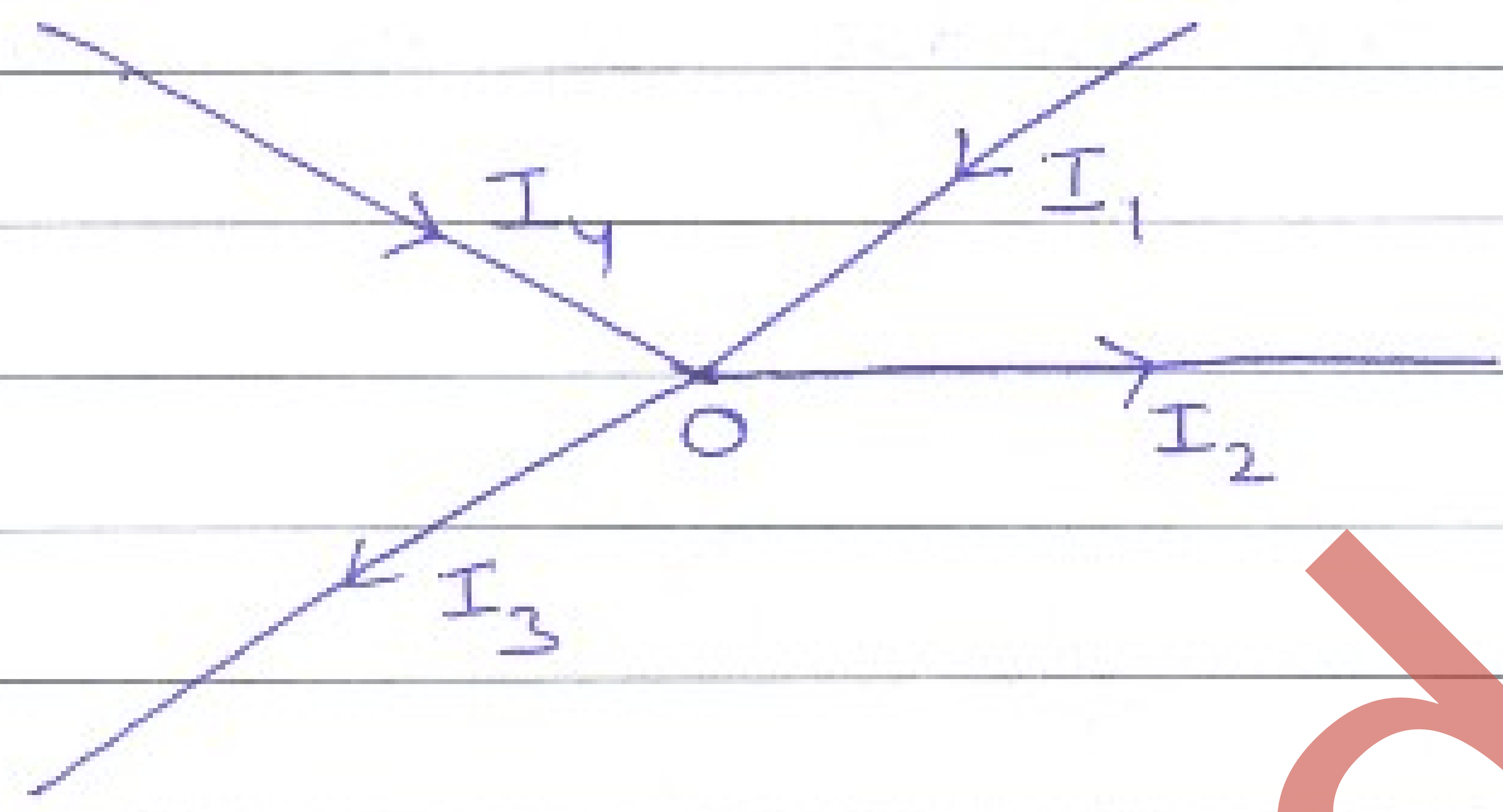
$$r = \frac{r_1 r_2}{r_1 + r_2}$$

Electrical Measurements

Kirchhoff's laws

Kirchhoff's first law (Junction Rule)

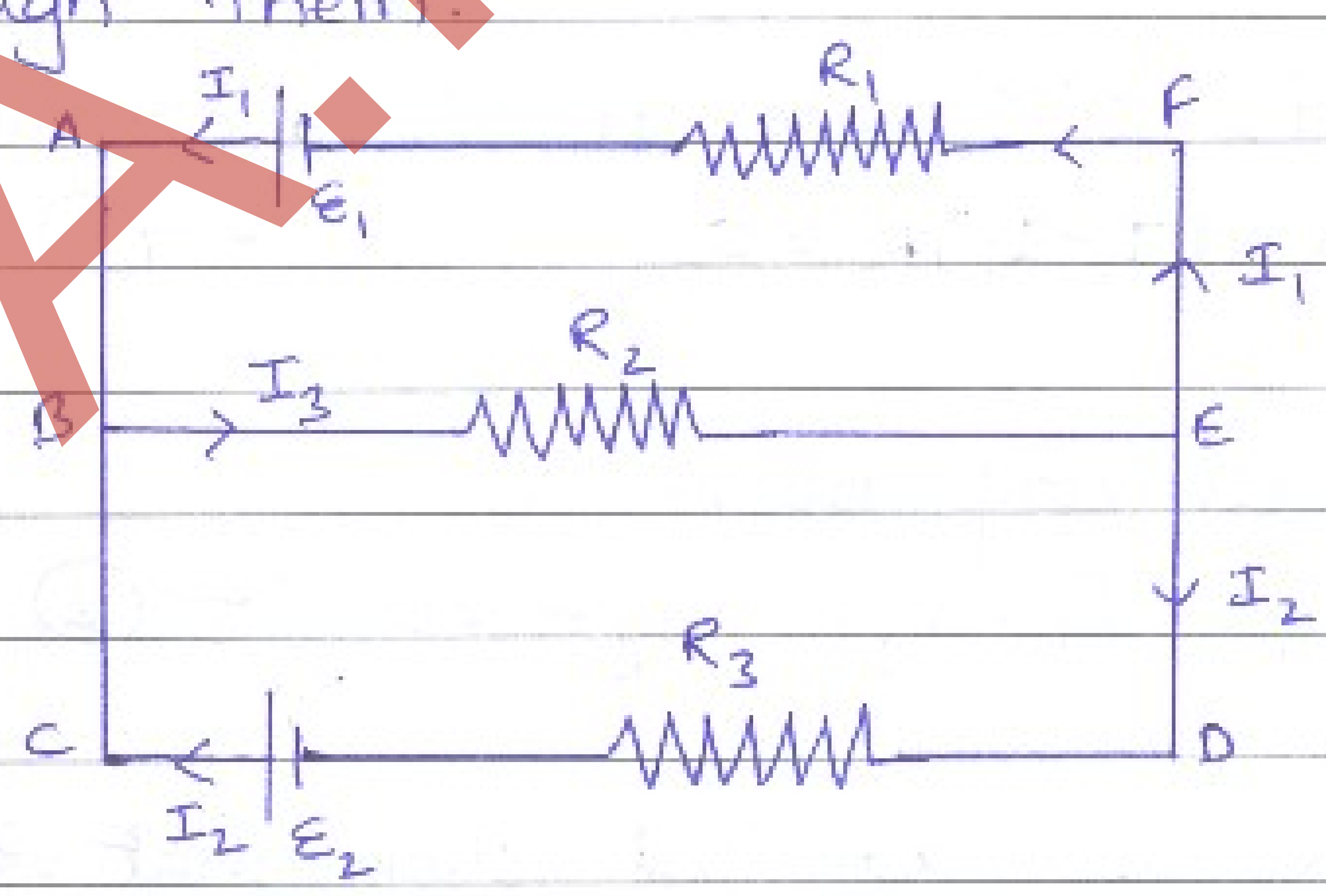
The algebraic sum of the currents meeting at a pt. in an electrical circuit is always zero.



$$I_1 + (-I_2) + (-I_3) + I_4 = 0$$
$$I_1 - I_2 - I_3 + I_4 = 0$$

Kirchoff's second law (Loop Rule)

In any closed part of an electrical circuit, the algebraic sum of the e.m.fs is equal to the algebraic sum of the products of resistances & currents flowing through them.



In closed loop ABEFA
$$\varepsilon_1 = I_1R_1 + I_3R_2$$

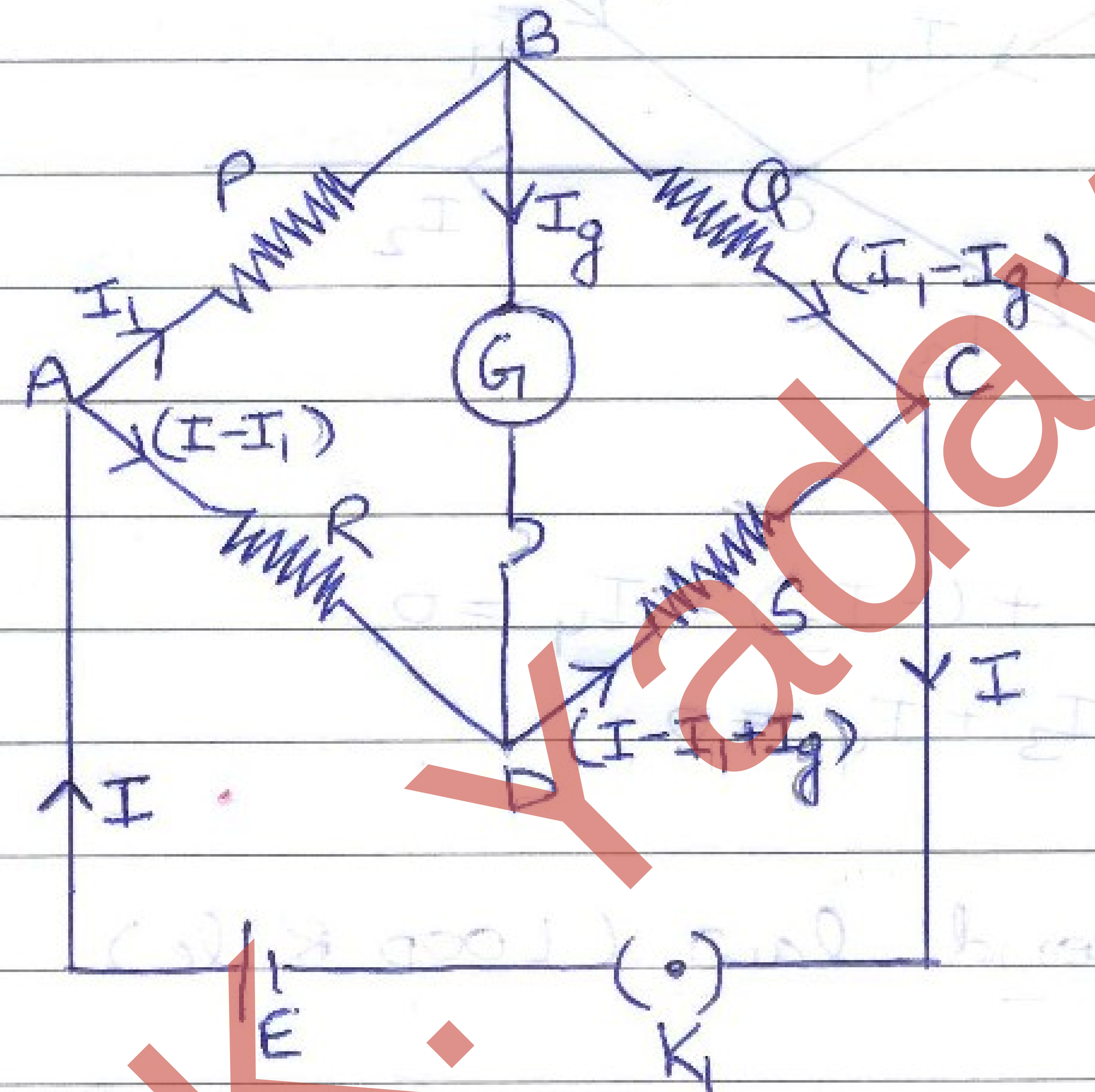
In closed loop CBEDC
$$\varepsilon_2 = I_3R_2 + I_2R_3$$

In closed loop ABCDEFA
$$\varepsilon_1 - \varepsilon_2 = I_1R_1 - I_2R_3$$

* Above relation only true if there is no capacitor in circuit.

Wheatstone Bridge Principle

If 4 resistances P, Q, R & S are arranged to form a bridge with a cell E & one way key K_1 between the points A & C and a galvanometer G and a tapping key K_2 betn the points B & D, then on closing K_1 first and K_2 later on, if galvanometer shows no deflection, the bridge is balanced.



$$\frac{P}{Q} = \frac{R}{S}$$

Proof:

Applying Kirchhoff's 2nd rule to closed circuit ABDA

$$I_1P + I_gG - (I-I_1)R = 0 \quad —(1)$$

Similarly, for closed loop BCDB

$$(I_1-I_g)Q - (I-I_1+I_g)S - I_gG = 0 \quad —(2)$$

The value of R is adjusted such that the galvanometer shows no deflection i.e. $I_g = 0$

So, $I_1P - (I-I_1)R = 0$ or $I_1P = (I-I_1)R$ —(3)

& $I_1Q - (I-I_1)S = 0$ or $I_1Q = (I-I_1)S$ —(4)

③ ÷ ④

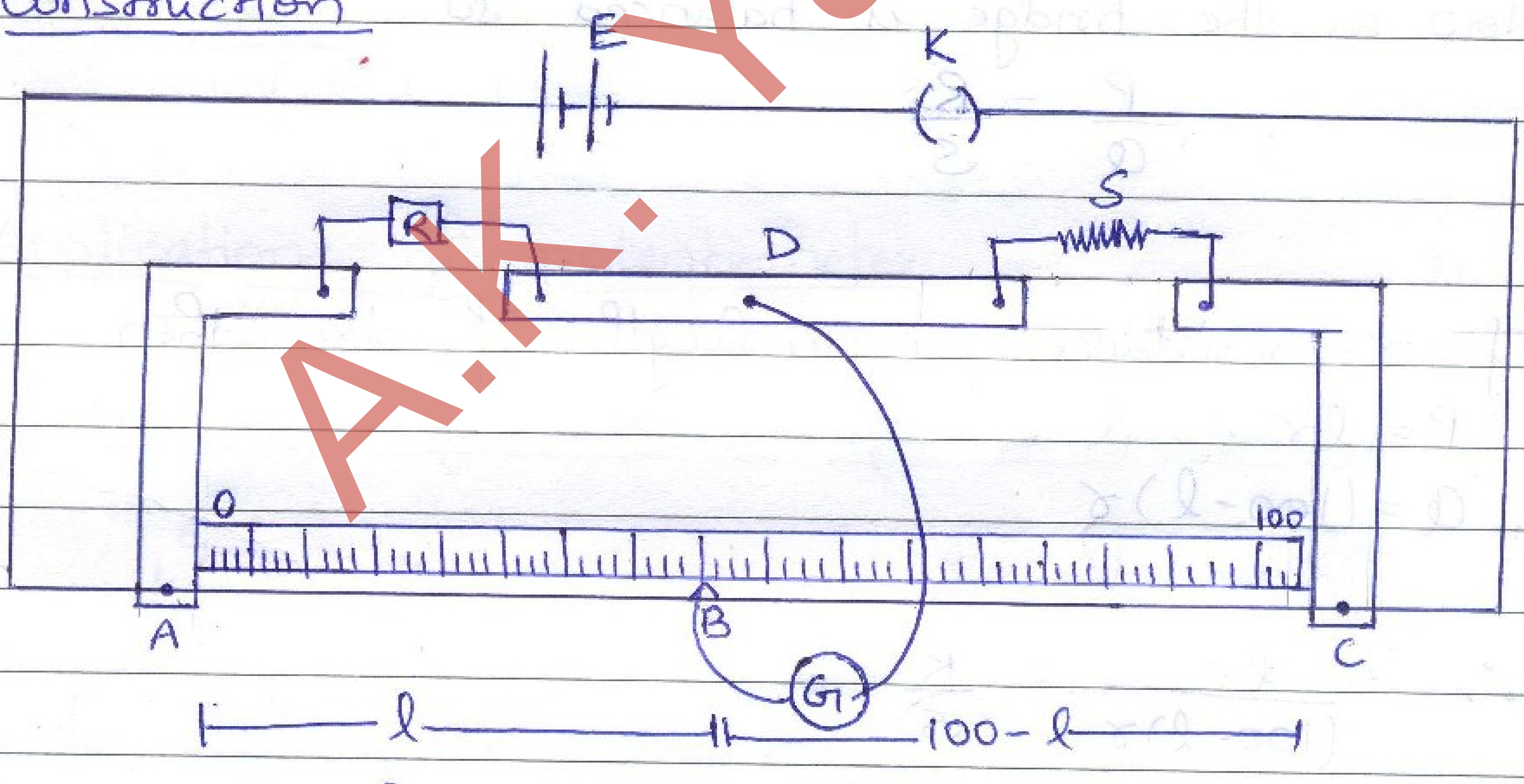
$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

Note → Arms AB (P) & BC (Q) – ratio arm
AD (R) – variable resistance arm
DC (S) – unknown " "

Slide wire bridge or meter bridge (practical form of Wheatstone bridge)

Principle : It is constructed on the principle of Wheatstone bridge i.e. when balanced $\frac{P}{Q} = \frac{R}{S}$

Construction



(i) It consists of wire AC (constantan or magnanin) of 1m length & uniform cross-sectional area, streched betⁿ the 2 copper strips on a horizontal wooden board.

(ii) A metre scale is also fitted on the wooden board parallel to the length of wire.

(iii) Another central copper strip is fitted on the wooden board to provide 2 gaps across which a resistance box R & unknown resistance S are connected.

(iv) One terminal of a sensitive galvanometer G is connected to the terminal D & the other to a jockey B, which can be slided over the wire to balance the bridge.

(v) The +ve pole of battery E is connected to A & -ve pole to C through K.

(vi) The circuit is now exactly the same as that of Wheatstone bridge.

Working

→ Close key K & take out suitable resistance R from resistance box

→ Adjust the position of jockey on wire (say at B) where on pressing, the galvanometer shows no deflection

→ Note the length $AB = l$

$BC = 100 - l$

→ Now as the bridge is balanced, so

$$\frac{P}{Q} = \frac{R}{S}$$

If r - resistance per cm length of wire, then

$P = lr$

$Q = (100 - l)r$

$$\therefore \frac{lr}{(100-l)r} = \frac{R}{S}$$

$$\boxed{S = \left(\frac{100 - l}{l}\right) \times R}$$

* Metre bridge can't be used to measure very high or very low resistances.
* The balanced position of meter bridge is not affected on interchanging the positions of battery & galvanometer.
* If 'l' is close to 50, % error in R is minimum.

Potentiometer

It is an instrument used to compare e.m.fs of 2 cells or to measure internal resistance of a cell accurately.

Principle

When a constant current is passed through a wire of uniform area of cross-section, the potential drop across any portion of wire is directly proportional to the length of that portion.

Let V be potential difference across certain portion of wire, whose resistance is R.

So, $V = IR$

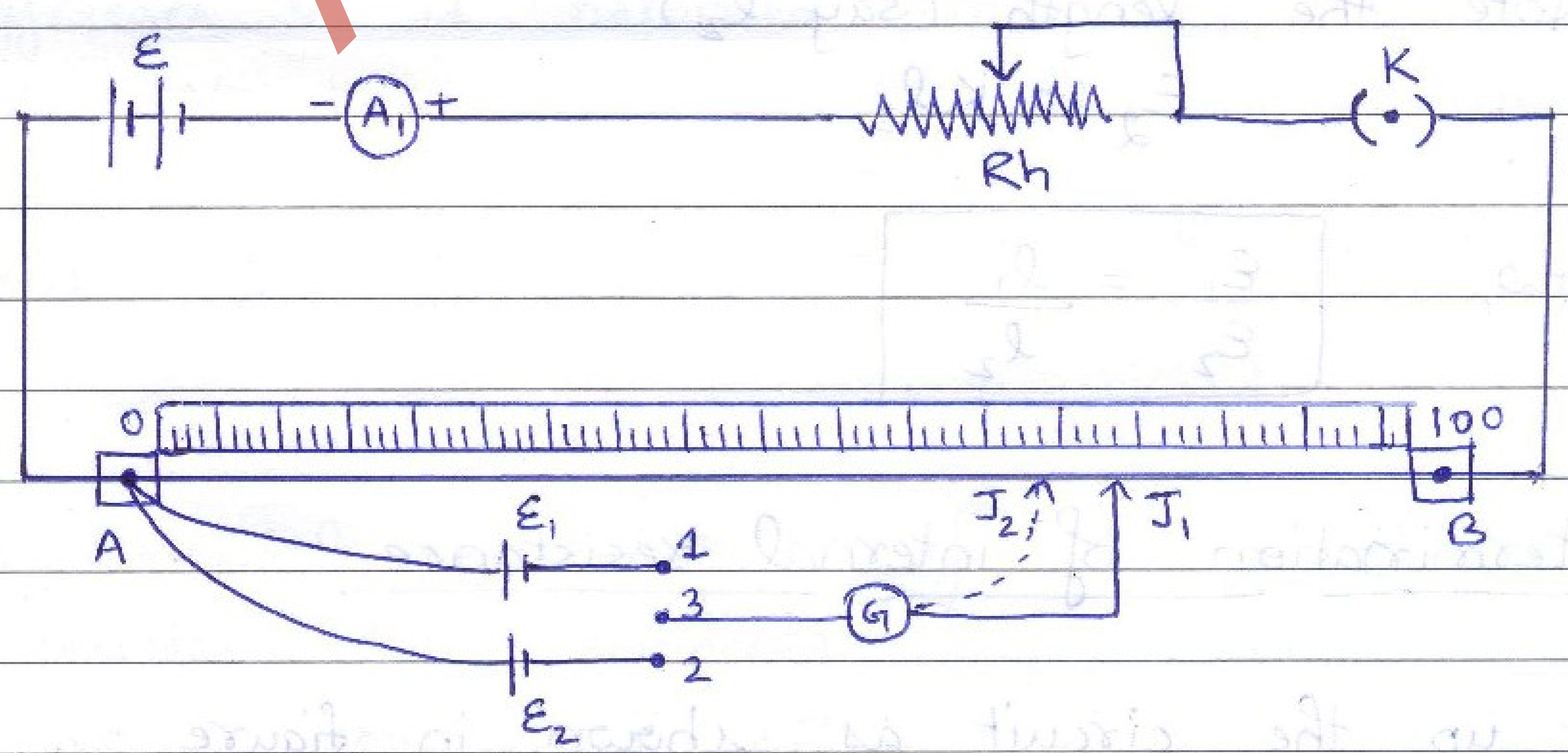
$= I \rho \frac{l}{A}$

$V = Kl$

$$\boxed{V \propto l}$$

Applications of potentiometer

(a) Comparison of e.m.fs of 2 cells



- A battery of emf ε is connected betⁿ the end terminals A & B of potentiometer with rheostat Rh, ammeter A & Key K in series.

→ The positive terminals of the 2 cells (having emfs ε_1 & ε_2) are connected to A & negative terminals to terminals 1 & 2 of 2 way key.

→ The common terminal 3 of the two way key is connected to jockey J through galvanometer G.

Working

(i) Close key K & adjust suitable constant current in the potentiometer wire with the help of rheostat.

(ii) Plug is inserted in the gap betn terminals 1 & 3 so that cell of emf ε_1 is in the circuit.

(iii) Adjust the position of jockey on potentiometer wire such that there is no deflection in galvanometer.

(iv) Note the length (say l_1)

$$\varepsilon_1 = K l_1$$

(v) Remove plug from 1 & 3 & insert betn 2 & 3 so that cell of emf ε_2 comes into circuit.

(vi) Again find the position of jockey on potentiometer wire, where galvanometer shows no deflection.

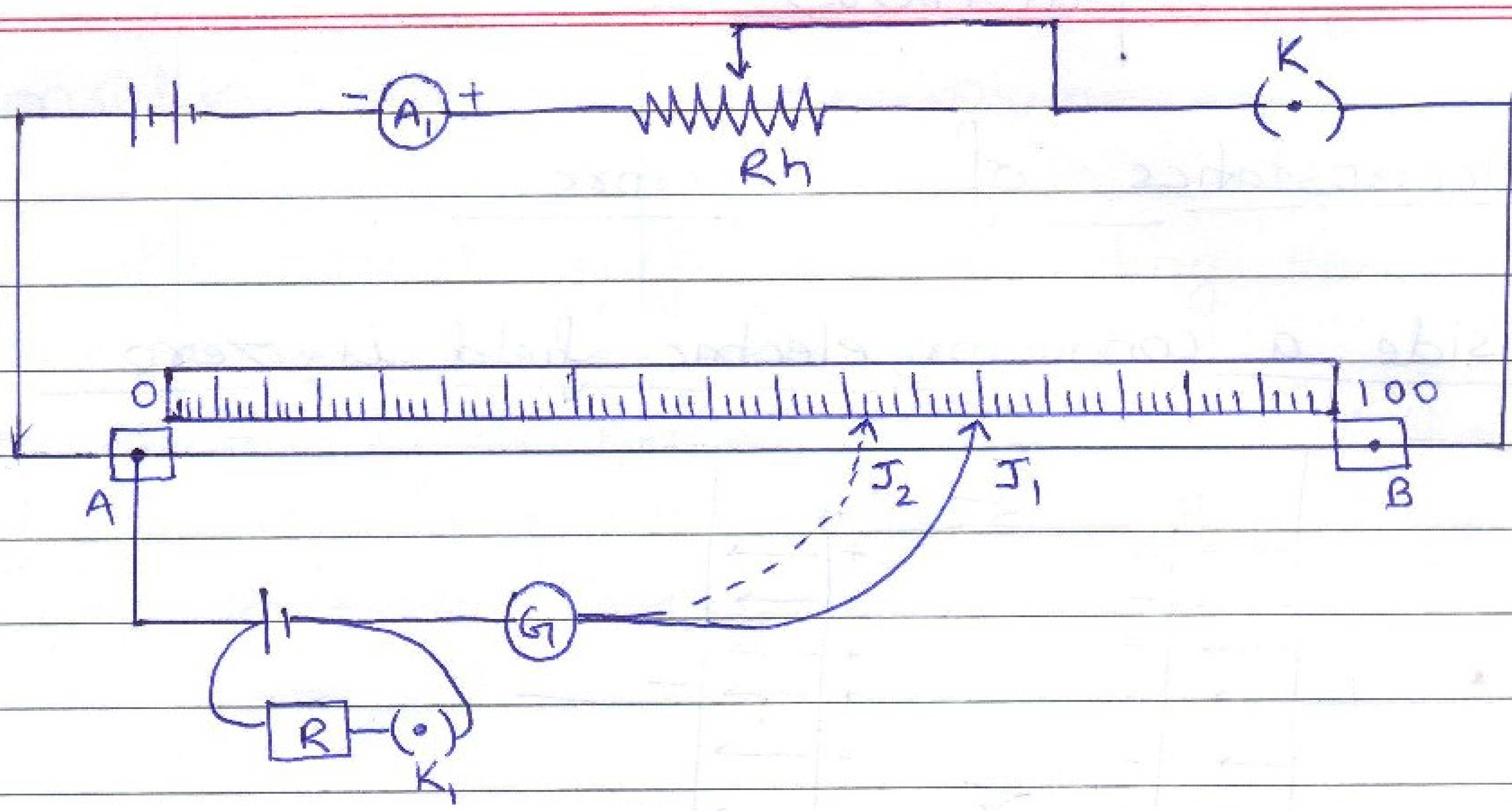
(vii) Note the length (say l_2)

$$\varepsilon_2 = K l_2$$

Now, $\boxed{\dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{l_1}{l_2}}$

(b) Determination of internal resistance

Set up the circuit as shown in figure



(i) Close key K & maintain constant current in the potentiometer wire with the help of rheostat.

(ii) Adjust the position of jockey on potentiometer such that galvanometer shows no deflection.

(iii) Note the length l_1. So e.m.f of cell

$$\varepsilon = K l_1$$

(iv) Close key K_2 & take out suitable resistance R from the resistance box.

(v) Again find the position of jockey where galvanometer shows no deflection. Note the length l_2.

(vi) As current is drawn from cell, its terminal potential difference V is balanced (not ε)

$$\therefore \quad V = K l_2$$

Now $$\frac{\varepsilon}{V} = \frac{l_1}{l_2}$$

The internal resistance 'r' of a cell of emf ε, when a resistance 'R' is connected in its circuit is

$$r = \frac{\varepsilon - V}{V} \times R = \left(\frac{\varepsilon}{V} - 1\right) R$$

$$\boxed{r = \left(\frac{l_1}{l_2} - 1\right) R = \frac{l_1 - l_2}{l_2} \times R}$$