Electrostatics

Introduction

Consider the following examples:

(i) When we take off our synthetic shirts or nylon sweaters in a dry weather, we see a spark or hear a crackle

(ii) Sensation of electric shock while opening the door of our car after sliding from our seat.

(iii) Lightning

The reason (in (i) & (ii)) is discharge of electric charges through our body, which were accumulated due to rubbing of insulating surfaces.

* **Interesting Case**

→ The metallic bodies of cars & truck also get charged because of friction betⁿ them & air passing them.

→ This charge can produce a spark which can be dangerous in case of petrol tanker.

→ So, petrol tankers have a metal chain dragging along the ground, which leaks the charge produced to the ground.

→ Now-a-days, the tyres are made by adding a carbon compound to the rubber, which also facilitates the charge buildup on the body of vehicle to leak to the ground.

Electrostatics

The branch of Physics, which deals with the study of charges at rest i.e. static charges, forces betⁿ the static charges, fields & potentials due to these charges is called Electrostatics or static electricity.

Electric charge

"Charge is something possessed by material objects that makes it possible for them to exert electrical force & to respond to electrical force."

As we know, $m_e = 9.1 \times 10^{-31}$ kg

$m_p = 1.67 \times 10^{-27}$ kg

$m_n = 1.67 \times 10^{-27}$ kg

The gravitational force of attraction bet^n 2 electrons 1 cm apart is

$$F_g = G \frac{m_1 m_2}{r^2}$$

$$= \frac{6.67 \times 10^{-11} \times (9.1 \times 10^{-31})^2}{(10^{-2})^2}$$

$$F_g = 5.5 \times 10^{-67} \text{ N} \quad \text{—} (1)$$

Also, an electron repels another electron at a distance of 1 cm with a force called electric force given by

$$F_e = 2.3 \times 10^{-24} \text{ N} \quad \text{—} (2)$$

from (1) & (2)

$$F_e >> F_g$$

→ It means electrons must have some additional property other than mass which is responsible for electric force bet^n them.

→ This additional property of electron, which gives rise to electric force bet^n 2 electrons is called electric charge.

→ Mass responsible for Gravitational force.
charge " electric "

Kinds of charges

→ Benjamin Franklin named the 2 kinds of charges as +ve & -ve.

→ By convention, charge acquired by glass rod or cat's fur is +ve & charge acquired by ebonite rod or silk cloth is -ve.

* Read charging by induction & GLE.

Quantization of electric charge

1st suggested by – Faraday

demonstrated experimentally by – Millikan

"It is the property by virtue of which all free charges are integral multiple of a basic unit of charge of an electron/proton (e)."

$$\boxed{q = ne}$$

where q – charge of body

n – integer

e = 1.6×10^{-19} C → charge.

Note

→ Quantisation of charge is only meaningful at microscopic level, where charges involved are of the order of tens or hundreds of e.

→ At microscopic level, the charges involved are of the order of 10^{13}e, which is very large.

→ At this scale, the fact that charge can increase or decrease only in units of 'e' is not visible.

→ The grainy nature of charge is lost & it appears to be continuous.

→ For example, a dotted line, viewed from a distance, appears continuous but is not continuous in reality.

→ As many points very close to one another give the impression of a continuous line, many small charges taken together appear as a continuous charge distribution.

→ So, at macroscopic level quantisation of charge is useless.

Conservation of charge

"Total charge of an isolated system always remains constant or conserved."

or

"Charges can be created or destroyed in equal and unlike pairs only."

Examples

(a) Pair production - a γ-ray photon materialise into an electron & positron.

$$\gamma = e^- + e^+$$

(b) Annihilation of matter

$$e^- + e^+ = \gamma + \gamma$$

(c) Nuclear transformations

$$_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_2He^4$$

Coulomb's Law

The force of interaction bet^n any 2 point charges is directly proportional to the product of the charges & inversely proportional to the square of the distance bet^n them.

$$F \propto \frac{|q_1||q_2|}{r^2}$$

$$F = k \frac{|q_1||q_2|}{r^2}$$

k - electrostatic force constant (whose value depend on the nature of medium separating the charges, & the system of units).

When charges are in free space, then
in cgs, $k = 1$
SI, $k = 9 \times 10^9 \, Nm^2C^{-2} = \frac{1}{4\pi\varepsilon_0}$

ε_0 - absolute electrical permitivity of free space.

$$\boxed{F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|q_1||q_2|}{r^2}}$$

Unit, Dimension & Value of ε_0

$$\varepsilon_0 = \frac{1}{4\pi F} \cdot \frac{q_1 q_2}{r^2}$$

Unit $\varepsilon_0 = \frac{1}{N} \cdot \frac{C^2}{m^2} = C^2N^{-1}m^{-2}$

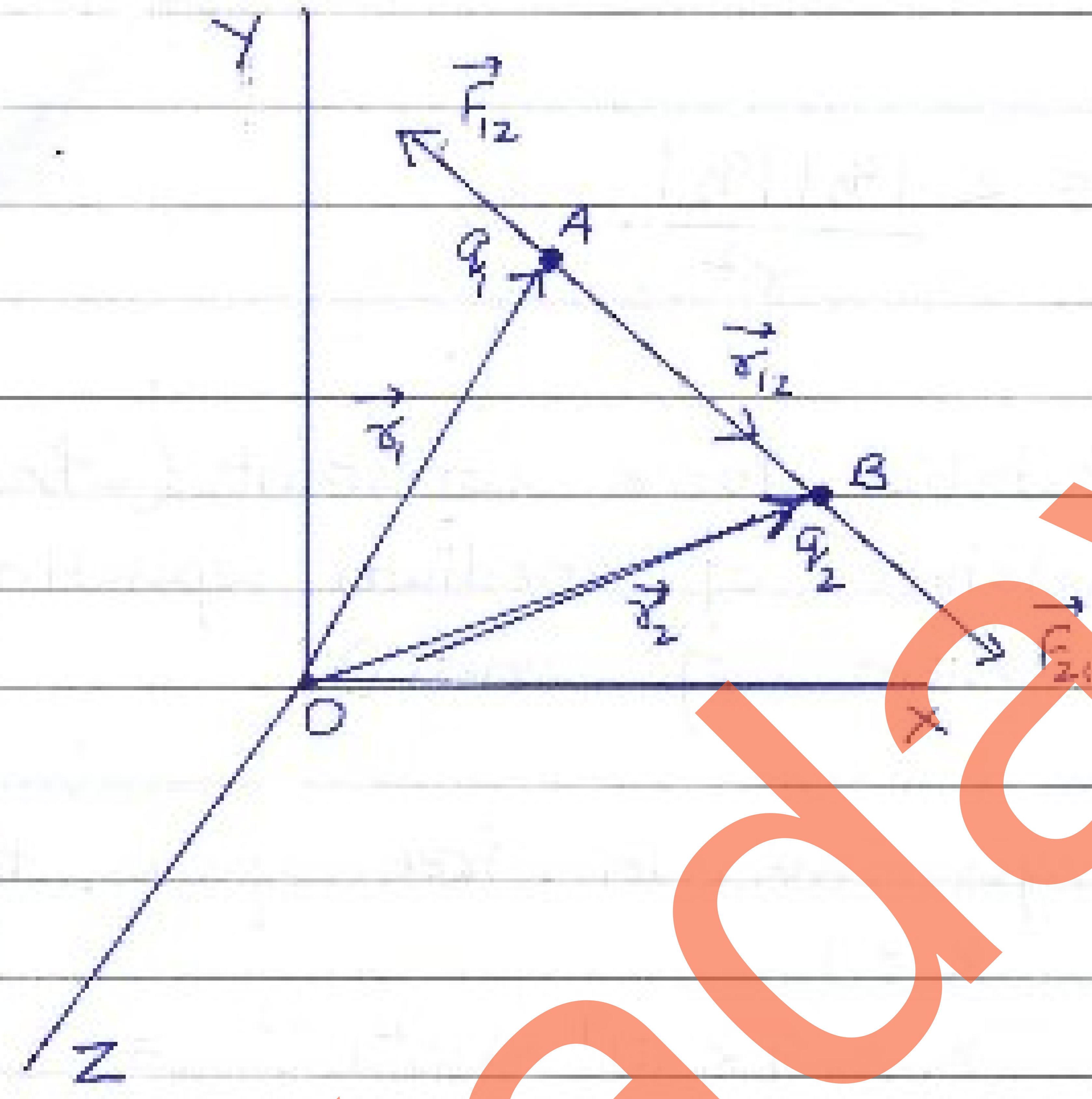
Dimension $= \frac{[AT][AT]}{[MLT^{-2}][L^2]} = [M^{-1}L^{-3}T^4A^2]$

Now, $\varepsilon_0 = \frac{1}{4\pi k}$ $\quad [\because k = \frac{1}{4\pi\varepsilon_0}]$

$$= \frac{1}{4 \times 3.14 \times 9 \times 10^9}$$

$$= 8.85 \times 10^{-12} \, C^2N^{-1}m^{-2}$$

Coulomb's law in vector form



$\vec{r}_1 = \vec{OA}$ – position vector of q_1
$\vec{r}_2 = \vec{OB}$ – " " " q_2

So, vector leading from q_1 to q_2, $\vec{AB} = \vec{r}_{12} = \vec{r}_2 - \vec{r}_1$

" " " q_2 to q_1, $\vec{BA} = \vec{r}_{21} = \vec{r}_1 - \vec{r}_2$

Also, $\hat{r}_{12} = \dfrac{\vec{r}_{12}}{r_{12}}$ & $\hat{r}_{21} = \dfrac{\vec{r}_{21}}{r_{21}}$

If $\vec{F}_{12}$ – force on q_1 due to q_2
$\vec{F}_{21}$ – " " q_2 " " q_1 then.

Coulomb's force betn 2 pt. charges q_1 & q_2 located at $\vec{r}_1$ & $\vec{r}_2$ in vacuum is

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{AB^2} \quad \text{along } AB$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}^2} \times \hat{r}_{12}$$ [$\hat{r}_{12}$ for direction]

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}^3} \times \vec{r}_{12}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1)$$

Special Cases

(1) If q_1, q_2 are of same sign

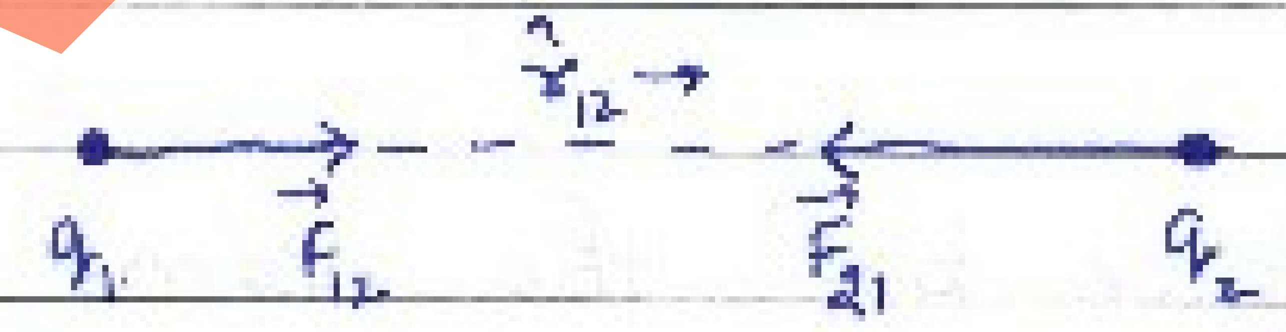
$$q_1 q_2 > 0$$

$\vec{F}_{21}$ is along $\hat{r}_{12}$ [Repulsion of like charges]

(2) If q_1, q_2 are of opp. sign

$$q_1 q_2 < 0$$

$\vec{F}_{21}$ is opp. to $\hat{r}_{12}$ or $\vec{F}_{21}$ is along $-\hat{r}_{12}$ [Attraction of unlike charges]



(3) $$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}^2} \times \hat{r}_{21}$$

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

Newton's 3rd law of motion verified.

Units of charge

S.I unit - Coulomb

cgs unit - electrostatic unit (e.s.u) or stat coulomb or franklin

Relation betⁿ Coulomb & stat coulomb

In S.I, charge on electron $= 1.6 \times 10^{-19}$ C

cgs " " " $= 4.8 \times 10^{-10}$ stat coulomb

So, 1.6×10^{-19} C $= 4.8 \times 10^{-10}$ stat coulomb

$$1\,C = \frac{4.8 \times 10^{-10}}{1.6 \times 10^{-19}} \text{ stat coulomb}$$

$$\boxed{1\,C = 3 \times 10^{9} \text{ stat coulomb}}$$

Dielectric constant

The electrostatic force betⁿ 2 charges in vacuum is

$$F_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad \text{—(1)}$$

The electrostatic force betⁿ the same 2 charges in a medium is

$$F_m = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r^2} \quad \text{—(2)}$$

where ε - absolute electrical permitivity of medium.

① ÷ (a)

$$\frac{F_0}{F_m} = \frac{\epsilon}{\epsilon_0} = \epsilon_r \text{ or } K \quad \text{---} ③$$

ϵ_r – relative electrical permittivity of medium
Dielectric constant (K)

Dielectric constant:

It is defined as a ratio of absolute electrical permittivity of medium to absolute electrical permittivity of free space.

OR

It is the ratio of force of interaction b/w 2 pt. charges separated by a certain distance in air & the force of interaction b/w the same 2 pt. charges, held the same distance apart in a medium.

From ③ $\boxed{F_m = \frac{1}{K} F_0}$

Eg force b/w charges in water is $\frac{1}{81}$ times of its value in air as $K = 81$ for water it means the force of attraction b/w oppositely charged ions decreases in water & it makes water a good solvent.

K depends only on nature of medium.

Principal of superposition

The total force on any charge due to a no. of other charges at rest is the vector sum of all the forces on that charge due to other charges, taken one at a time.

So, total force $\vec{F}_0$ on a test charge q_0 at position $\vec{r}_0$ due to all the 'n' discrete charges is

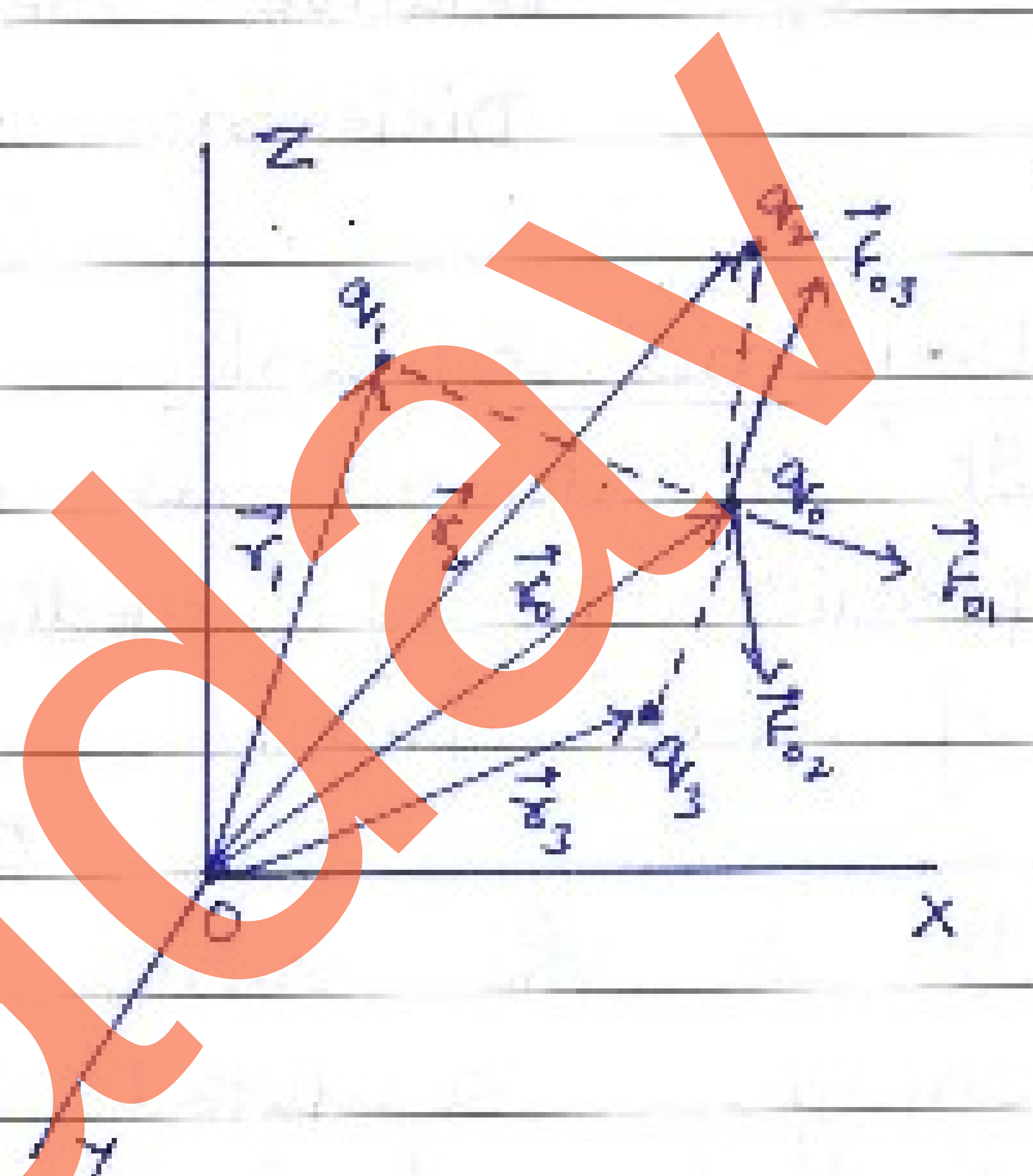
$$\boxed{\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \vec{F}_{03} + \ldots\ldots + \vec{F}_{0n}}$$

where,

$\vec{F}_{01}$ = force on q_0 due to q_1

$\vec{F}_{02}$ = " " " " " q_2

$\vec{F}_{0n}$ = " " " " " q_n



Acc. to Coulomb's law

$$\vec{F}_{01} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_0 q_1}{r_{10}^2} \hat{r}_{10}$$

$$\vec{F}_{02} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_0 q_2}{r_{20}^2} \hat{r}_{20}$$

$$\vec{F}_{0n} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_0 q_n}{r_{n0}^2} \hat{r}_{n0}$$

$$\therefore \vec{F}_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_0 q_1}{r_{10}^2}\hat{r}_{10} + \frac{q_0 q_2}{r_{20}^2}\hat{r}_{20} + \ldots\ldots + \frac{q_0 q_n}{r_{n0}^2}\hat{r}_{n0}\right]$$

$$\boxed{\vec{F}_0 = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_0 q_i}{r_{i0}^2}\hat{r}_{i0}}$$

Continuous charge distribution

→ As charge can exist only as an integral multiple of e so charge distribution is always discrete (discrete).

→ But it is impractical to work in terms of discrete charges.

→ For example, on the surface of a charged conductor, we will specify the charge distribution in terms of microscopic charged particle locations.

→ But we can consider a small area element ΔS on the surface of conductor

→ This area element is very small on macroscopic level but big enough to include a large no. of electrons.

→ If ΔQ is amount of charge on the element, then surface charge density is

$$\sigma = \frac{\Delta Q}{\Delta S}$$

Repeating the process at different points on the surface of conductor we arrive at a continuous function σ called surface charge density.

→ At microscopic level, charge distribution is discontinuous as there are discrete charges separated by intervening space where there is no charge.

→ So σ represents macroscopic surface charge density.

→ When charge is distributed along a line straight or curved,

linear charge density, $\lambda = \frac{\Delta Q}{\Delta l}$, unit = Cm^{-1}

Δl = small line element of wire on macroscopic level

→ When charge is distributed in volume

$$\rho = \frac{\Delta Q}{\Delta V} \quad , \text{unit} = Cm^{-3}$$

ΔV = small volume element

Force due to continuous distribution of charges

Suppose a continuous charge distribution in space of a volume charge density ρ. W.r.t. to O, let the position vector of any pt. in the charge distribution be $\vec{r}$.
Inside the charge distribution take small volume element of size Δv.
So charge in the volume element, $\Delta q = \rho\, \Delta v$

Consider a pt. P (inside or outside) in the charge distribution with position vector $\vec{OP} = \vec{r}_0$

From Coulomb's law, force on q_0 (small test charge) at P due to Δq is

$$d\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{q_0 (\Delta q)}{r'^2}\hat{r}'$$

$$= \frac{q_0}{4\pi\epsilon_0} \cdot \frac{\rho\, \Delta v}{r'^2}\hat{r}'$$

By superposition principle, total force due to total volume charge distribution is obtained by summing the forces due to different volume elements

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \sum_{\text{all } \Delta V} \frac{\rho \Delta V}{r'^2} \hat{r}'$$

when $\Delta V \to 0$

$$\boxed{\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_V \frac{\rho \Delta V}{r'^2} \hat{r}'}$$

* Doing similarly we get

$$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_L \frac{\lambda \, \Delta l}{r'^2} \hat{r}'$$

& $$\vec{F} = \frac{q_0}{4\pi\epsilon_0} \int_S \frac{\sigma \, ds}{r'^2} \hat{r}'$$

Electric field

Electric field

Electric field due to a given charge is the space around the charge in which electrostatic force of attraction or repulsion due to the charge can be experienced by any other charge.

Source charge (Q) - produces electric field
Test charge (q_0) - test the effect of Q.

Electric field intensity $(\vec{E})$:

It is the strength of electric field at a point. It is defined as the force experienced by a unit +ve charge placed at a pt.

$$\vec{E}(\vec{r}) = \frac{\vec{F}(\vec{r})}{q_0}$$

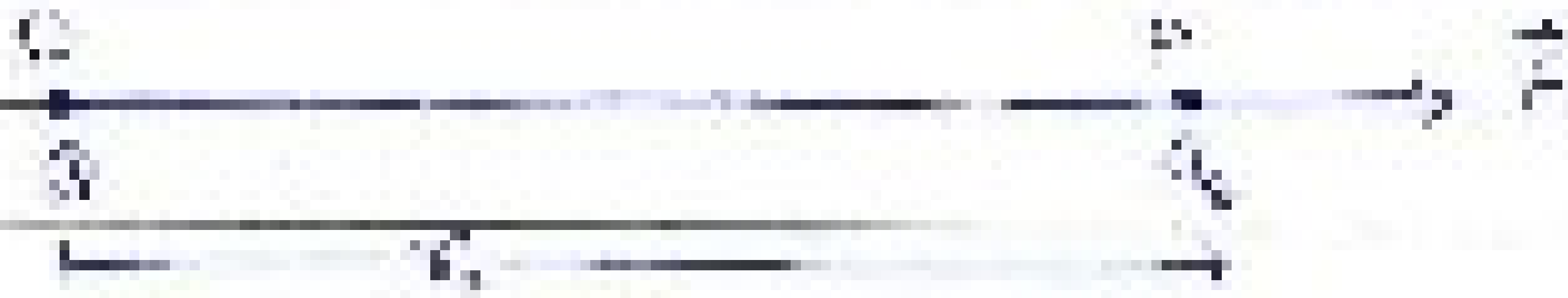
[illegible]

S.I. unit = NC^{-1}

As the test charge $+q_0$ may have its own electric field, it can modify the original field of source charge, so to minimize this effect & completely remove it we can write

$$\vec{E}(\vec{r}) = \lim_{q_0 \to 0} \frac{\vec{F}(\vec{r})}{q_0}$$

Electric field intensity due to a point charge



Force on charge q_0 at P is

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q q_0}{r^2} \hat{r}_0$$

$\hat{r}_0$ = unit vector directed from Q to P

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r_0^2} \cdot \hat{r}_0}$$

From the above eq^n it is clear magnitude of $\vec{E}$ due to a given charge Q depends only on distance r.

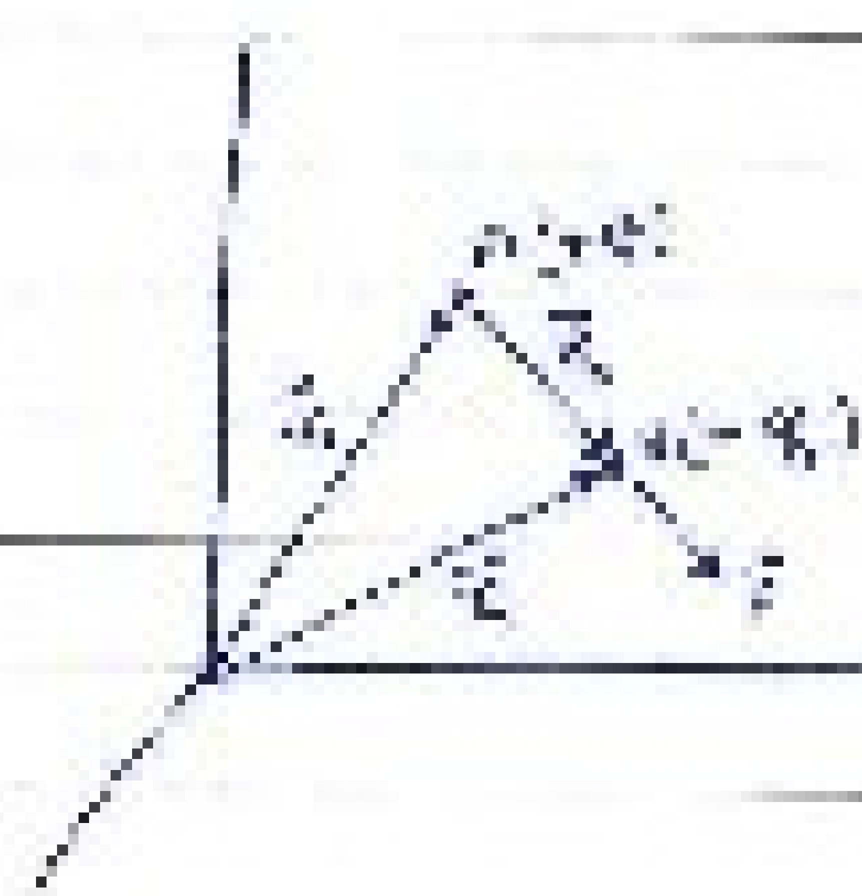
So, at equal distance from Q, E is same.

If a pt charge was at the centre of a sphere, magnitude of $\vec{E}$ at every pt on the surface of sphere would be same.

It means that electric field due to a single charge is spherically symmetric.

$$\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q q_0}{r_0^2} \hat{r}_0 = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q q_0}{r_0^3} \vec{r}_0$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{Q q_0}{|\vec{r}_0 - \vec{r}'|^3} (\vec{r}_0 - \vec{r}')$$



$$\therefore \vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{|\vec{r}_0 - \vec{r}'|^3} (\vec{r}_0 - \vec{r}')$$

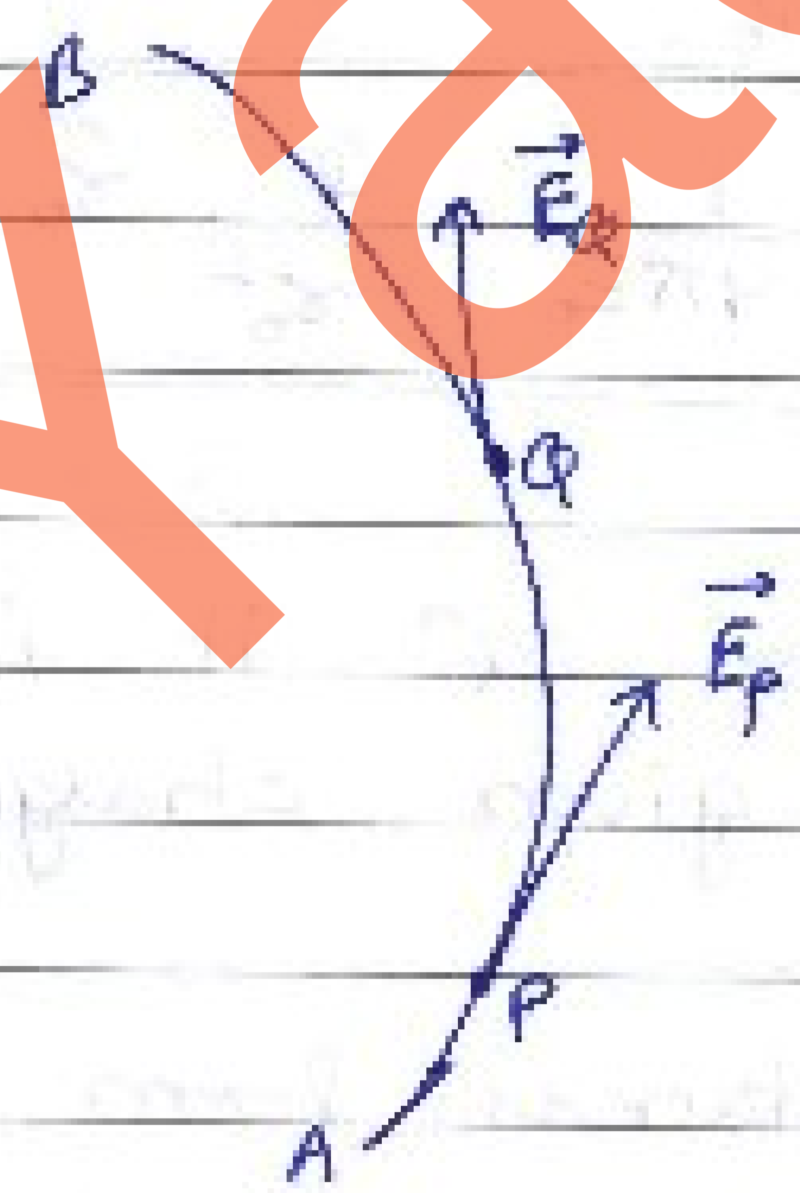
Physical significance of electric field

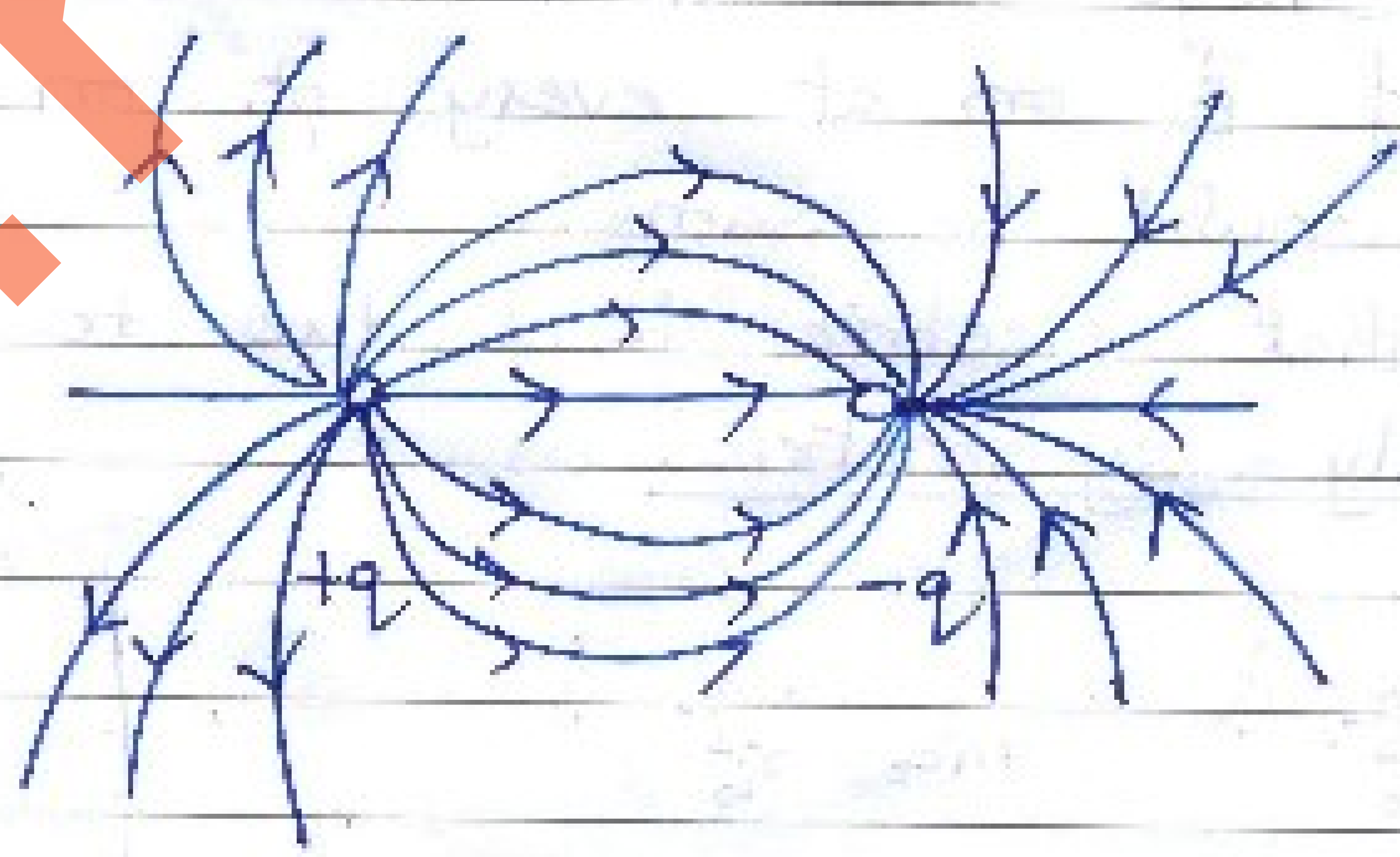
From the knowledge of electric field intensity $\vec{E}$ at any pt. $\vec{r}$, we can readily calculate the magnitude & direction of force experienced by any charge q_0 held at that pt.

$$\vec{F}(\vec{r}) = q_0 \vec{E}(\vec{r})$$

Electric field lines

An electric field line is a path, straight or curved in electric field, such that tangent to it at any point gives the direction of electric field intensity at that point.





Properties of Electric field lines

(1) They are continuous curves (+ve to −ve) but not continuous closed loops (i.e. no electric field lines inside the charged body)

(2) Tangent to the electric field line at any point gives the direction of electric field intensity at that point.

(3) No 2 electric field lines intersect each other.

(4) They contract longitudinally on account of attraction betⁿ unlike charges.

(5) They are always normal to the surface of conductor

Note

1. The magnitude of field is represented by the density of field lines.
   Near the charge, lines are closer, density more & so $\vec{E}$ is strong near the charge.
2. $\vec{E}$ (e.f.i) at a pt. is equal to number of field lines crossing a unit area around that point.

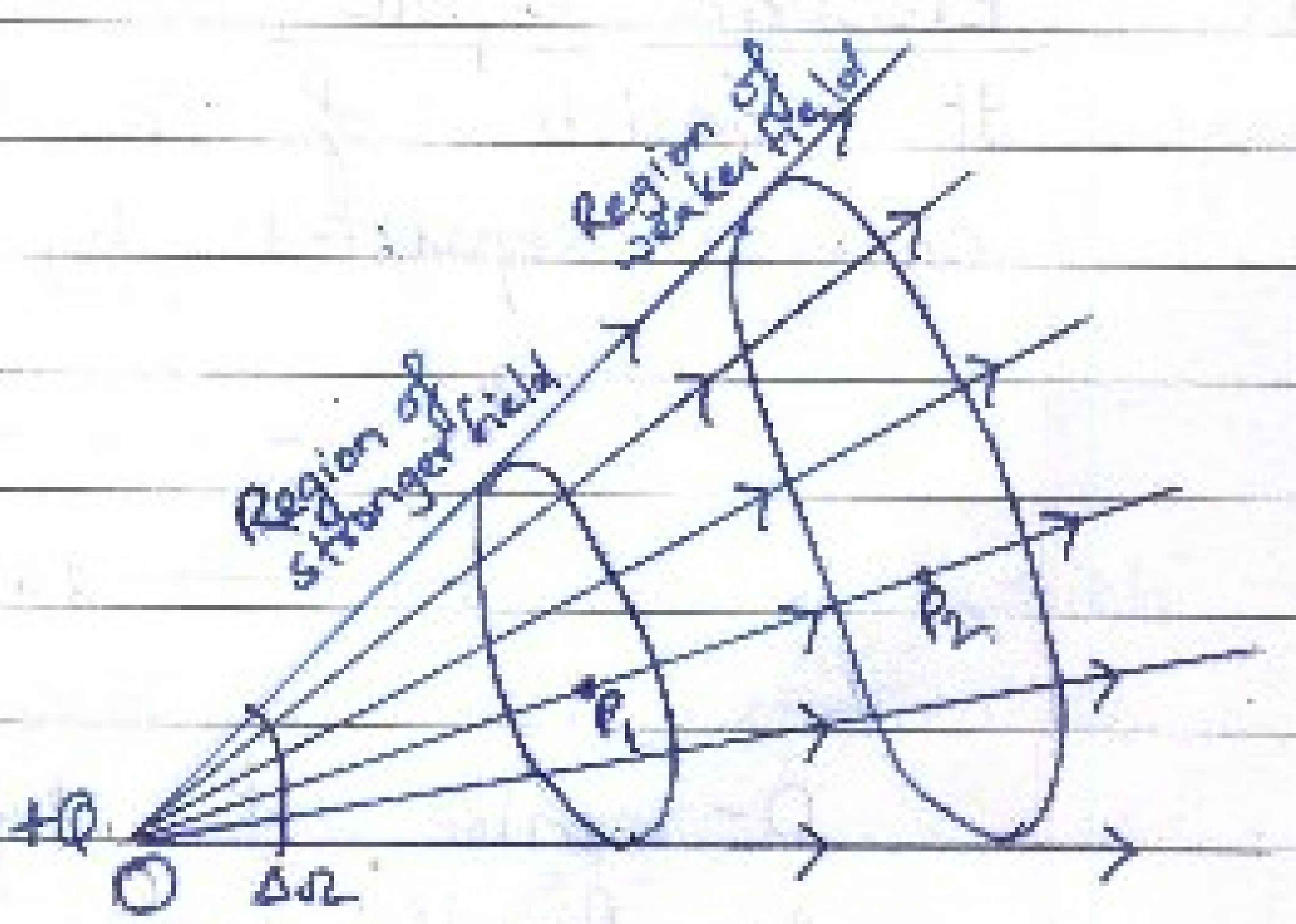
Q. Show that electric field intensity is more near the charge.

Ans.

As we know

$$\text{solid angle} = \frac{\text{area}}{\text{distance}^2}$$

$$\Delta\Omega = \frac{\Delta S}{r^2}$$



$OP_1 = r_1$, $OP_2 = r_2$

With a charge +Q at the apex of cone, the no. of radial field lines in a given solid angle are same.

Consider 2 pts P_1 & P_2 at distances r_1 & r_2 from the charge $+q$.

At P_1, area subtended by $\Delta\Omega$ = $r_1^2 \Delta\Omega$

P_2 = $r_2^2 \Delta\Omega$

The no of field lines cutting these area element are same. Let it be n.

∴ No of field lines crossing unit area placed at P_1
= electric intensity at P_1

i.e. $E_1 = \dfrac{n}{r_1^2 \Delta\Omega}$

Similarly, at P_2 $E_2 = \dfrac{n}{r_2^2 \Delta\Omega}$

Now $\dfrac{E_1}{E_2} = \dfrac{r_2^2}{r_1^2}$

∴ $\boxed{E \propto \dfrac{1}{r^2}}$

Hence proved

Electric dipole

It consists of a pair of equal & opposite pt charges separated by some small distance.

O - centre of dipole

2a - length " "

eg. Atoms of CO_2, NH_3, HCl, H_2O (because centres of +ve & -ve charges are separated by small distance)

Dipole moment ($\vec{p}$)

It measures the strength of electric dipole. It is equal to the product of magnitude of either charge & the distance betn them.

$$\vec{p} = q.(2\vec{a})$$

$$\boxed{|\vec{p}| = q(2a)}$$

By convention, direction of $\vec{p}$ → -ve to +ve.

S.I. unit = C-m.

* If q gets larger, & 2a gets smaller & smaller to keep $\vec{p}$ const. we get an ideal or pt. dipole

or

An ideal dipole is the smallest dipole having almost no size.

Physical significance of electric dipoles

It tells about polar & non-polar molecules.

(a) If the centre of mass of +ve charges coincides with the centre of mass of -ve charges, the molecule behaves as non-polar molecule.

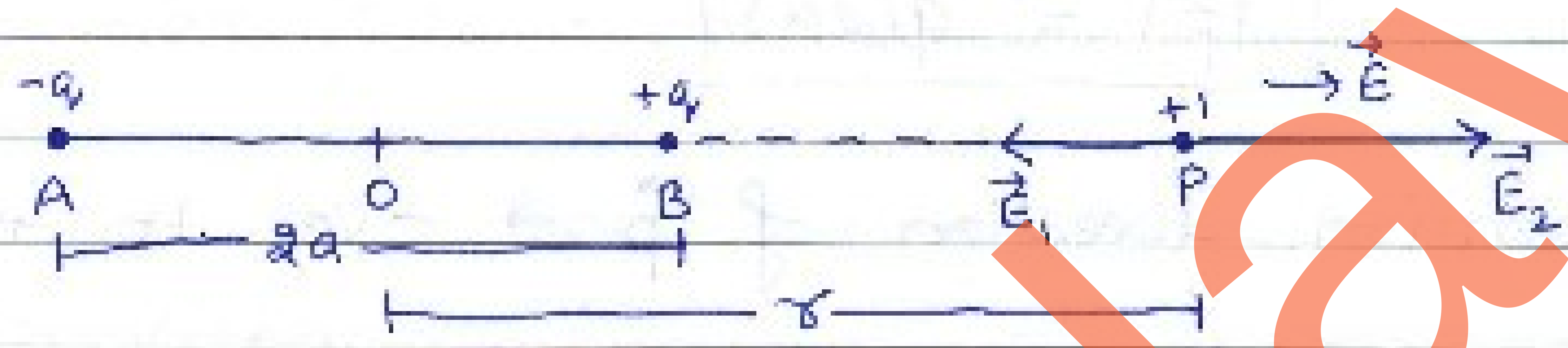
(b) If the centre of mass of +ve charges does not coincides with the centre of mass of -ve charges, the molecule behaves as polar molecules.

Dipole Field

It is the space around the dipole in which the electric effect of dipole can be experienced.

Field Intensity on axial line of electric dipole

Consider an electric dipole having 2 pt. charges $-q$ & $+q$ separated by a small distance '$2a$'. Let P be a pt. on the axial line of dipole at a distance $OP = r$ from the centre of dipole.



If $\vec{E}_1$ is the electric field intensity at P due to charge $-q$ at A, then

$$|\vec{E}_1| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r+a)^2} \quad \text{(along PA)}$$

$|||^{ly}$ E.F.I at P due to $+q$ at B is

$$|\vec{E}_2| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r-a)^2} \quad \text{(along BP)}$$

As $\vec{E}_1$ & $\vec{E}_2$ are collinear vectors acting in opp directions & $|\vec{E}_2| > |\vec{E}_1|$, so, resultant intensity at P (acting along BP) is

$$\vec{E} = |\vec{E}_2| - |\vec{E}_1|$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot q \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot q \left[\frac{(r+a)^2 - (r-a)^2}{(r^2-a^2)^2} \right]$$

$$= \frac{q}{4\pi\varepsilon_0} \cdot \frac{4ar}{(r^2-a^2)^2}$$

$$= \frac{q \times 2a \times 2r}{4\pi\varepsilon_0 (r^2-a^2)^2}$$

$$\boxed{\vec{E} = \frac{|\vec{p}|}{4\pi\varepsilon_0} \cdot \frac{2r}{(r^2-a^2)^2}}$$

if dipole is short, $2a << r$

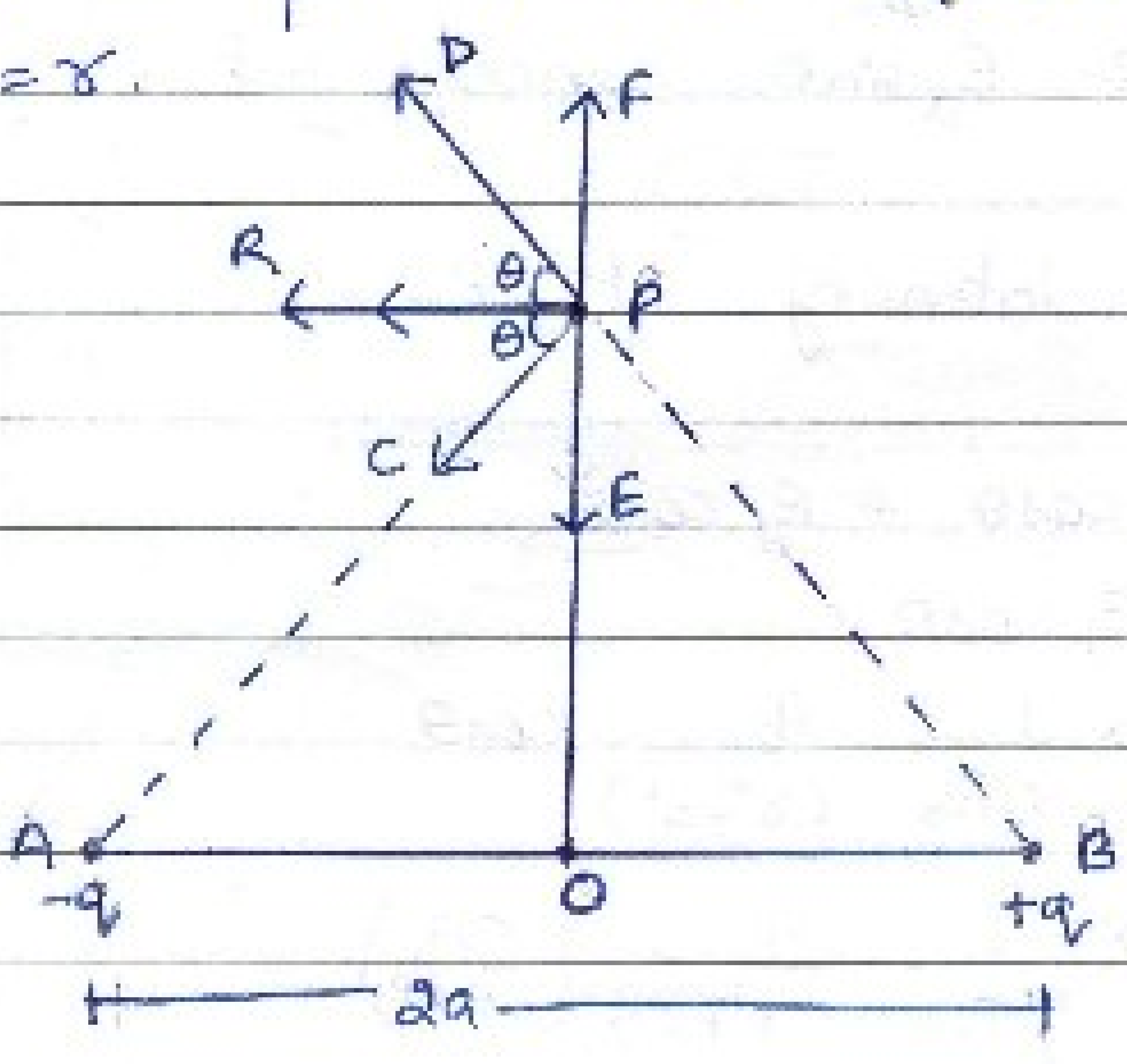
$$\vec{E} = \frac{|\vec{p}|}{4\pi\varepsilon_0} \cdot \frac{2r}{r^4}$$

$$\boxed{\vec{E} = \frac{2|\vec{p}|}{4\pi\varepsilon_0 r^3}}$$

Field Intensity on equatorial line of dipole

Consider an electric dipole having 2 pt. charges $-q$ & $+q$ separated by small distance $AB = 2a$ with centre at O.

Let P be a pt. on the equatorial line of dipole with $OP = r$.



If $\vec{E}_1$ is electric field intensity at P due to $-q$ at A then,

$$|\vec{E}_1| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{AP^2}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r^2+a^2)} \quad \text{--- (1)} \qquad [AP^2 = OP^2 + OA^2 = r^2 + a^2]$$

$\vec{E}_1$ has 2 rectangular components - $E_1 \cos\theta$ (along PR ∥ BA)
$E_1 \sin\theta$ (PE ⊥ BA)

$|||^{ly}$ E.F.I at P due to $+q$ at B is

$$|\vec{E}_2| = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{BP^2}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r^2+a^2)} \quad \text{--- (2)}$$

$\vec{E}_2$ has 2 rectangular components - $E_2 \cos\theta$ (PR ∥ BA)
$E_2 \sin\theta$ (along PF)

from (1) & (2)

$$|\vec{E}_1| = |\vec{E}_2|$$

So, $E_1 \sin\theta$ & $E_2 \sin\theta$ cancel out

∴ Resultant intensity at P

$$|\vec{E}| = E_1 \cos\theta + E_2 \cos\theta$$

$$= 2E_1 \cos\theta$$

$$= 2 \times \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r^2+a^2)} \cos\theta$$

$$= 2 \times \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(r^2+a^2)} \left(\frac{OA}{AP}\right)$$

$$= 2 \times \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2+a^2)} \cdot \frac{a}{(r^2+a^2)^{1/2}}$$

$$\boxed{\vec{E} = \frac{|\vec{p}|}{4\pi\epsilon_0 (r^2+a^2)^{3/2}}}$$

The direction of $\vec{E}$ is along $\vec{PR} \parallel \vec{BA}$ (opp to $\vec{p}$)

In vector form,

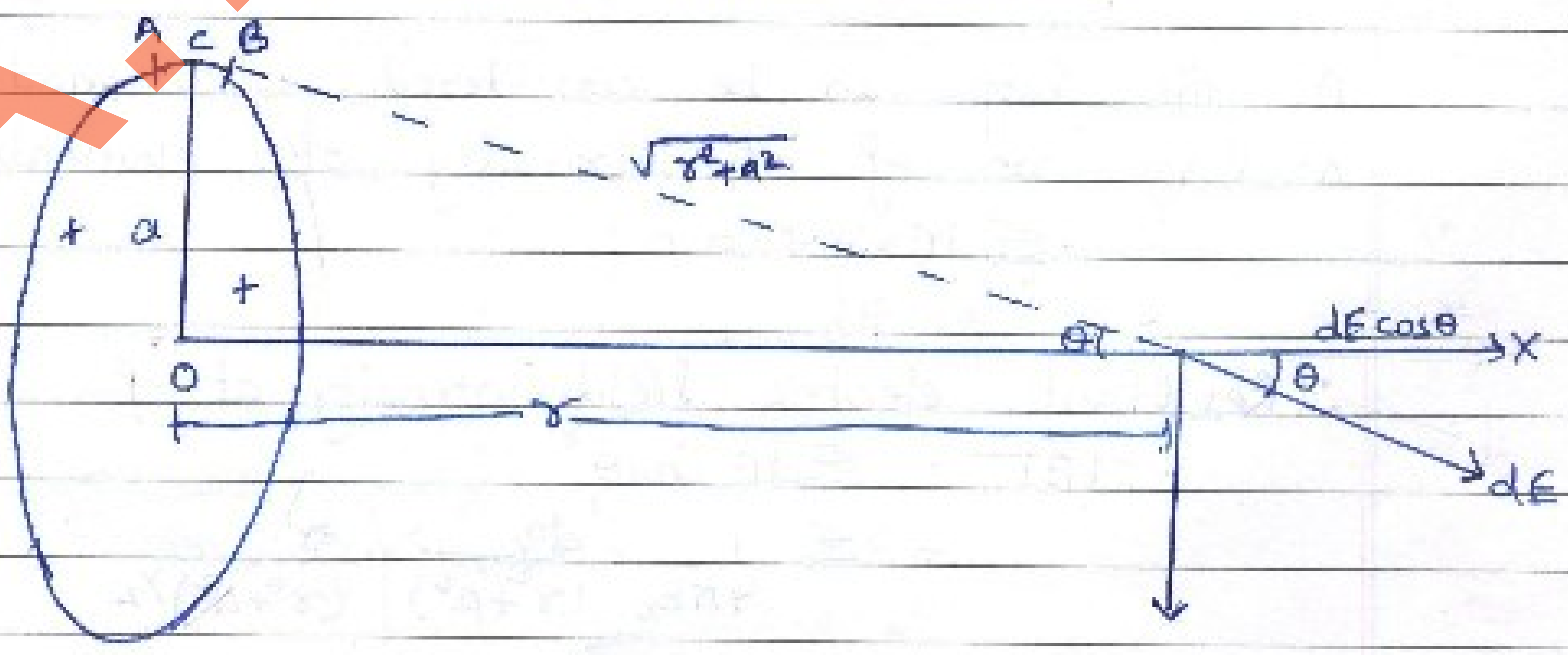
$$\boxed{\vec{E} = \frac{-\vec{p}}{4\pi\epsilon_0 (r^2+a^2)^{3/2}}}$$

if $2a << r$

$$\vec{E} = \frac{|\vec{p}|}{4\pi\epsilon_0 r^3}$$

Also, $\boxed{\frac{E_{axial}}{E_{equa}} = 2}$

Electric Field Intensity at any point on the axis of a uniformly charged ring



Consider a circular loop of wire of negligible thickness, radius a & centre O held $\perp^r$ to the plane of paper.
Let the loop carry a total charge $+q$ distributed uniformly over its circumference.
Let P be any pt. on the axis of loop such that $OP = r$.

Consider a small element $AB = dl$ of the loop and C be the centre of AB.
The charge on AB is

$$dq = \frac{q}{2\pi a} dl$$

Electric field intensity at P due to AB is

$$|d\vec{E}| = \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{CP^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{(r^2+a^2)}$$

$d\vec{E} \rightarrow dE\cos\theta$ along PX ($\parallel^l$ to axis)

$d\vec{E} \rightarrow dE\sin\theta$ along PY ($\perp^r$ to axis)

As the loop can be considered to be made up of a large no. of diametrically opp. elements, so

$$\Sigma dE\sin\theta = 0$$

$\therefore$ Resultant electric field intensity at P is

$$|\vec{E}| = \Sigma dE\cos\theta$$

$$= \Sigma \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{(r^2+a^2)} \cdot \frac{r}{(r^2+a^2)^{1/2}}$$

$$= \sum \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{2\pi a} dl \times \frac{r}{(r^2+a^2)^{3/2}}$$

$$= \frac{qr}{4\pi\varepsilon_0 \, 2\pi a \, (r^2+a^2)^{3/2}} \sum dl$$

$$= \frac{qr}{4\pi\varepsilon_0 \times 2\pi a \times (r^2+a^2)^{3/2}} \times 2\pi a$$

$$\boxed{|\vec{E}| = \frac{qr}{4\pi\varepsilon_0 (r^2+a^2)^{3/2}}}$$

Case 1 : When P lies at the centre of loop

$$r = 0$$

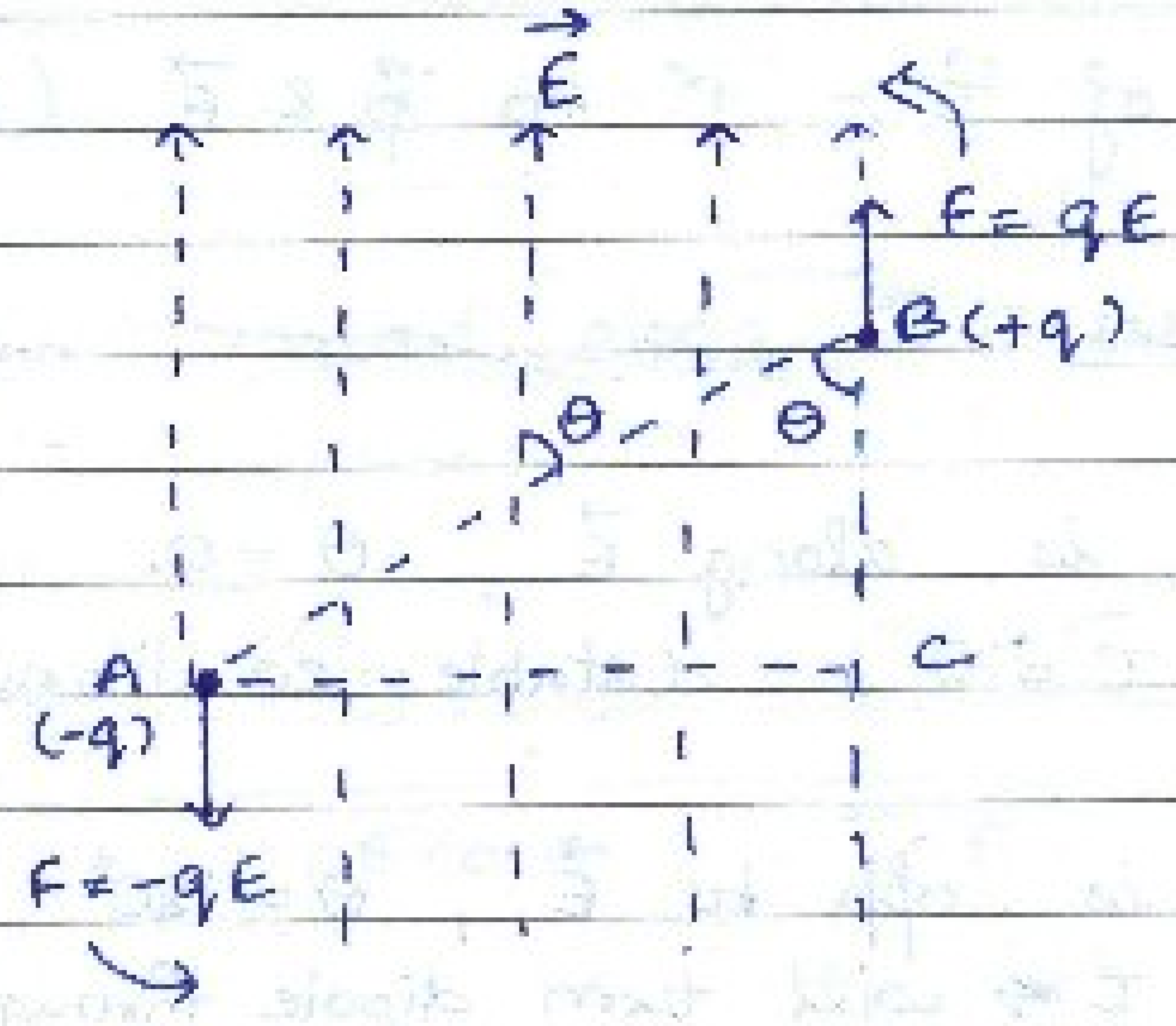
$$\vec{E} = 0$$

Case 2 : When $r >> a$

$$\vec{E} = \frac{qr}{4\pi\varepsilon_0 r^3} = \frac{q}{4\pi\varepsilon_0 r^2}$$

This is expression for $\vec{E}$ at a distance of r from a pt. charge q.

Electric dipole in a uniform electric field



Consider an electric dipole having charges $-q$ (at A) & $+q$ (at B) separated by small distance $2a$, having dipole moment $|\vec{p}| = q \times 2a$.

② Let this dipole be held in a uniform external electric field $\vec{E}$ at an angle θ with the direction of $\vec{E}$.

Force on charge at A $= -qE$ (opp to $\vec{E}$)
" " " " B $= qE$ (along $\vec{E}$)

As the forces are equal, unlike & parallel, acting at different points, they form a couple, which rotates the dipole in anti-clockwise direction.

$\therefore \quad \tau = \text{force} \times \text{arm of couple}$

$= F \times AC$

$= F \times AB \sin\theta$

$= F \times 2a \sin\theta$

$= qE \times 2a \sin\theta$

$$\boxed{\tau = pE\sin\theta}$$

or $\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$

Direction of $\vec{\tau}$ - $\perp$ to $\vec{p}$ & $\vec{E}$ (By right hand screw rule)

Special cases

① When $\vec{p}$ is along $\vec{E}$, $\theta = 0$
$\tau = 0$ (stable equilibrium)

② When $\vec{p}$ is opp to $\vec{E}$, $\theta = 180°$
$\tau \Rightarrow$ will turn dipole through 180° (unstable equilibrium)

③ When $\vec{p}$ is perpendicular to $\vec{E}$

$\tau = pE$ (max. value)

Potential Energy of dipole in uniform electric field

Suppose an electric dipole is oriented at an angle θ with the direction of uniform external electric field $\vec{E}$

The torque acting on the dipole is

$$\tau = pE\sin\theta$$

[τ tries to align the dipole with $\vec{E}$]

Small amount of work done in rotating the dipole through a small angle $d\theta$ against the torque is

$$dW = \tau\, d\theta$$
$$= pE\sin\theta\, d\theta$$

∴ Total work done in rotating the dipole from θ_1 to θ_2 is

$$W = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$$

$$= pE[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$\boxed{U = W = -pE(\cos\theta_2 - \cos\theta_1)}$$

Case 1 → dipole initially aligned along $\vec{E}$, $\theta_1 = 0°$, $\theta_2 = \theta$

$$U = -pE(\cos\theta - 1)$$

Case 2 → dipole initially aligned $\perp^r$ to $\vec{E}$, $\theta_1 = 90°$, $\theta_2 = \theta$

$$\boxed{U = -pE\cos\theta = -\vec{p}\cdot\vec{E}}$$

Area Vector

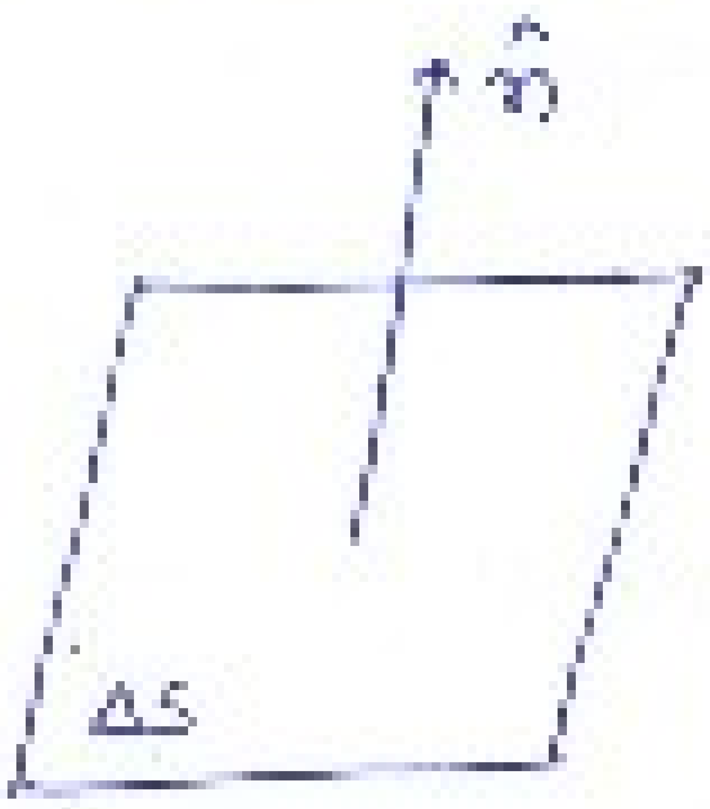
→ The orientation of area element & not only its magnitude is important in many contexts.

→ For example, in a stream, the amount of water flowing through a ring depends upon how you hold the ring.

- If you hold it normal to the flow, max. water will flow through it.
- If you hold it with some other orientation, less water will flow through it.

→ This shows that an area element should be treated as a vector.

It has a magnitude & direction (normal to the plane & outwards).



$$\Delta\vec{S} = \Delta S\,\hat{n}$$

where ΔS - magnitude of area element

$\hat{n}$ - unit vector in a direction normal (outward) at that point.

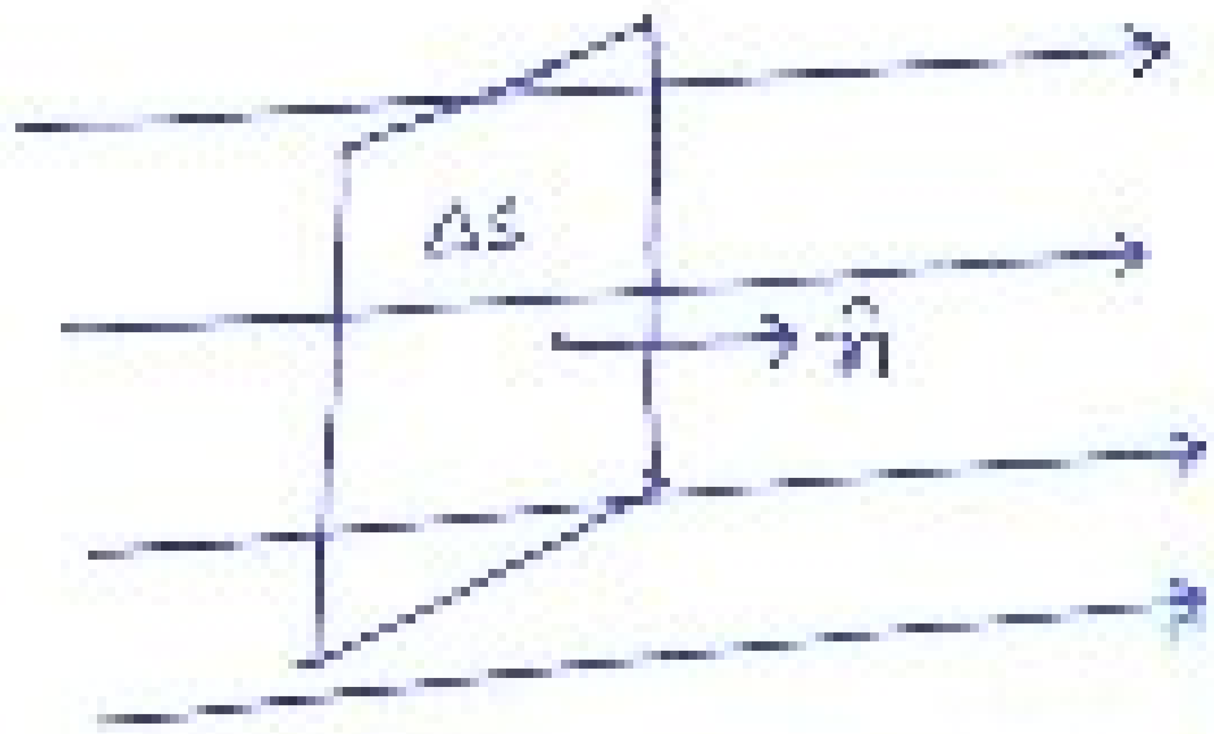
* How to associate a vector to the area of a curved surface ?

1. Imagine dividing the surface into a large number of very small area elements.
2. Each small area element may be treated as a planar & a vector associated with it.

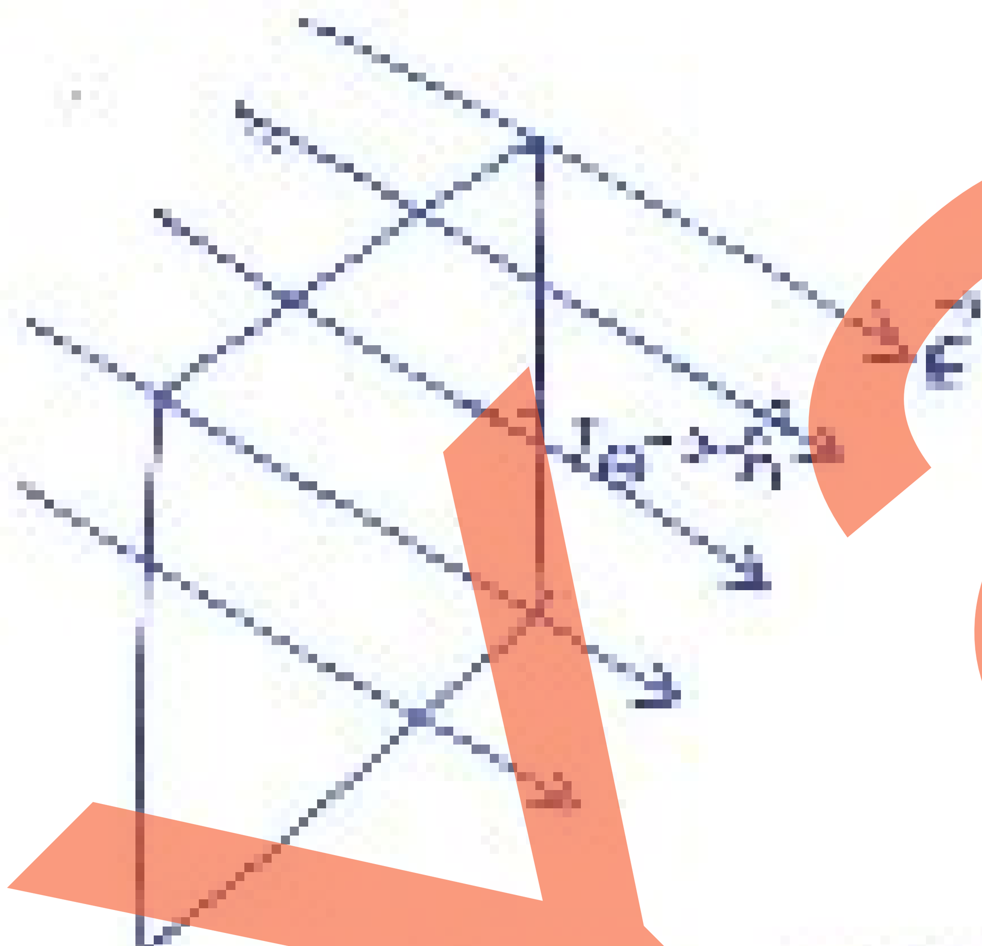
* By convention, the vector associated with every area element of a closed surface is taken to be in the direction of outward normal.

Electric flux (ϕ)

Electric flux through an area element ΔS is proportional to the no. of field lines crossing the area element.



If we tilt $\vec{E}$ w.r.t area element by angle θ, the projection or component of $\vec{E}$ normal to area element is $E\cos\theta$



$\therefore$ No. of electric field lines crossing ΔS is proportional to $E\,\Delta S\cos\theta$

So, electric flux through an area element $\Delta\vec{S}$ in an electric field $\vec{E}$ is defined as

$$\boxed{\Delta\phi = \vec{E}\cdot\Delta\vec{S} = E(\Delta S)\cos\theta}$$

where $\theta \rightarrow$ small angle betn $\vec{E}$ & $\Delta\vec{S}$ & outward normal ($\hat{n}$) to ΔS

$\rightarrow$ angle betn $\vec{E}$ & outward normal (for a closed (curved) surface)

* $\Delta\phi = E\,\Delta S\cos\theta = E(\Delta S\cos\theta) \rightarrow E$ times the projection of area element to $\vec{E}$.

$= (E\cos\theta)\,\Delta S \rightarrow$ component of $\vec{E}$ along normal to the area element times the magnitude of area element.

* When $\vec{E}$ is normal to area element, $\theta = 0°$ [angle bet^n $\vec{E}$ & $\hat{n}$]

ϕ - max ($E\,\Delta S$)

* When $\vec{E}$ is along area element, $\theta = 90°$ (angle bet^n $\vec{E}$ & $\hat{n}$)

$\phi = 0$ (min)

* $\theta > 90°$, $\phi = -ve$

* Total electric flux through any given surface (divide the surface into small area elements, find flux of each element & add them) is

$$\boxed{\phi_E \approx \sum \vec{E}\cdot\vec{ds} = \oint \vec{E}\cdot\vec{ds}}$$

$\oint$ - closed integral (surface of integration is a closed surface)

* Electric flux is a scalar quantity

* S.I unit - Nm^2C^{-1}

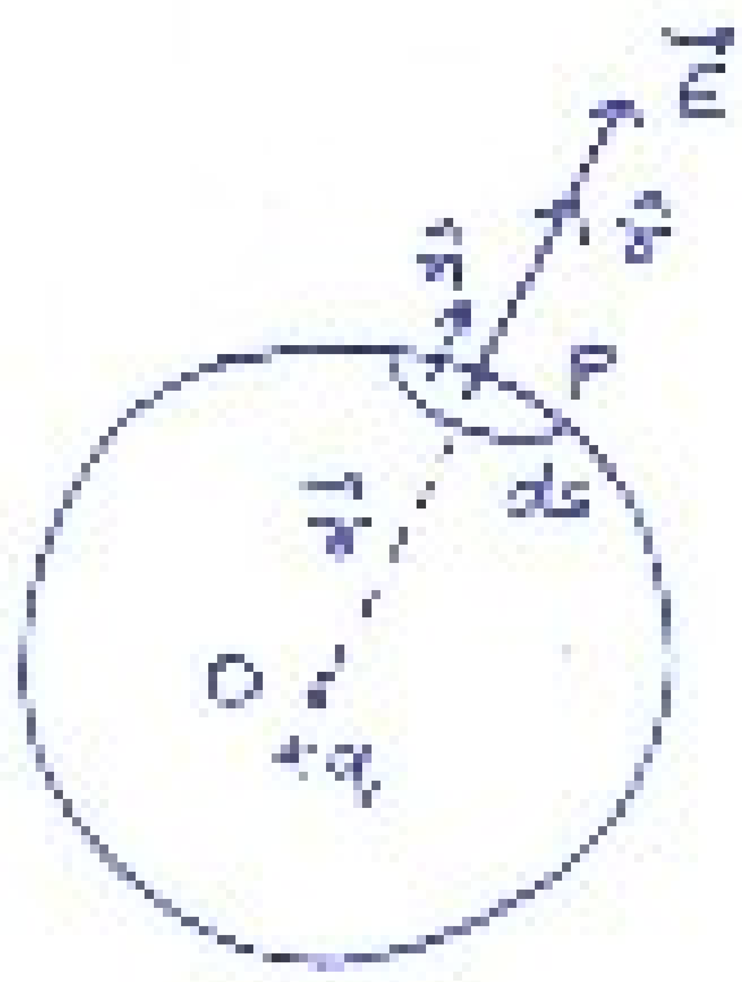
Dimensions - $ML^3T^{-3}A^{-1}$

Gauss Theorem

"Total electric flux over the closed surface S in vacuum is $1/\epsilon_0$ times the total charge (Q) enclosed inside S."

$$\boxed{\phi_E = \oint_S \vec{E}\cdot\vec{ds} = \frac{Q}{\epsilon_0}}$$

Proof:



Suppose an isolated positive point charge q is situated at the centre O of a sphere of radius r.

Electric field intensity at any pt P on the surface of sphere is

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\,\frac{\hat{r}}{r^2}$$

$\hat{r}$ - unit vector from O to P

Consider a small area element 'ds' of the sphere around P

$$d\vec{s} = \hat{n}\, ds$$ ($\hat{n}$ – unit vector along outward normal to area element)

$\therefore$ Electric flux over area element

$$d\phi_E = \vec{E} \cdot d\vec{s}$$

$$= \frac{q}{4\pi\varepsilon_0} \cdot \frac{\hat{r}}{r^2} \cdot \hat{n}\, ds$$

$$= \frac{q}{4\pi\varepsilon_0} \cdot \frac{ds}{r^2} \qquad [\because \hat{r} \cdot \hat{n} = 1]$$

Total normal electric flux over the entire sphere is

$$\phi_E = \oint_S \vec{E} \cdot d\vec{s}$$

$$= \frac{q}{4\pi\varepsilon_0 r^2} \oint_S ds$$

$$= \frac{q}{4\pi\varepsilon_0 r^2} \cdot 4\pi r^2$$

$$\boxed{\phi_E = \oint_S \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0}}$$

* If $q_1, q_2, \ldots\ldots q_n$ are point charges lying inside the surface, each will contribute to the electric flux, independent of the others, so

$$\phi_E = \phi_{E_1} + \phi_{E_2} + \ldots\ldots + \phi_{E_n}$$

$$= \frac{1}{\varepsilon_0}(q_1 + q_2 + \ldots\ldots + q_n)$$

$$\boxed{\phi_E = \frac{Q}{\varepsilon_0}}$$

So, total electric flux over a closed surface in vacuum is $\frac{1}{\varepsilon_0}$ time the total charge enclosed irrespective of how the charge is distributed.

Important points regarding Gauss law

① It is true for any closed surface irrespective of its size or shape.

② Total electric flux through a closed surface is zero either if no charge is enclosed by the surface or the total sum of all the charges enclosed by the surface is zero

Explanation of 2nd part of above statement:

Consider a closed cylindrical surface placed in a uniform external ~~wate~~ electric field $\vec{E}$.

Total flux is $\phi = \phi_1 + \phi_2 + \phi_3$

$$\phi_1 = -E.S \quad , \quad \phi_2 = E.S \quad , \quad \phi_3 = 0$$

$$\therefore \phi = 0$$

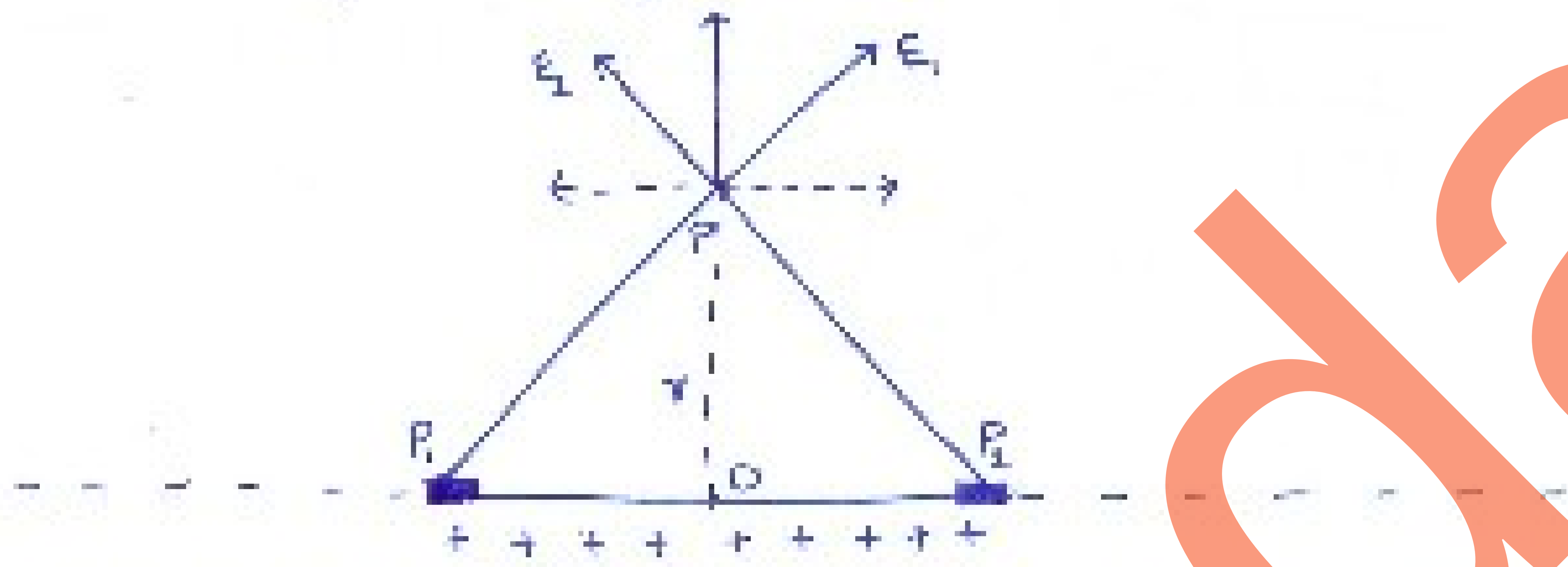
③ If the Gaussian surface is so chosen that there are some charges inside & some outside, while applying Gauss law, we shall only consider those charges, which lie inside the surface.

④ Gauss law is based on the inverse square dependence of distance contained in the Coulomb's law. Any violation of Gauss law indicate departure from the inverse square law.

Applications of Gauss Law

① Electric field due to infinitely long straight wire

Consider an infinitely long straight wire with uniform linear charge density λ.



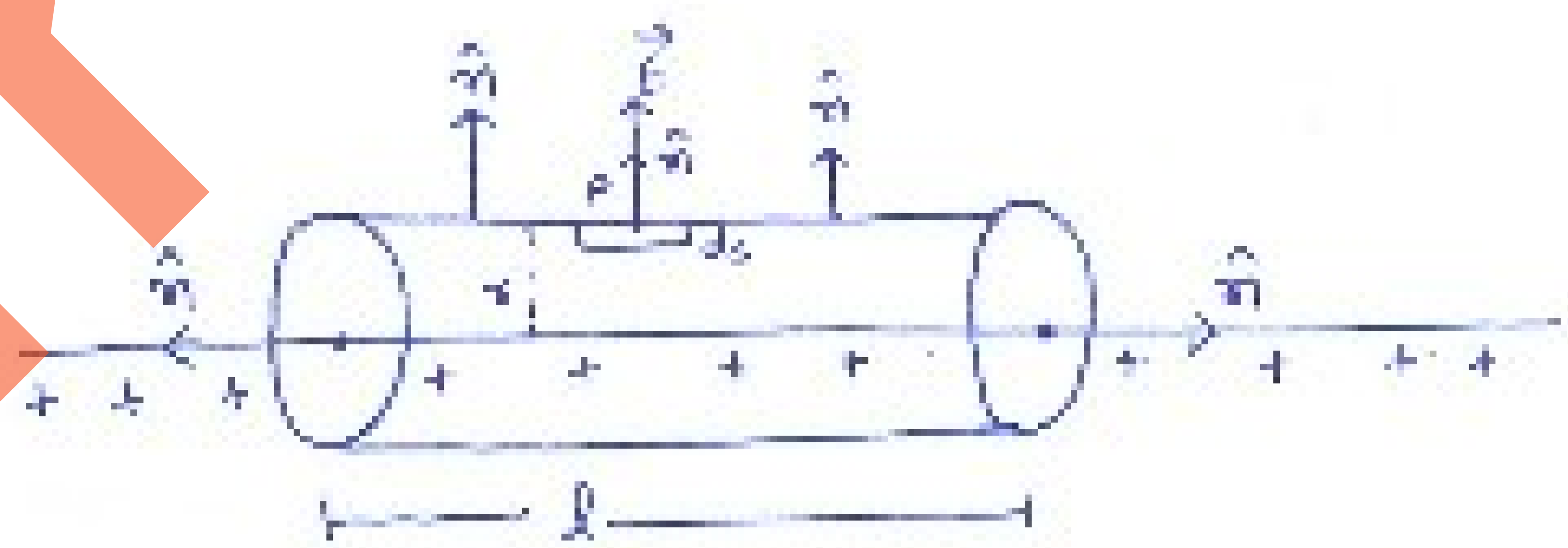
Consider a pair of line elements P_1 & P_2 of the wire at equal distances on either side of O.

Electric fields at P (pt. at which field is to be calculated) due to P_1 & P_2 are $\vec{E}_1$ & $\vec{E}_2$.

Components of E_1 & E_2 along horizontal cancels out while they get added up vertically.

So the total field at pt. P is radial

Consider a right circular closed cylinder of radius 'r' & length 'l' with the infinitely long straight uniformly charged wire as its axis.



Electric flux over the curved surface area of cylinder

$$\oint_S \vec{E}\cdot\vec{ds} = \oint_S \vec{E}\cdot\hat{n}\, ds = \oint_S E\, ds \cos 0 = E\oint_S ds = E \times 2\pi r l$$

Electric flux at the ends of the cylinder

$$\oint_S \vec{E}\cdot\vec{ds} = \oint_S E\,ds\cos 90^\circ = 0$$

$\therefore$ Total flux over the whole cylinder

$$\phi_E = E \times 2\pi r l$$

Acc. to Gauss theorem

$$\phi_E = \frac{q}{\epsilon_0}$$

$$E \times 2\pi r l = \frac{\lambda l}{\epsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

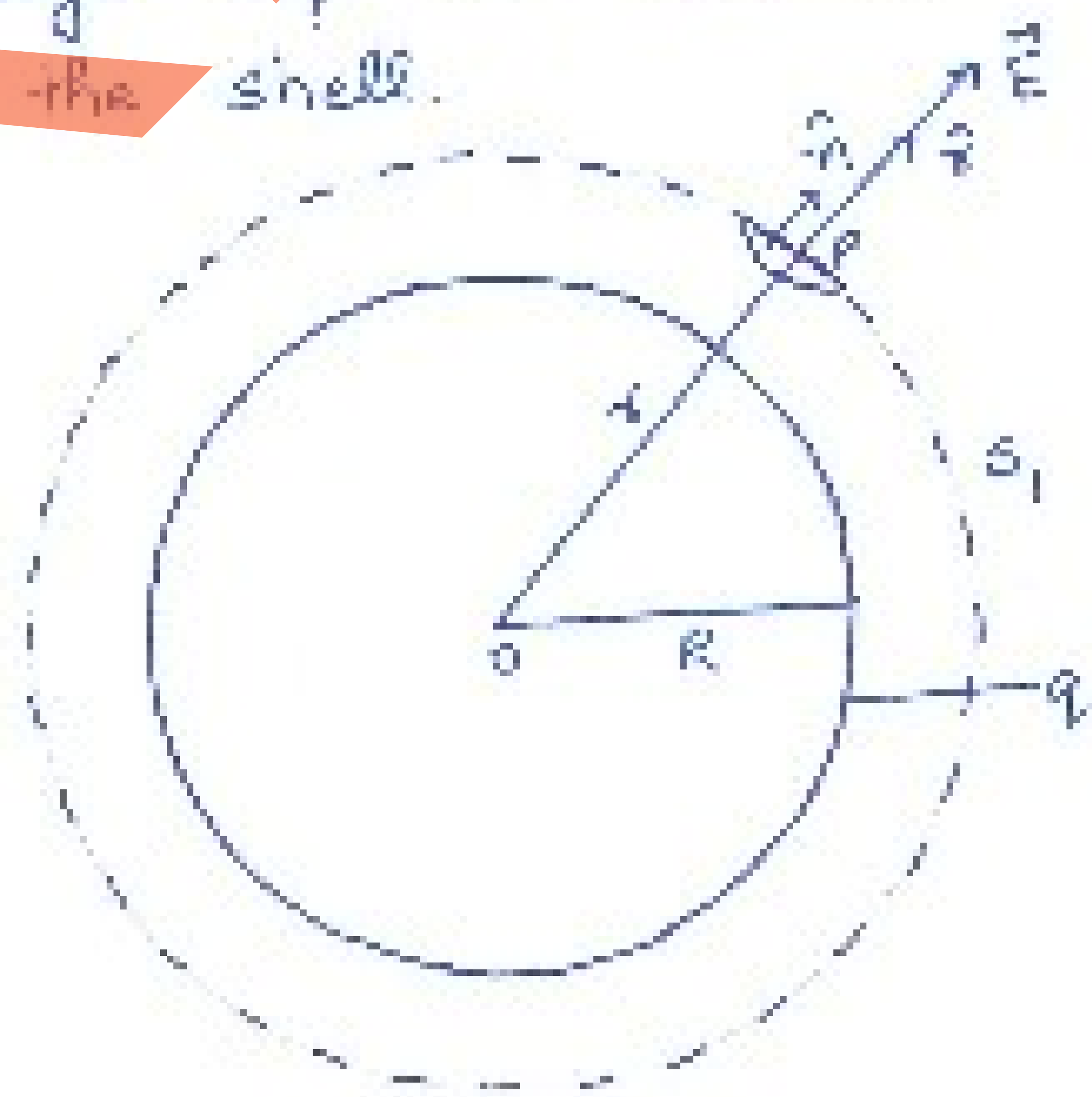
* $\lambda > 0$, direction of $\vec{E}$ at every point – radially outwards

$\lambda < 0$ " " " " " – radially inwards

ⓐ Electric field intensity due to a uniformly charged spherical shell

(a) Field outside the shell

Consider a thin spherical shell of radius R with centre O. Let a charge +q be distributed uniformly over the surface of the shell.



Consider/suppose a sphere S_1 with centre O & radius r (=OP). The surface of this sphere is a Gaussian surface.

Acc. to Gauss theorem

$$\oint_S \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$$

$$E \oint_S \vec{ds} = \frac{q}{\varepsilon_0}$$

$$E \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$

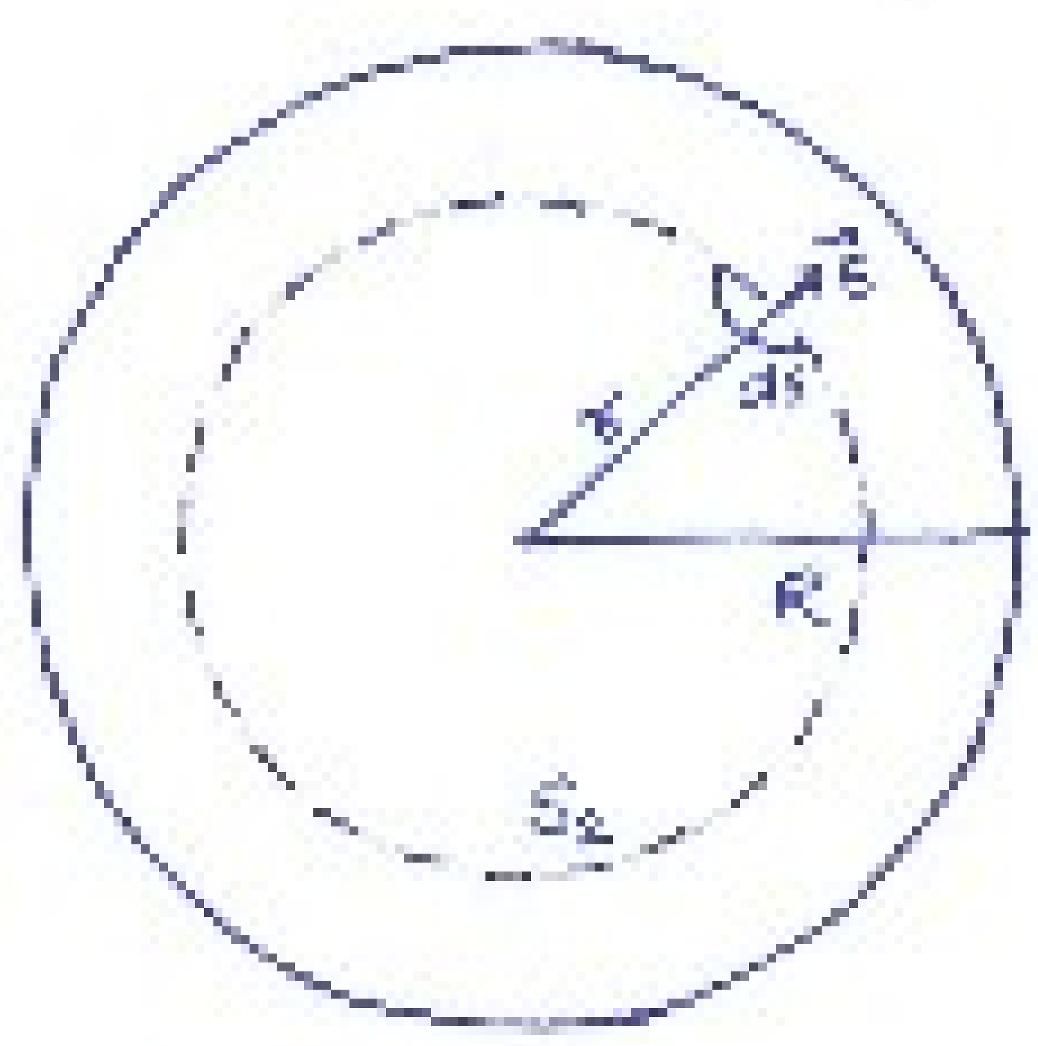
* At a point on the surface of the shell, $r = R$

$$E = \frac{q}{4\pi\varepsilon_0 R^2}$$

As $\sigma = \frac{q}{A} = \frac{q}{4\pi R^2}$ | σ - surface charge density

$$\therefore \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

(b) Field inside the shell



Gaussian surface is the surface of sphere S_2 passing through P & centre O.

Flux through Gaussian surface

$$\phi = E \times 4\pi r^2$$

As charge inside a spherical shell is zero, the Gaussian surface encloses no charge. i.e. $q = 0$

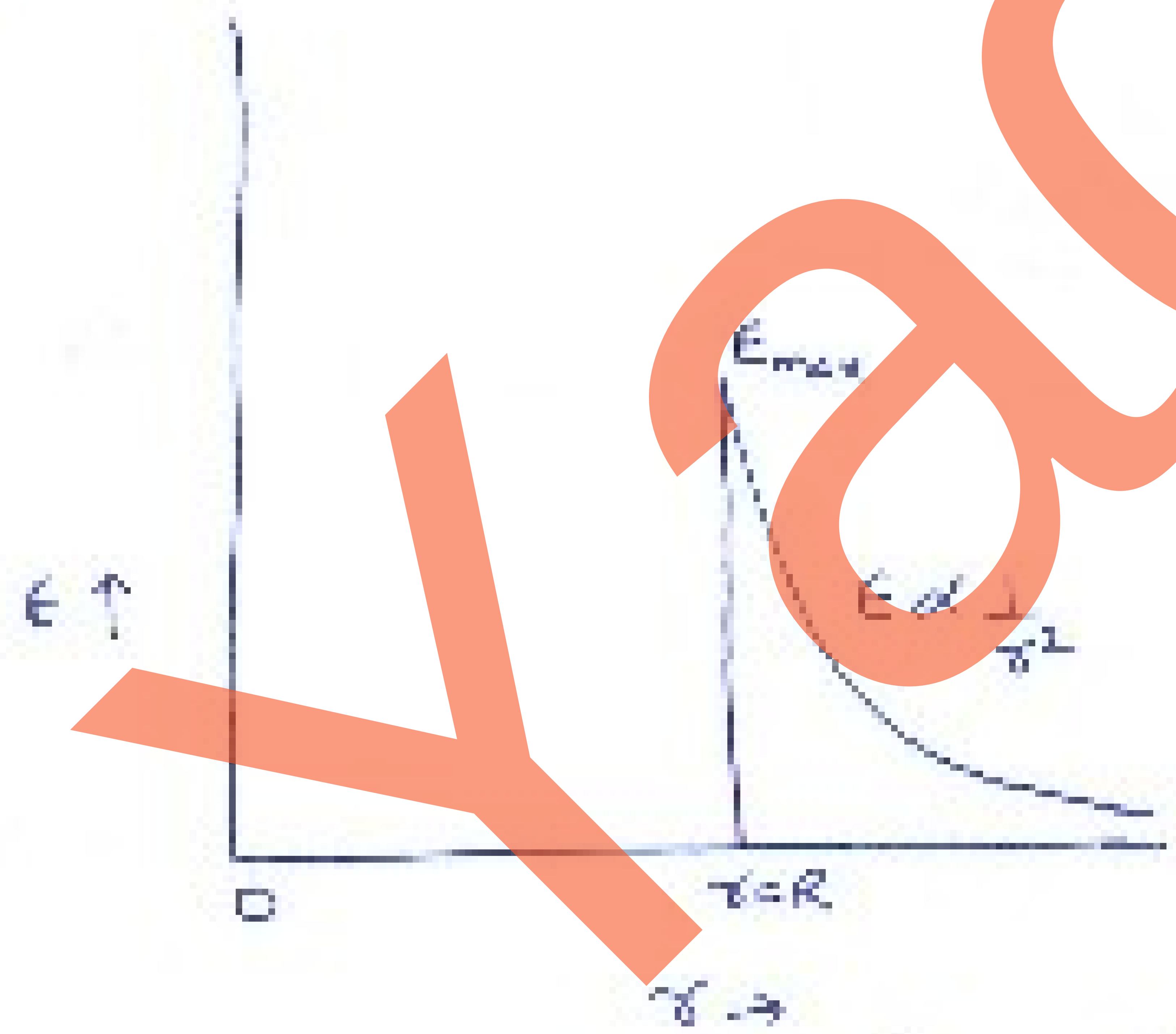
Acc. to Gauss theorm

$$\phi = \frac{q}{\varepsilon_0}$$

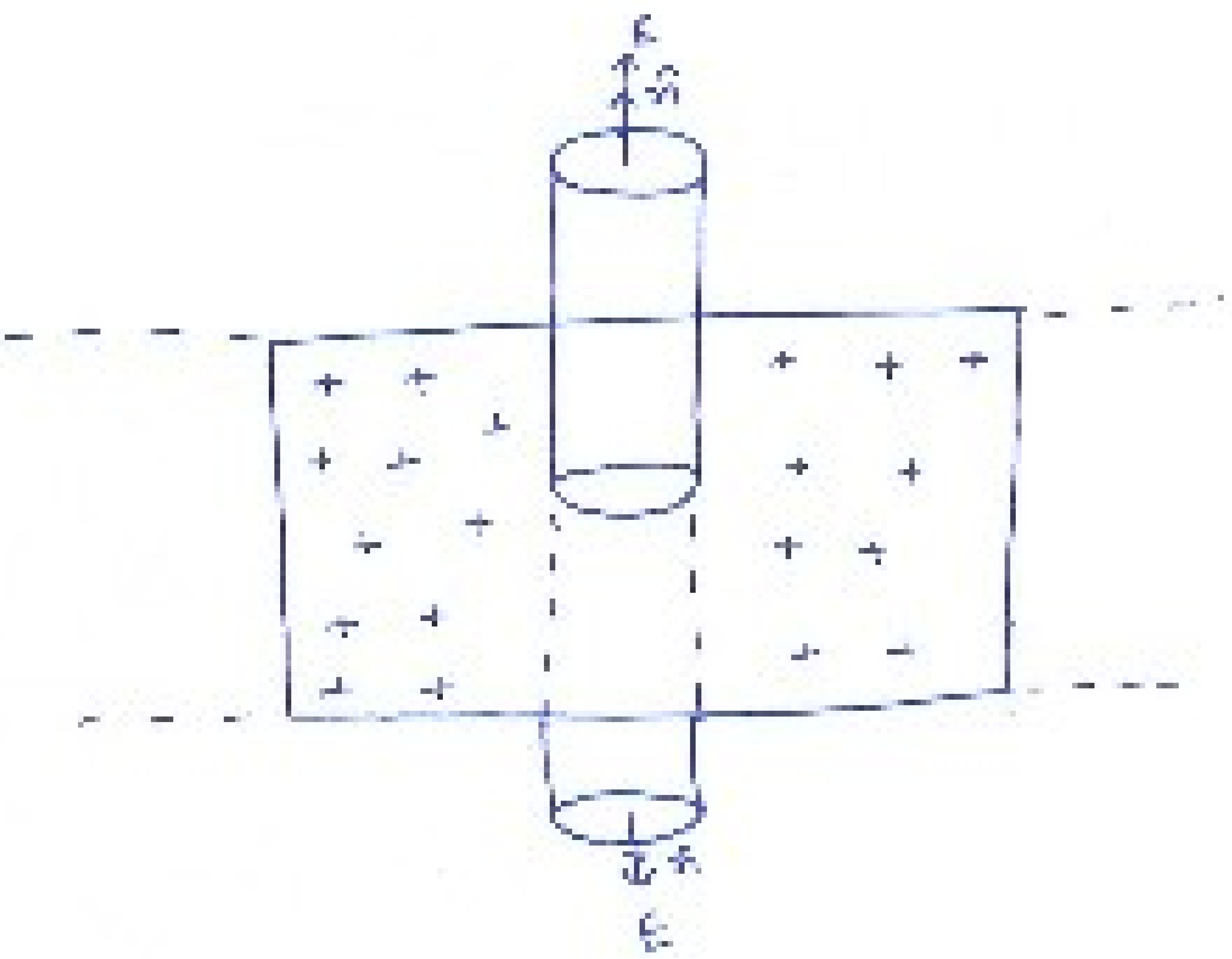
$$E \times 4\pi r^2 = 0$$

$$\boxed{E = 0} \quad \text{for } r < R$$

* Variation of E with distance (r) from centre of shell



③ Electric field intensity due to a thin infinite plane sheet of charge



Consider a very thin and infinite plane sheet on which charge q is uniformly distributed over its surface

Let σ - surface charge density of sheet

$$\sigma = \frac{q}{ds}$$

Consider a cylinder of cross-sectional area 'ds' around P & length '2r' through the sheet.

Electric flux around circular ends

$$\phi_c = 2\vec{E}.\hat{n}\, ds = 2E\, ds$$

Electric flux around curved surface

$$\phi_s = \vec{E}.\hat{n}\, ds = 0 \qquad | \because E \perp \hat{n}$$

$\therefore$ Total electric flux over the entire surface of cylinder

$$\phi = 2E\, ds$$

Acc. to Gauss theorem

$$\phi = \frac{q}{\varepsilon_0}$$

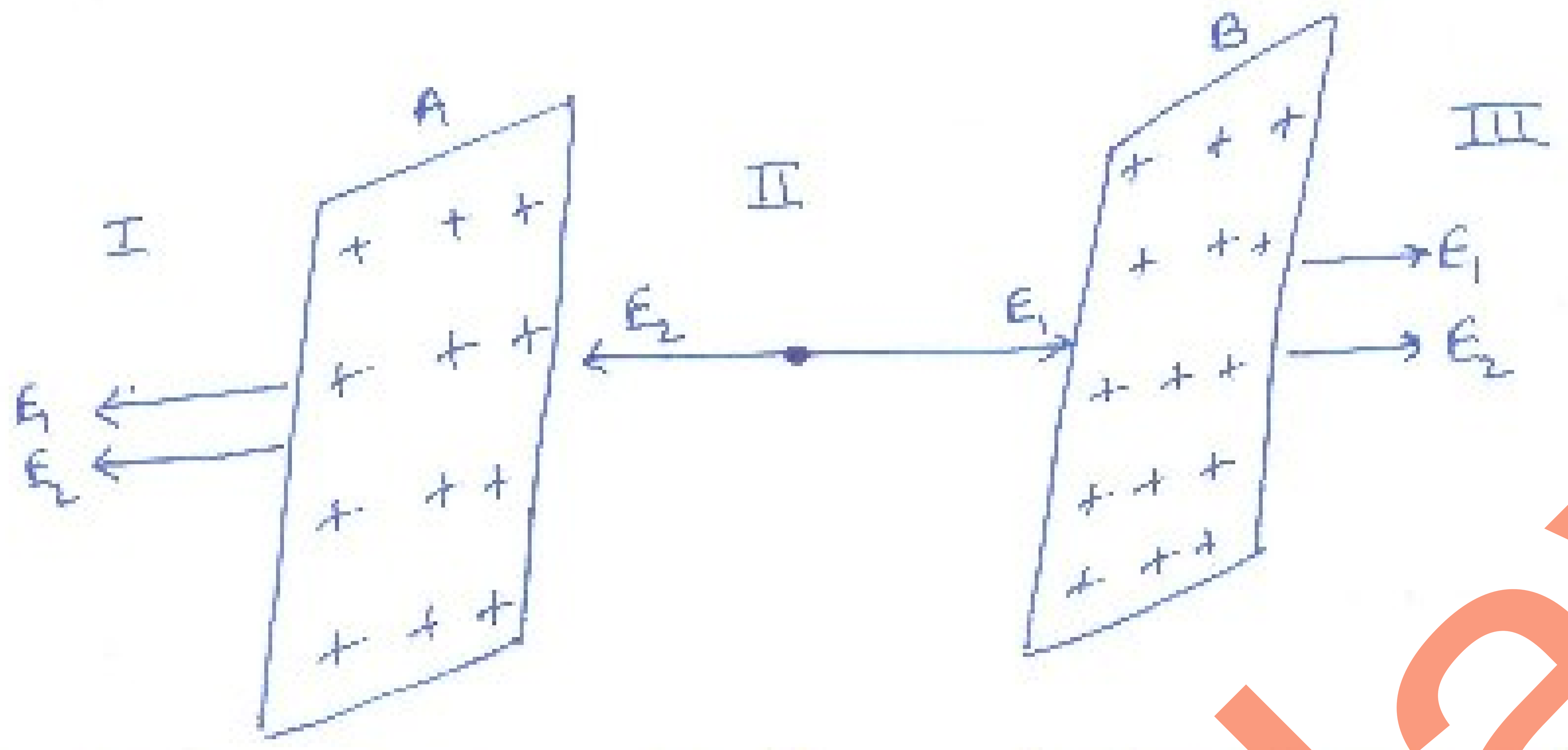
$$2E\, ds = \frac{q}{\varepsilon_0} = \frac{\sigma\, ds}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

* E is independent of r
* $\sigma > 0$, E - uniform, normal, outwards

  $\sigma < 0$, E - " " inwards

Electric field intensity due to 2 thin infinite parallel sheets of charge



Let A & B be 2 thin infinite plane charged sheets held parallel to each other.

Electric field intensity at a pt. due to A, $E_1 = \dfrac{\sigma_1}{2\epsilon_0}$

" " " " " " B, $E_2 = \dfrac{\sigma_2}{2\epsilon_0}$

Let $\sigma_1 > \sigma_2 > 0$

Region I : $E = -E_1 - E_2 = -\dfrac{\sigma_1}{2\epsilon_0} - \dfrac{\sigma_2}{2\epsilon_0} = -\dfrac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)$

Region II : $E = E_1 - E_2 = \dfrac{\sigma_1}{2\epsilon_0} - \dfrac{\sigma_2}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(\sigma_1 - \sigma_2)$

Region III : $E = E_1 + E_2 = \dfrac{\sigma_1}{2\epsilon_0} + \dfrac{\sigma_2}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(\sigma_1 + \sigma_2)$

Special case : If $\sigma_1 = \sigma$, $\sigma_2 = -\sigma$

Region I : $E = 0$

Region II : $E = \dfrac{2\sigma}{2\epsilon_0} = \dfrac{\sigma}{\epsilon_0} =$ constant

Region III : $E = 0$

Electrostatic Potential

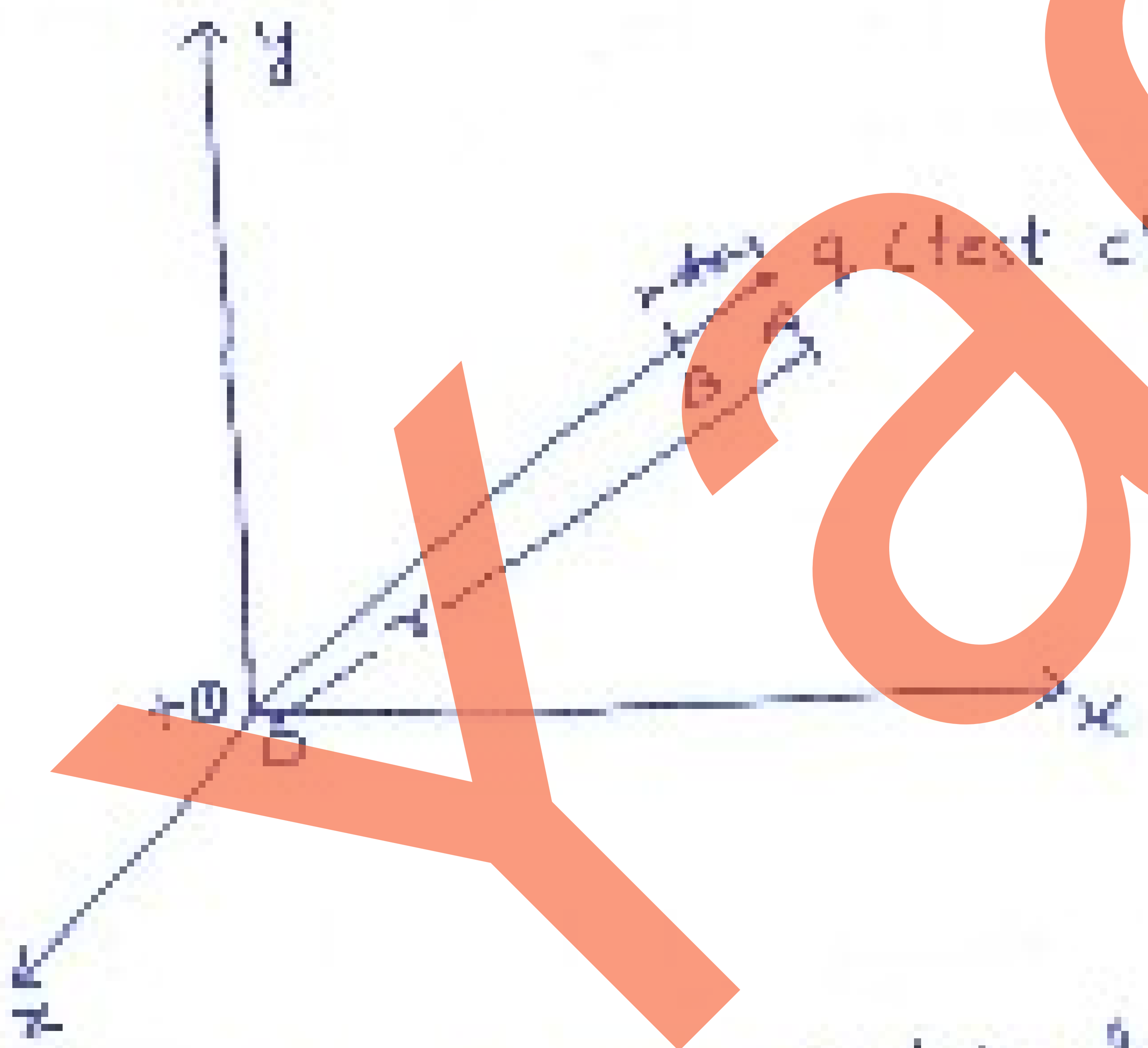
→ Electrostatic potential of a charged body represents the degree of electrification of body.

→ It determines the direction of flow of charge betⁿ 2 charged bodies placed in contact with each other.

→ The charge always flow from a body at higher potential to another body at lower potential.

→ The flow of charge stops as soon as the potentials of the 2 bodies become equal.

Electrostatic potential Energy



→ Consider a charge +Q having electric field intensity $\vec{E}$ placed at the origin O.

→ Let a small test charge (+q) be brought from A to B, against the repulsive force of +Q.

→ We assume

(a) +q is so small that it does not disturb the configuration of +Q.

(b) External force $\vec{F}_{ext}$ applied is just sufficient to counter the repulsive force $\vec{F}_E$ on +q, so that net force on q is zero & it moves from A to B without acc.

→ In this situation, work done by $\vec{F}_{ext}$ is −ve of $\vec{F}_E$ & get fully stored in q as potential energy.

→ On reaching B, if $\vec{F}_{ext}$ is removed from +q, $\vec{F}_E$ will take +q away from +Q.

→ The stored potential energy is used to provide K.E to q in such a way that the sum of K.E & P.E at every point is conserved.

Work done by $\vec{F}_{ext}$ in moving +q from A to B is

$$\boxed{W_{AB} = \int_A^B \vec{F}_{ext} \cdot d\vec{r} = -\int_A^B \vec{F}_E \cdot d\vec{r}}$$

" Electrostatic potential energy betn 2 points is the minimum work required to be done by an external force in moving without acceleration a test charge q from A to B".

* $\Delta U = W_{AB} = U_B - U_A$

If pt. A is at infinity, $U_A = U_\infty = 0$

$\therefore \boxed{W_{\infty B} = U_B - U_\infty = U_B}$

$\therefore$ Electrostatic potential energy of a charge 'q' at a pt. (B) in Electrostatic field due to any charge configuration is the work done by the external force (equal & opposite to electric force) in bringing the charge q from ∞ to that point without any acceleration.

Electrostatic potential (V)

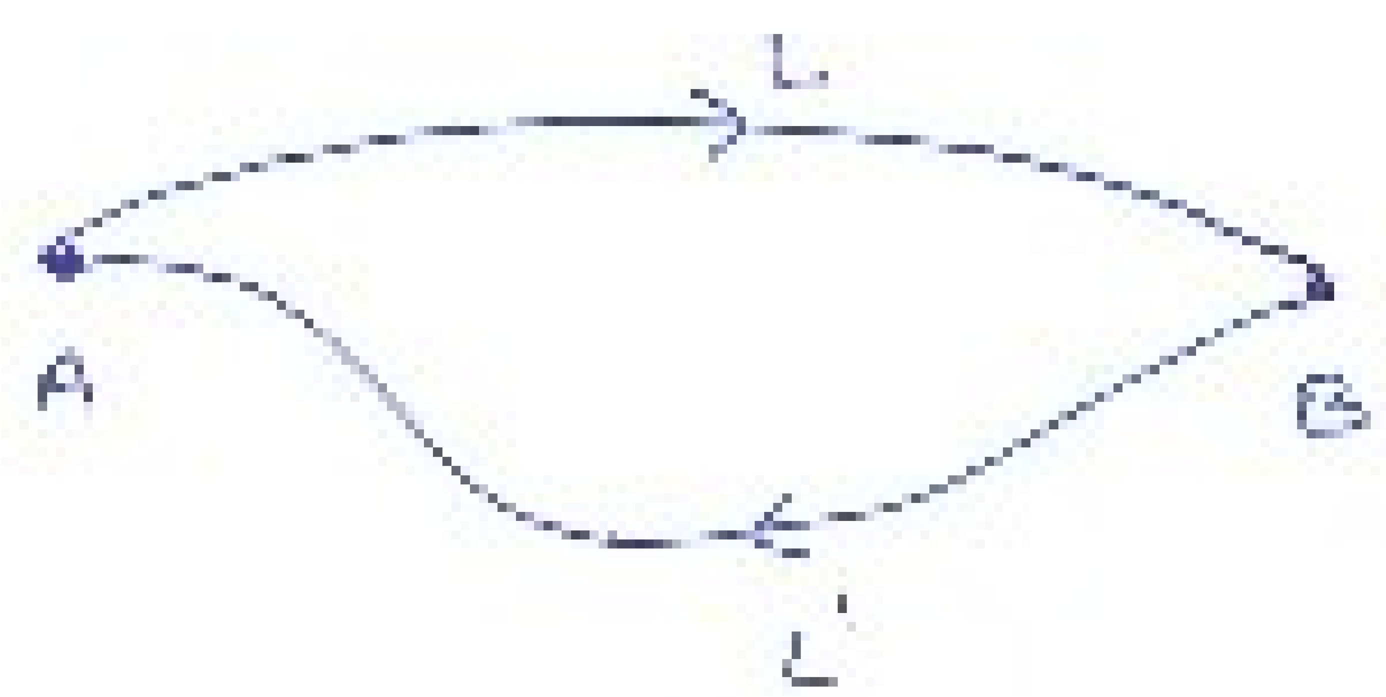
Electrostatic potential at any pt. in a region of electrostatic field is the amount (minimum) of work done in moving a unit positive charge without acc from ∞ to that point.

$$\boxed{V = \frac{W}{q}}$$

Unit → $1V = JC^{-1} = NmC^{-1}$

Dimensional formula → $ML^2T^{-3}A^{-1}$

Electrostatic forces are conservative



Work done in carrying unit positive charge from A to B

$$\frac{W_{AB}}{q_0} = V_B - V_A \quad \text{---} (1)$$

Work done in moving unit positive charge from B to A

$$\frac{W_{BA}}{q_0} = V_A - V_B \quad \text{---} (2)$$

Total work done in carrying unit +ve charge over a closed path $A \to B \to A$ is

$$\frac{W_{AB}}{q_0} + \frac{W_{BA}}{q_0} = V_B - V_A + V_A - V_B = 0$$

Electrostatic potential due to a point charge

The electrostatic force (i.e. electric field intensity) on a unit positive charge at A is

$$E = \frac{1}{4\pi\epsilon_0} \frac{1}{x^2} \quad \text{along OA}$$

Small work done in moving the unit positive charge from A to B is

$$dW = \vec{E} \cdot \vec{dx} = E\,dx \cos 180^\circ = -E\,dx$$

$\therefore$ Total work done in moving the unit positive charge from ∞ to P is

$$W = \int_{\infty}^{r} -E\,dx$$

$$= -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x^2}\,dx$$

$$= \frac{-q}{4\pi\varepsilon_0}\int_{\infty}^{r} x^{-2}\,dx$$

$$= \frac{-q}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}-\frac{1}{\infty}\right]$$

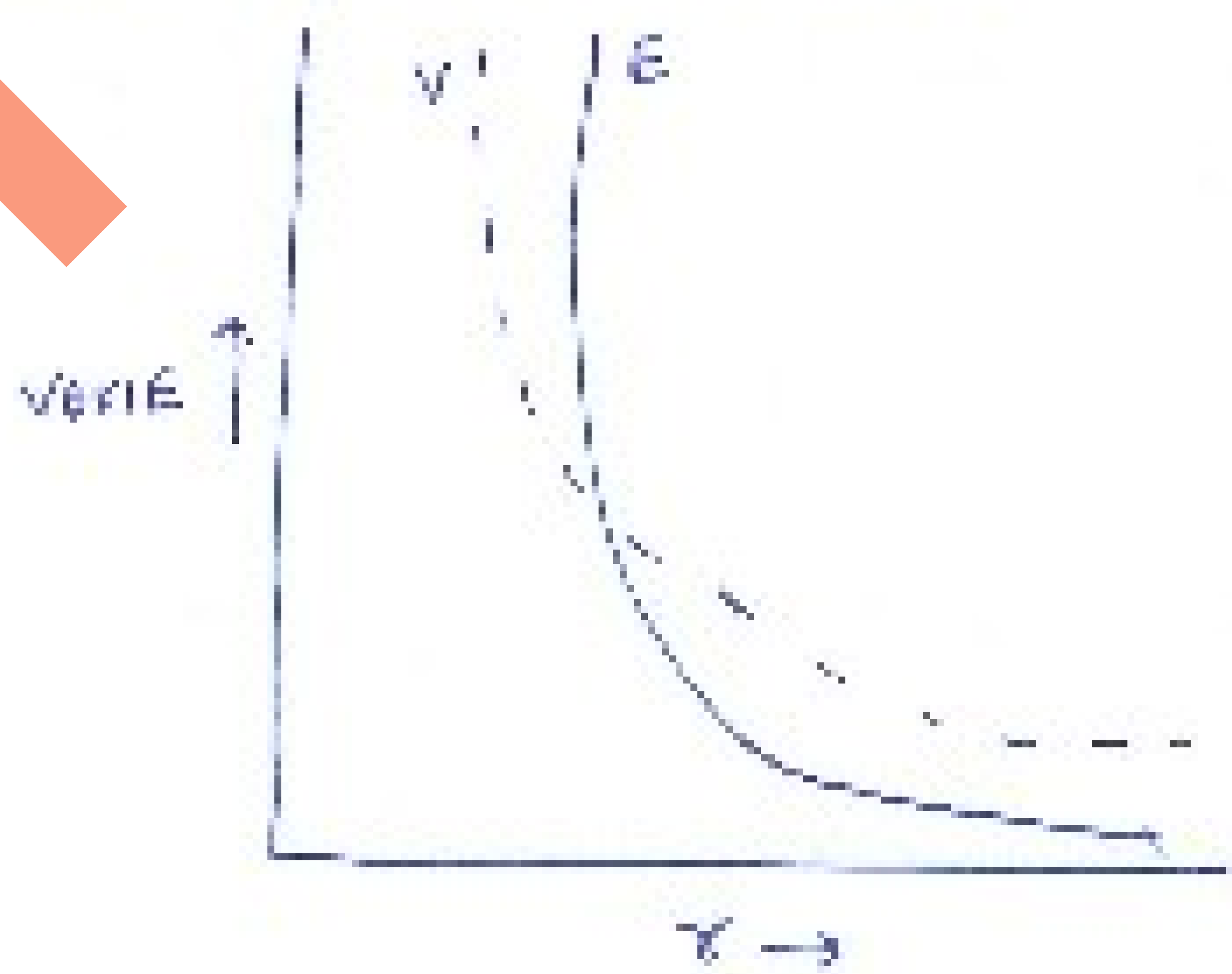
$$\boxed{V = W = \frac{q}{4\pi\varepsilon_0 r}}$$

(i) $q = +ve$, $V = +ve$
$= -ve$, $V = -ve$

(ii) At $r = \infty$, $V = 0$

(iii) Electrostatic potential due to a single charge is spherically symmetric as at equal distances from q, V is same.

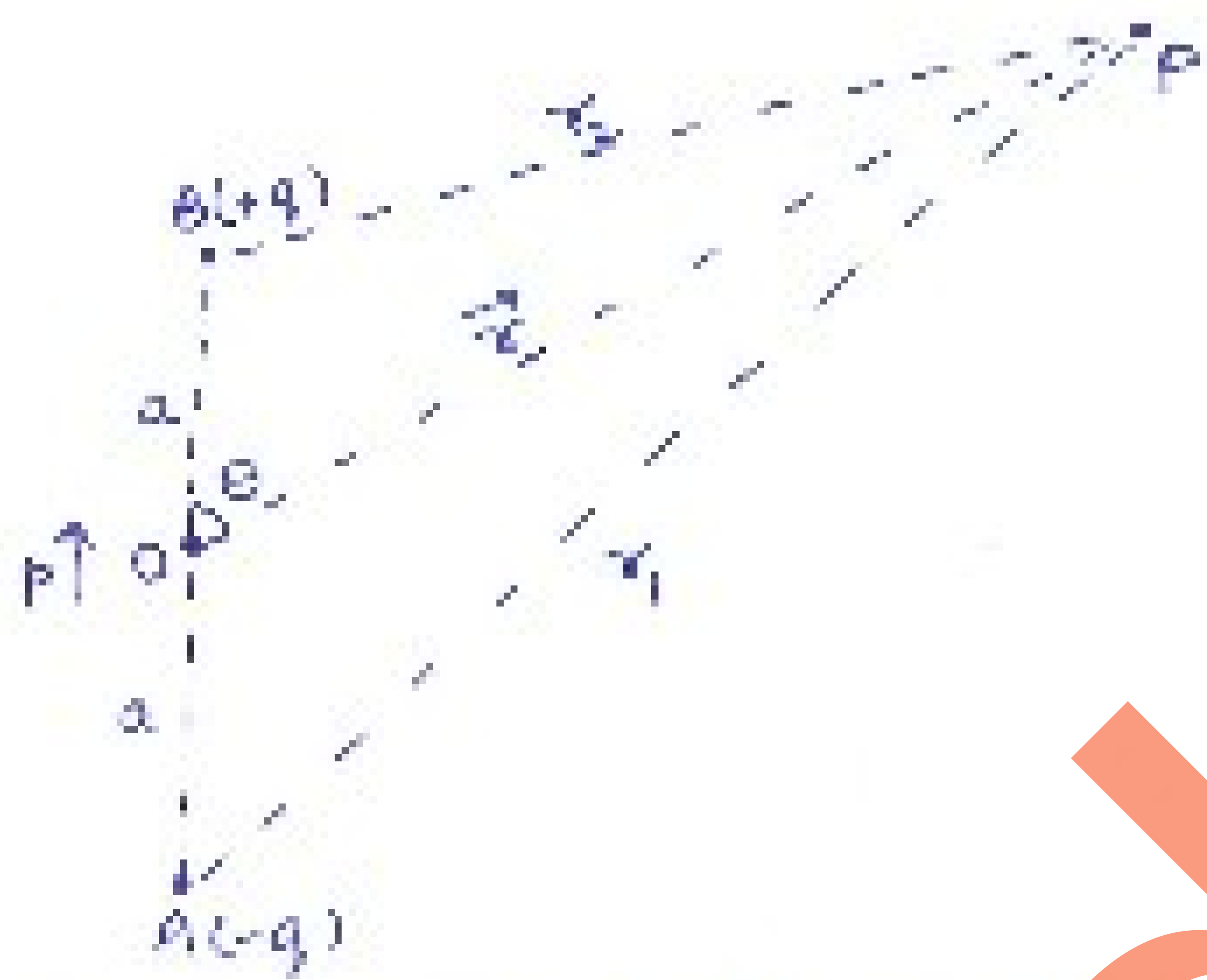
(iv) Variation of V & E with r



$V \propto \frac{1}{r}$

$E \propto \frac{1}{r^2}$

Electrostatic potential at a point due to a dipole



Consider a point P such that $AP = r_1$, $BP = r_2$ & $OP = \vec{r}$

Electrostatic potential at P due to (-q) charge at A

$$V_1 = \frac{-q}{4\pi\epsilon_0 r_1}$$

Electrostatic potential at P due to (+q) charge at B

$$V_2 = \frac{q}{4\pi\epsilon_0 r_2}$$

$\therefore$ Potential at P due to the dipole is

$$V = V_1 + V_2 = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \text{ --- (1)}$$

By geometry

$$r_1^2 = r^2 + a^2 + 2ar\cos\theta$$

$$r_2^2 = r^2 + a^2 - 2ar\cos\theta$$

Explanation of above step (only for understanding)

Draw $AC \perp OP$ (on extension) & $BD \perp OP$

In $\triangle BDO$, $OD = a\cos\theta$

$\triangle ACO$, $OC = a\cos\theta$

Now, $r_1 = AP = CP = OP + OC = r + a\cos\theta$

$$r_1^2 = r^2 + a^2 + 2ar\cos\theta$$

$r_2 = BP = DP = OP - OD = r - a\cos\theta$

$$r_2^2 = r^2 + a^2 - 2ar\cos\theta$$

Now, $r_1^2 = r^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)$

If $a << r$, $\frac{a^2}{r^2}$ can be neglected

$$r_1^2 = r^2\left(1 + \frac{2a}{r}\cos\theta\right)$$

$$r_1 = r\left(1 + \frac{2a}{r}\cos\theta\right)^{1/2}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{2a}{r}\cos\theta\right)^{-1/2}$$

Similarly: $\frac{1}{r_2} = \frac{1}{r}\left(1 - \frac{2a}{r}\cos\theta\right)^{-1/2}$

$$\therefore V = \frac{q}{4\pi\varepsilon_0 r}\left[\left(1 - \frac{2a}{r}\cos\theta\right)^{-1/2} - \left(1 + \frac{2a}{r}\cos\theta\right)^{-1/2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left[1 + \frac{a}{r}\cos\theta - 1 + \frac{a}{r}\cos\theta\right]$$

$$= \frac{q \times 2a\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\boxed{V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2}}$$

$\because p\cos\theta = \vec{p}\cdot\hat{r}$

$\hat{r}$ - unit vector along $\vec{OP}$

Special cases

① On the dipole axis

$\theta = 0^\circ$, $V = \frac{p}{4\pi\varepsilon_0 r^2}$

$= 180^\circ$, $V = \frac{-p}{4\pi\varepsilon_0 r^2}$

② At any point in the equatorial plane

$\theta = \frac{\pi}{2}$; $\cos\frac{\pi}{2} = 0$

$\therefore V = 0$

Equipotential Surfaces

- An equipotential surface is that surface at every point of which electric potential is same.

P.D. betⁿ 2 points B & A = Work done in carrying unit positive charge from A to B

$$V_B - V_A = W_{AB}$$

If A & B lie on equipotential surface, then $V_B = V_A$

$$\therefore W_{AB} = V_B - V_A = 0$$

So, no work is done in moving the test charge from one point of equipotential surface to the other.

- If dl is the small distance over the equipotential surface through which unit positive charge is carried, then

$$dW = \vec{E} \cdot \vec{dl}$$

$$0 = E \, dl \cos\theta$$

$$\cos\theta = 0$$

$\boxed{\theta = 90^\circ}$ i.e. $\boxed{\vec{E} \perp \vec{dl}}$

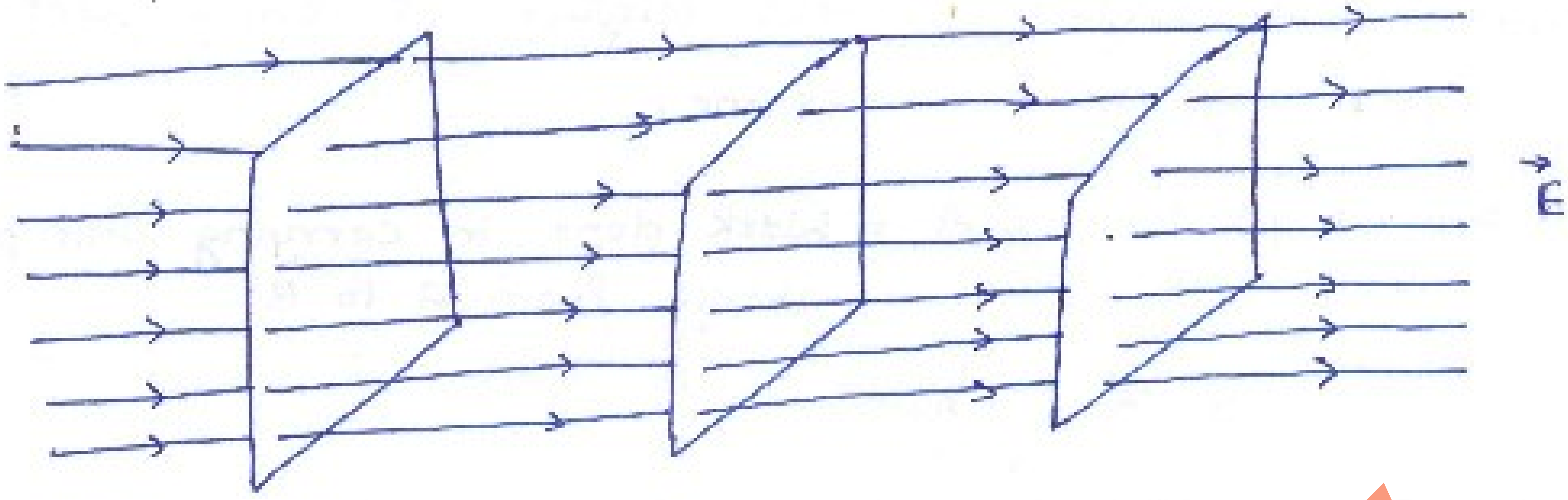
∴ Electric field intensity ($\vec{E}$) is always normal to the equipotential surface.

- For a single charge, $V = \dfrac{q}{4\pi\varepsilon_0 r}$

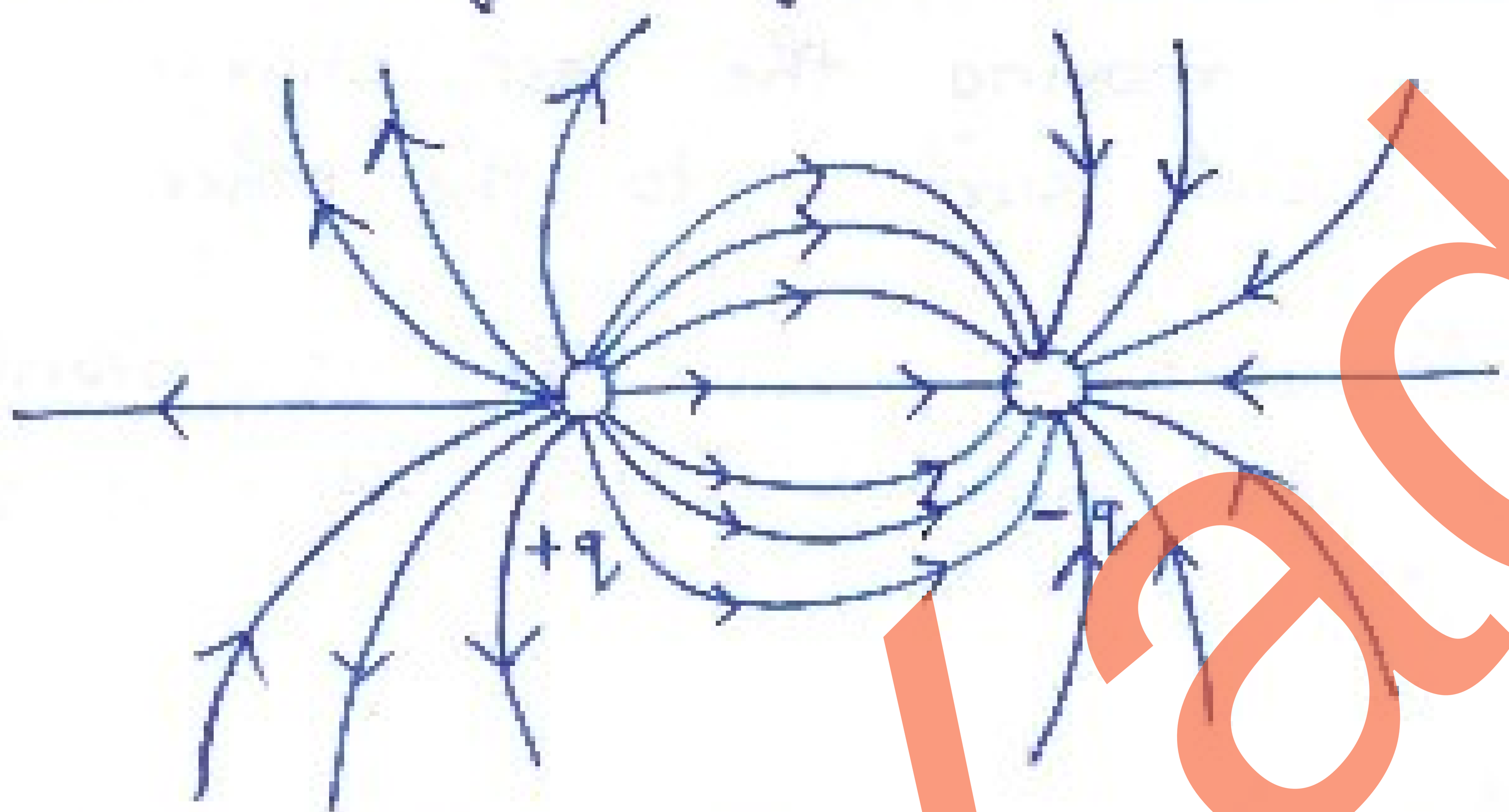
If r - constant, then V - constant

Hence, equipotential surfaces of a single point charge are concentric spherical surfaces centred at the charge.

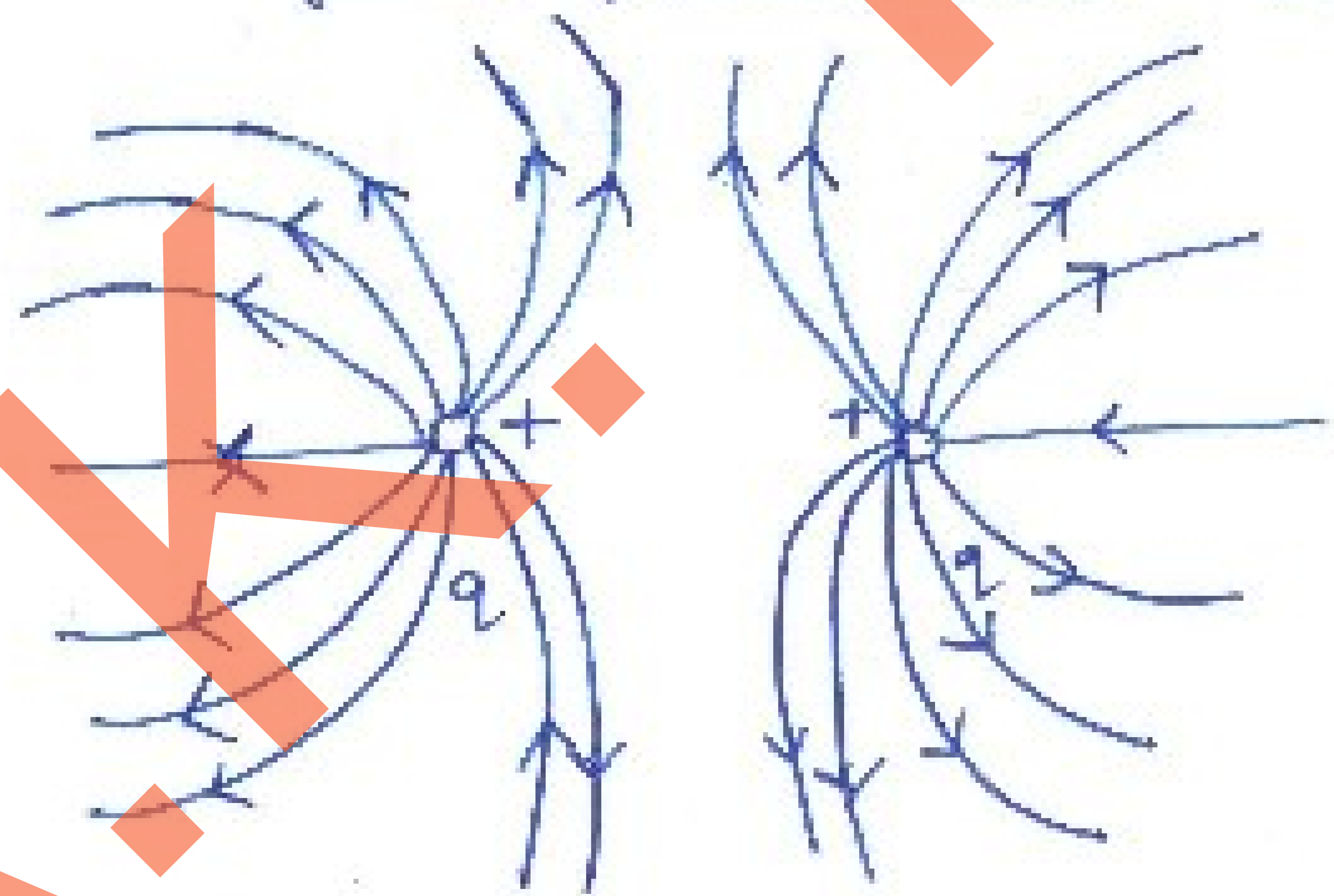
- Equipotential surfaces for a uniform electric field



- Equipotential surfaces for an electric dipole



- Equipotential surfaces for 2 identical positive charges



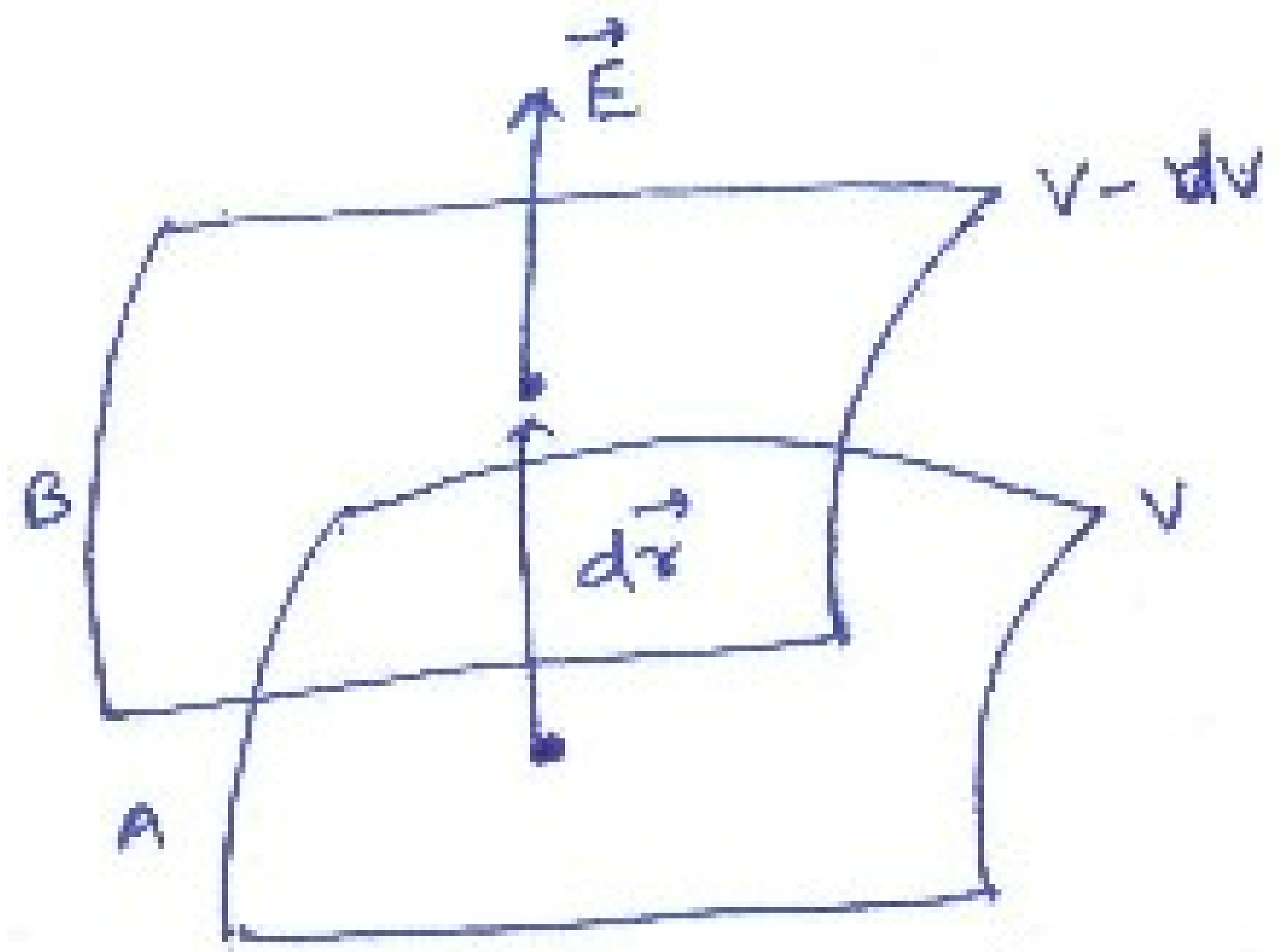
Relation betn electric field & electric potential

Consider 2 equipotential surfaces A & B spaced closely.

Let $V_A = V$ (potential at A)

$V_B = V - dV$ (potential at B)

dV - change in potential in the direction of $\vec{E}$



Work done in moving a unit positive charge along perpendicular distance ($d\vec{r}$) from B to A against the electric field is

$$W_{BA} = -E\,dr$$

Also, $W_{BA} = V_A - V_B = V - V + dV = dV$

$$\therefore \; -E\,dr = dV$$

$$\boxed{E = -\frac{dV}{dr}}$$

<u>Conclusions</u>

(i) Negative sign shows that the direction of $\vec{E}$ is the direction of decreasing potential.

(ii) Magnitude of E is given by change in magnitude of potential per unit displacement normal to the equipotential surface at that point.

$$|\vec{E}| = -\frac{|dV|}{dr} = -(\text{potential gradient})$$

<u>Potential energy of a system of charges</u>

(a) <u>System of 2 charges</u>

Suppose a point charge q_1 is at a pt. $P_1(\vec{r}_1)$ & point charge q_2 is at ∞.

Electrostatic potential at position P_2 due to charge q_1 is

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{12}}$$

Work done in carrying charge q_2 from ∞ to P_2 is

$$W = V \times q_2$$

$$\boxed{U = W = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}}$$

(b) System of n-point charges

Work done in bringing the first charge q_1 to position $P_1(\vec{r}_1)$ is

$$W_1 = 0 \quad \text{[As all other charges are at } \infty \text{ \& no field is there]}$$

Work done in bringing q_2 from ∞ to $P_2(\vec{r}_2)$ at a distance r_{12} from q_1 is

$$W_2 = [\text{potential due to } q_1] \times q_2$$

$$= \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

Work done in bringing q_3 from ∞ to $P_3(\vec{r}_3)$ is

$$W_3 = [\text{potential due to } q_1 \,\&\, q_2] \times q_3$$

$$= \left[\frac{q_1}{4\pi\varepsilon_0 r_{13}} + \frac{q_2}{4\pi\varepsilon_0 r_{23}}\right] q_3$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

Similarly, work done in bringing q_4 from ∞ to $P_4(\vec{r}_4)$ is

$$W_4 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_4}{r_{14}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right]$$

So, electrostatic potential energy of a system of 4 charges is

$$U = W_1 + W_2 + W_3 + W_4$$

$$= 0 + \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}}\right] + \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right] + \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_4}{r_{14}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right]$$

For n-point charges

$$U = \frac{1}{4\pi\varepsilon_0} \sum_{\text{all pairs}} \frac{q_j q_k}{r_{jk}}$$

$$\boxed{U = \frac{1}{2}\sum_{j=1}^{N}\sum_{\substack{k=1 \\ j\neq k}}^{N} \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_j q_k}{r_{jk}}}$$

($\frac{1}{2}$ is used because we should include only one term for each pair of charges
eg: $j=1, k=2$ & $j=2, k=1$ are same thing)

Potential energy of a single charge in external field

If $V(\vec{r})$ is external potential at any point P of position vector $\vec{r}$, then work done in bringing a unit positive charge from ∞ to P is V

$\therefore$ Work done in bringing a charge q from ∞ to P in the external field is

$$W = q \cdot V$$

$$\therefore \boxed{U = q \cdot V}$$

Potential energy of a system of 2 charges in external field

Consider 2 point charges q_1 & q_2 at position vectors $\vec{r}_1$ & $\vec{r}_2$ resp. in a uniform electric field (external) $\vec{E}$.

Work done in bringing q_1 from ∞ to $\vec{r}_1$ against $\vec{E}$ is

$$W_1 = q_1 V(\vec{r}_1)$$

[$V(\vec{r}_1)$ - potential at $\vec{r}_1$ due to $\vec{E}$]

Work done in bringing q_2 from ∞ to $\vec{r}_2$ against $\vec{E}$ is

$$W_2 = q_2 \cdot V(\vec{r}_2)$$

[$V(\vec{r}_2)$ - potential at $\vec{r}_2$ due to $\vec{E}$]

Work done in bringing q_2 from ∞ to $\vec{r}_2$ against field due to q_1 is

$$W_3 = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$

[r_{12} - distance betn r_1 & r_2]

$\therefore$ Total work done (P.E.) is

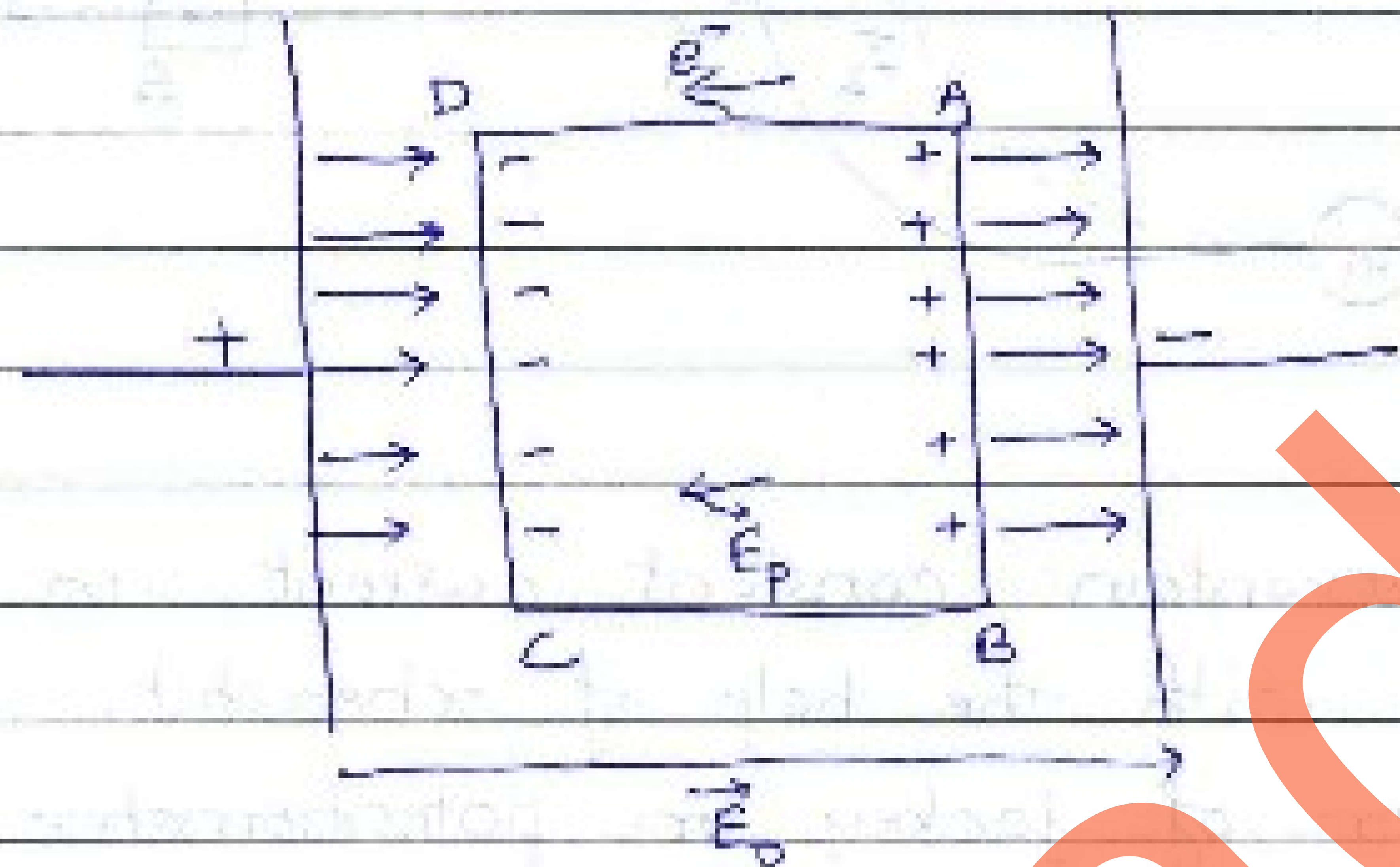
$$U = W_1 + W_2 + W_3$$

$$\boxed{U = q_1 \cdot V(\vec{r}_1) + q_2 \cdot V(\vec{r}_2) + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}}$$

Capacitors

Electrostatics of conductors

① <u>Inside a conductor, electric field is zero</u>



→ Suppose a conductor ABCD is held in an external electric field $\vec{E}_0$

→ Free electrons in the conductor moves from AB to CD.

→ As a result, some net negative charge appears on CD & equal positive charge on AB.

These are called induced charges.

→ They produce any induced electric field $\vec{E}_p$ which opposes the flow of electrons from AB to CD.

→ The flow stops as soon as $\vec{E}_p = \vec{E}_0$

→ As the applied field & induced electric field inside the conductor are equal & opposite, so net field inside the conductor is zero.

② <u>The interior of a conductor can have no excess charge in static situation</u>

- A neutral conductor has equal +ve & -ve charges in every small volume or surface element.
- When the conductor is charged the excess charge can reside only on the surface in static condition.

(a) Consider any arbitrary volume element v inside a conductor

(b) On the closed surface S bounding the volume element v, electric field is zero.

(c) So, electric flux(ϕ) through S = 0 $[\because \phi = E\,ds\cos\theta]$

(d) By Gauss law, no charge is enclosed by S

$$\left[\because \phi = \frac{Q}{\varepsilon_0},\ \text{here } \phi = 0 \quad \text{So, } Q = 0\right]$$

(e) Even if S is made as small as possible, we will not find any charge at any point inside the conductor

(f) The excess charge, if any, must reside on the outer surface of conductor.

③ Electric field just outside a charged conductor is $\perp^r$ to the surface of the conductor at every point

→ Under electrostatic conditions, flow of charge stops.

→ So, component of $\vec{E}$ along the tangent to the surface of conductor, must be zero.

i.e. $E\cos\theta = 0$

$E \neq 0$, $\cos\theta = 0$, $\theta = 90°$

④ Electrostatic potential is constant throughout the volume of the conductor & has the same value as on its surface

(i) As $\vec{E} = 0$, inside the conductor

↓

No work is done in moving a ~~test~~ charge ~~inside conductor~~

↓

No p.d. bet^n any 2 points inside the conductor

↓

Electrostatic potential is const. throughout the volume of conductor.

(b) $\vec{E}$ $\perp^r$ to surface of conductor

↓

No tangential component on its surface

↓

No work done in moving a test charge on the surface of conductor

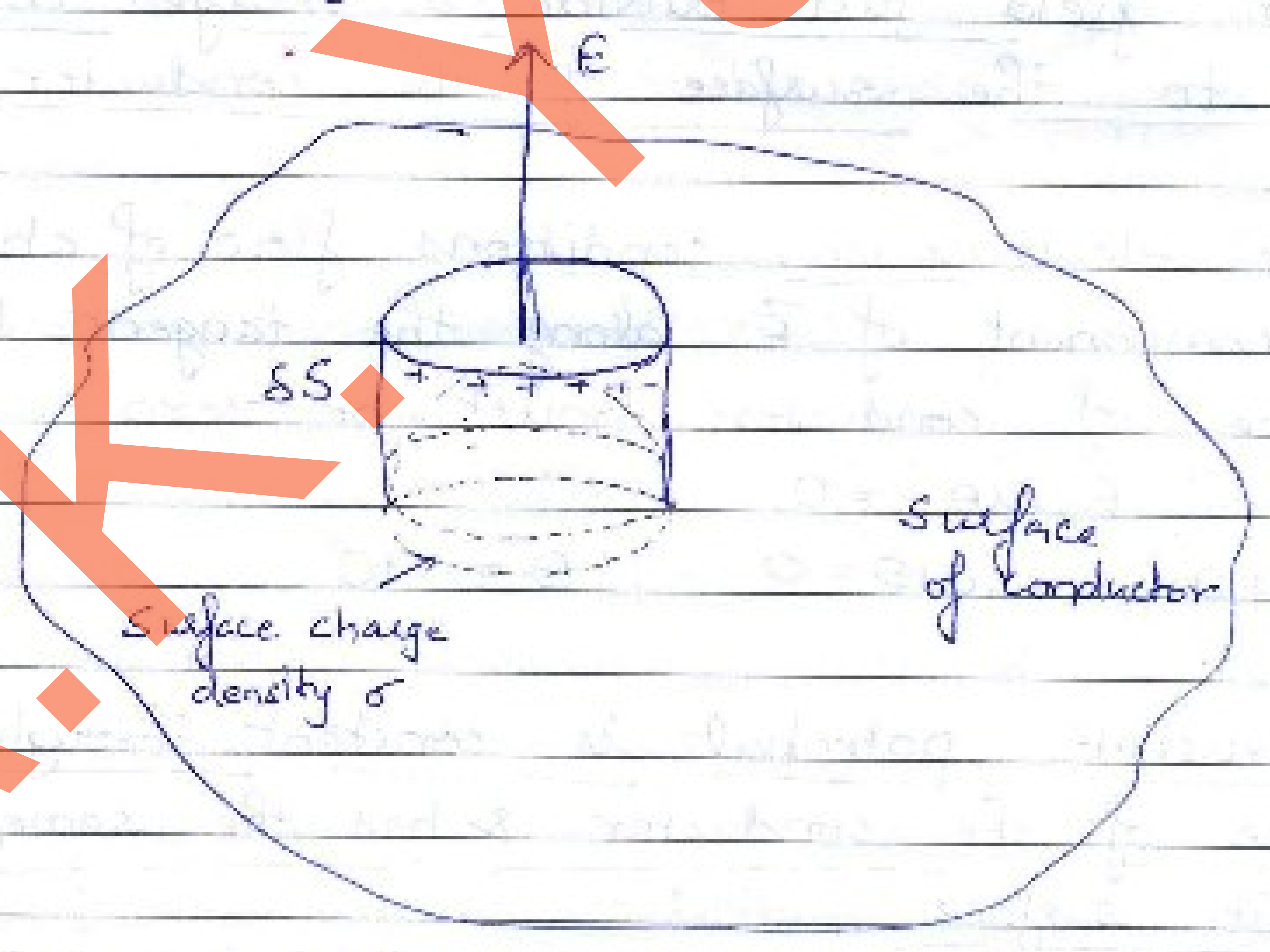
↓

So, no p.d. betn any 2 points on surface of conductr.

↓

Hence electrostatic potential is constant throughout the volume of conductor.

(c) Electric field at the surface of charged conductor is $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$



Consider a short cylinder as a Gaussian surface partly inside & partly outside the surface of cylinder

Let δS - small area of cross-section

Now, just inside the surface $\vec{E} = 0$
& just outside $E \perp$ to surface
So, the total flux through the cylinder comes from outside (circular) cross-section of cylinder

i.e. $\phi = \pm E\,\delta S \quad \begin{bmatrix} +ve \text{ for } \sigma > 0 \\ -ve \quad \sigma < 0 \end{bmatrix}$

Charge enclosed by cylinder, $q = \sigma\,\delta S$

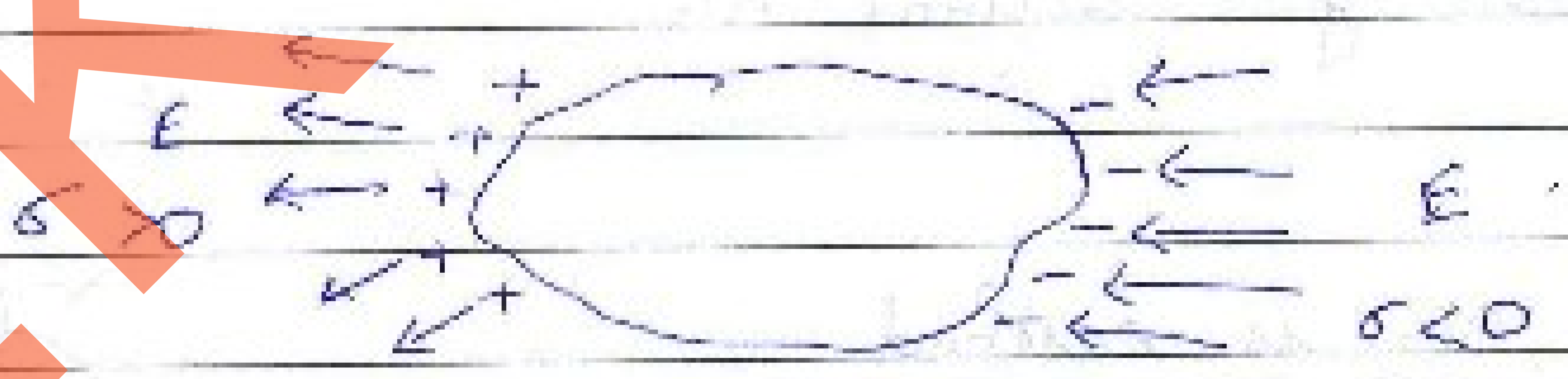
By Gauss' law

$$E\,\delta S = \frac{\sigma\,\delta S}{\varepsilon_0}$$

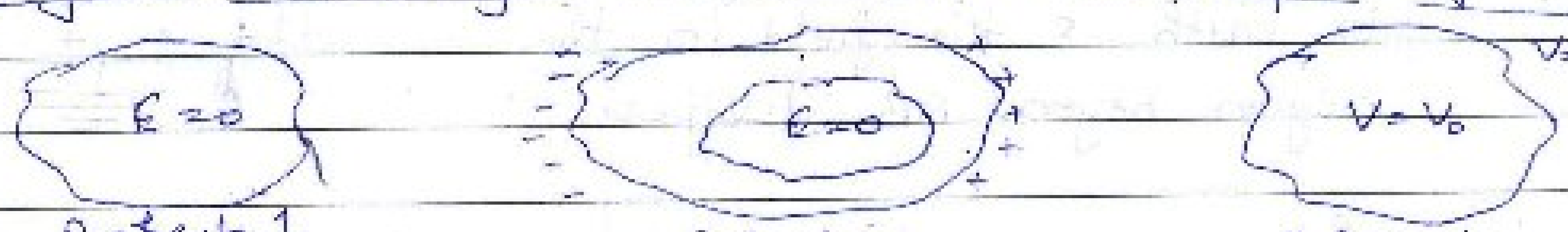
$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

As, $E \perp$ surface $\boxed{\vec{E} = \frac{\sigma}{\varepsilon_0}\,\hat{n}}$

The result can be represented as



* Diagrams showing the electrostatic properties of conductors



Property 1 Property 2 Property 4

Electrostatic Shielding

The phenomenon of protecting a certain region of space from external electric field.

Example → (i) Electric field inside a conductor is zero, so to protect delicate instruments from external electric fields, we enclose them in hollow conductors called Faradays Cages

(ii) During lightening, it is better to be inside a car as the metallic body of car provides electrostatic shielding from lightening.

(iii) It is used in designing of T.V. cables.

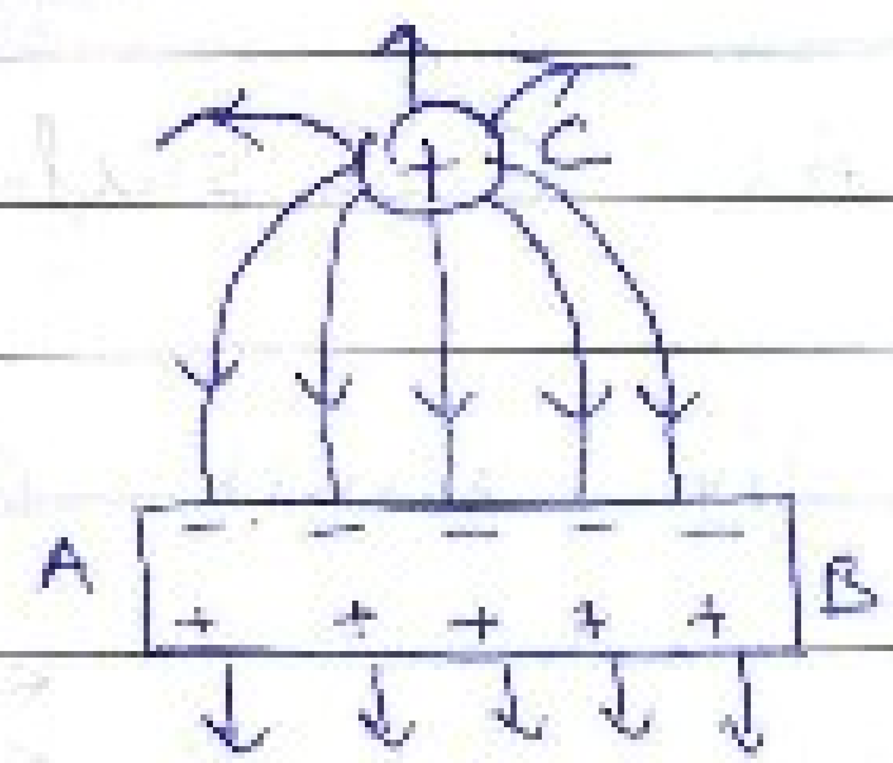
For knowledge

Q Why a high voltage generator is usually enclosed in such a cage which is earthed?

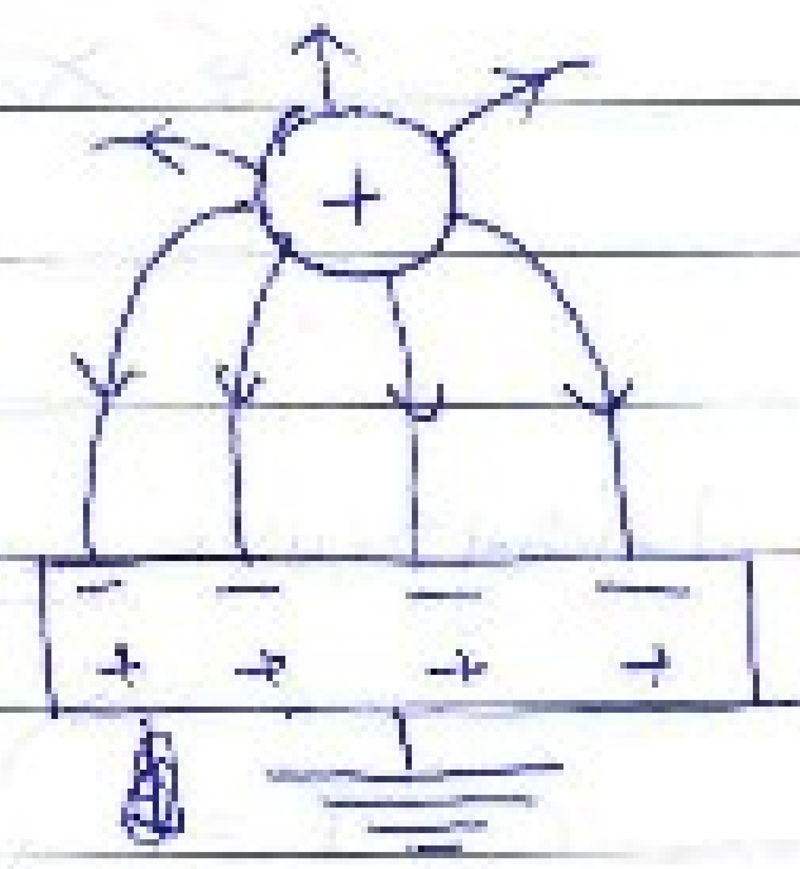
Ans An earthed conductor can act as a shield against electric field.

When AB is not earthed

Electric field of C due to induction goes beyond AB



When AB is earthed



The induced +ve charge flows to earth & the field in the region beyond AB disappears.

This prevents the electrostatic field of generator from spreading out of cage.

classmate
Date
Page

Electrical Capacitance

It is related to the ability of a conductor to store electric charge.

When a conductor is given some charge, its electric potential increases.
It is found that

$$Q \propto V$$

$$\boxed{Q = CV}$$

C - electrical capacitance

Electrical capacity of a conductor is the ratio of charge given to conductor to rise in its potential

$$C = \frac{Q}{V}$$

If $V = 1$, $C = Q$
Electrical capacity of a conductor is numerically equal to the charge required to raise its potential by unity.

S.I unit → farad

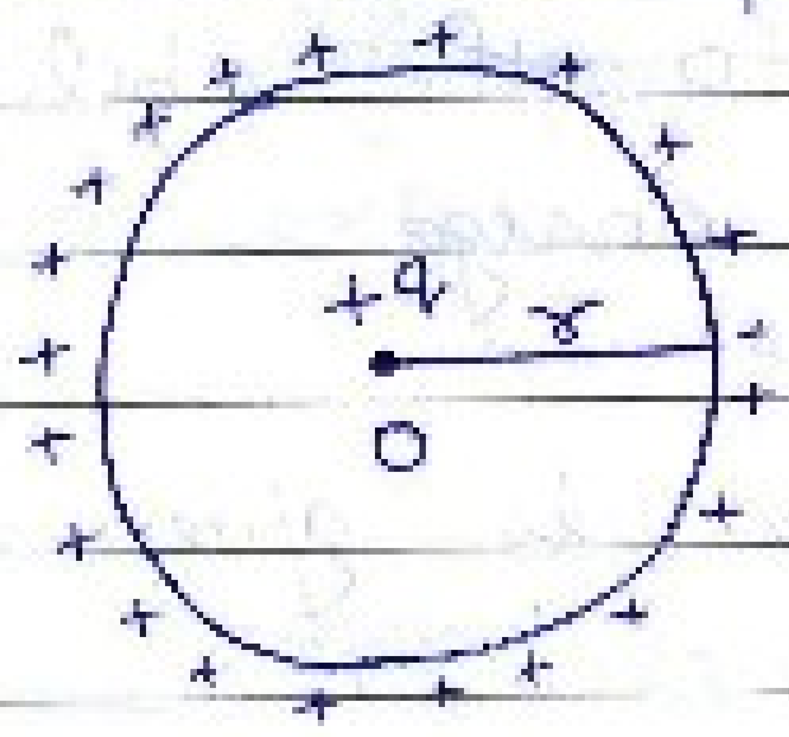
$$1 \text{ farad } (F) = \frac{1C}{1V}$$

Smaller units → $1\ \mu F = 10^{-6} F$
$1\ nF = 10^{-9} F$
$1\ pF = 10^{-12} F$

Dimensional formula

$$C = \frac{Q}{V} = \frac{Q^2}{W} = \frac{(AT)^2}{ML^2T^{-2}} = [M^{-1}L^{-2}T^4A^2]$$

Capacity of an isolated spherical conductor



→ Consider an isolated spherical conductor with centre O & radius r.

→ Let $+q$ charge be given to the sphere.

→ The charge spreads uniformly over the outer surface of sphere, so potential is same at every point on the surface of the sphere.

$$V = \frac{q}{4\pi\varepsilon_0 r}$$

Now, $C = \frac{q}{V} = \frac{q}{q} \times 4\pi\varepsilon_0 r$

$$\boxed{C = 4\pi\varepsilon_0 r}$$

$C \propto r$

Capacity of earth

$$C = 4\pi\varepsilon_0 r$$

$$= \frac{1}{9\times10^9} \times 6.4\times10^6$$

$$= 0.711 \times 10^{-3} F$$

$$= 0.711 \times 10^{-3} \times 10^6 \mu F$$

$$= 711 \mu F$$

As for earth (biggest sphere around us) C is $711 \mu F$ so, farad is very large unit so in routine units of capacitance are μF & pF.

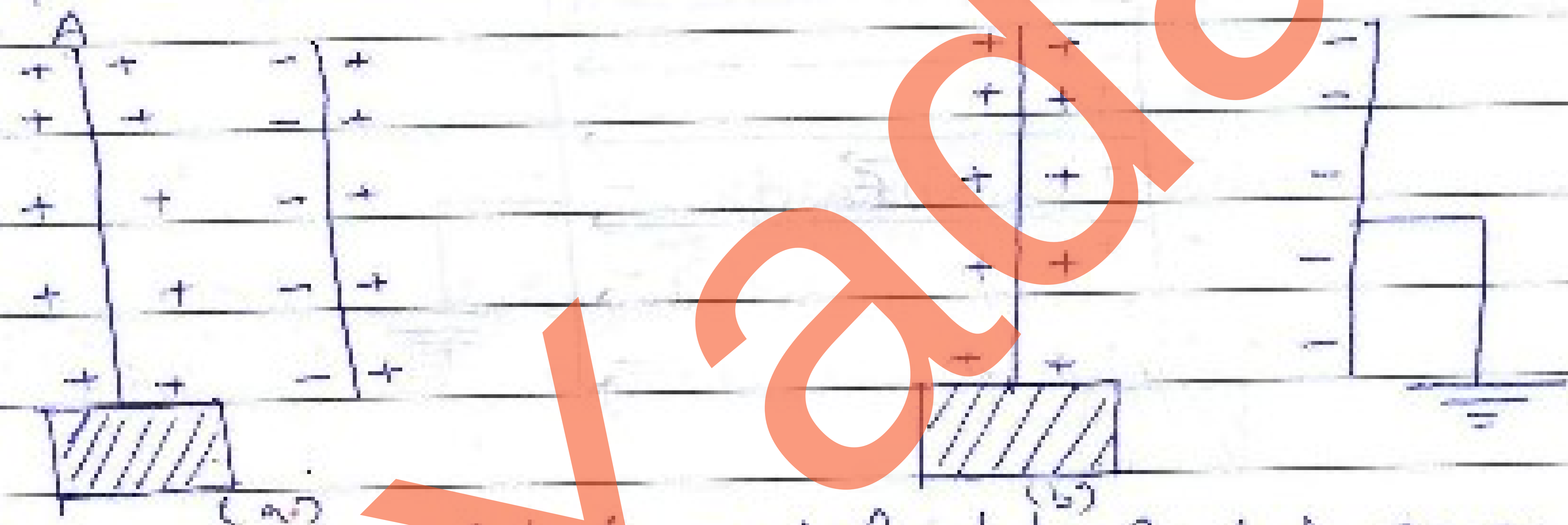
Capacitor

→ It is an arrangement for storing large amounts of electric charge.

→ Usually a capacitor consists of a system of 2 conductors separated by an insulating medium.

→ Normally they are charged by connecting them to the 2 terminals of a battery.

Principle of a capacitor



→ Consider an insulated metal plate A. Let some positive charge be given to it till its potential becomes maximum.

→ Consider another insulated metal plate B held near A.

→ By induction charge produced on B.

→ The induced negative charge tends to decrease the potential of A as it is closer.

→ The overall potential of A reduces & hence some more charge can be given to A to raise its potential to max. value.

→ So, capacity of a conductor can be increased by bringing another uncharged conductor near it.

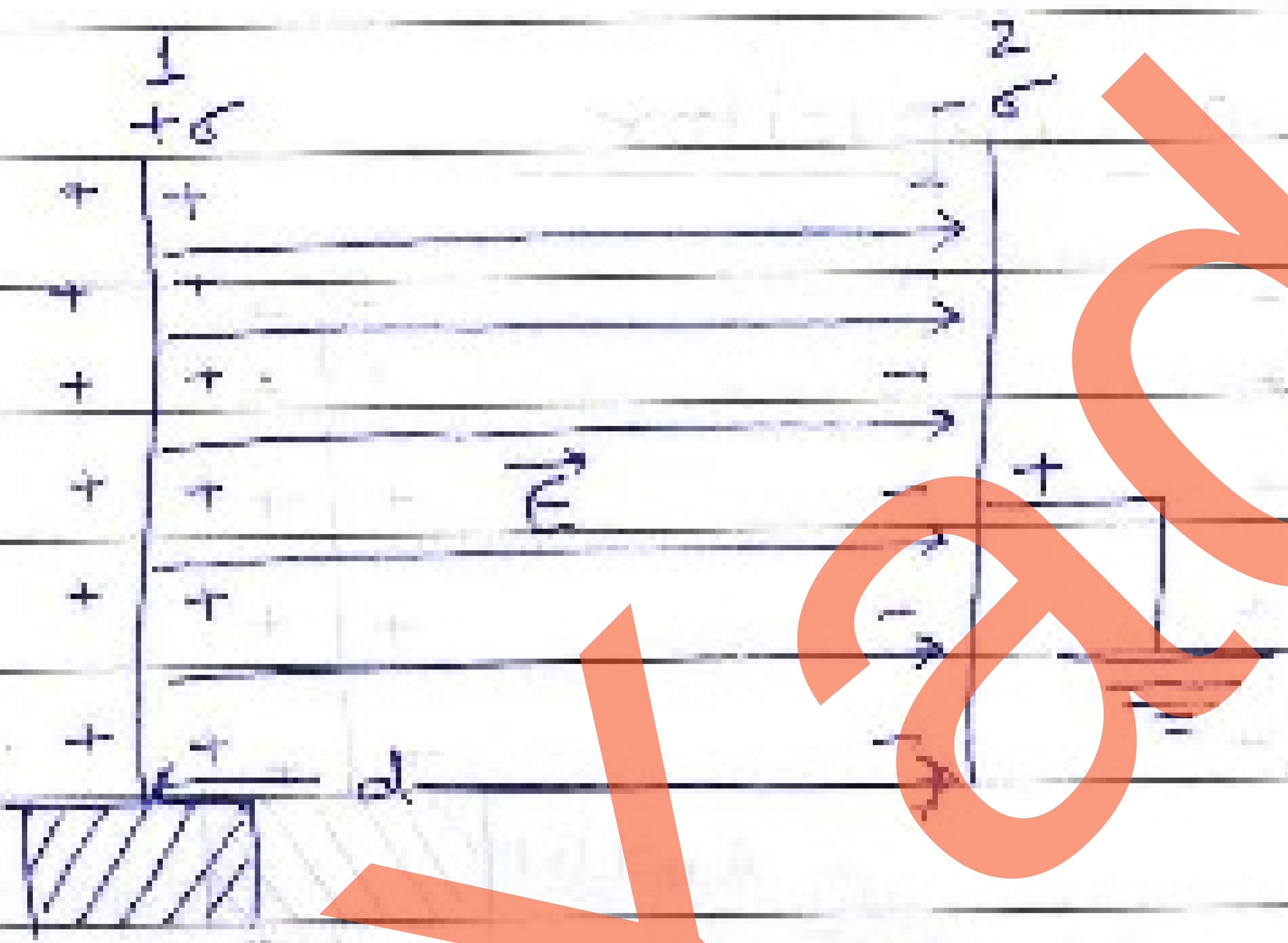
→ Now, connect B to earth. The induced positive charge flows to earth & only the induced negative charge stays.

→ Due to the induced negative charge on B, potential of A is greatly reduced.

→ So, a large amount of charge can be given to A to raise it to max. potential.

→ So, the capacitance of an insulated conductor is increased considerably by bringing an uncharged earthed conductor near it.

Parallel Plate Capacitor



Surface charge density of plate 1, $\sigma = \frac{Q}{A}$

" " " " " 2, $\sigma = \frac{-Q}{A}$

In the region betⁿ left of 1 & right of 2, $\vec{E} = 0$

" " " " 1 & 2, $\vec{E} = \frac{\sigma}{\varepsilon_0}$

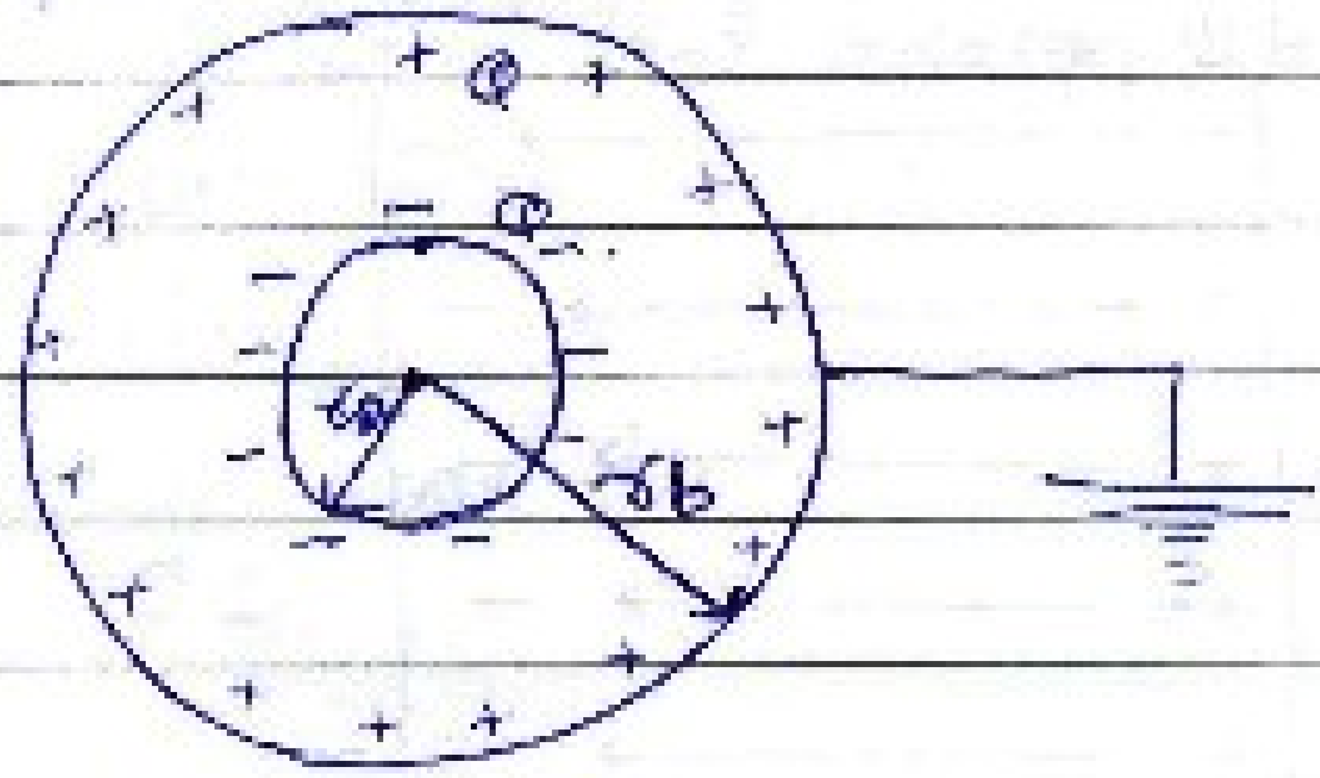
$$= \frac{1}{\varepsilon_0}\frac{Q}{A}$$

$[E = \frac{V}{d}]$

Now, $V = E \times d = \frac{1}{\varepsilon_0}\frac{Q}{A}d$

$$\boxed{C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}}$$

Spherical Capacitor



A spherical capacitor consists of a hollow conducting sphere A of radius r_a, surrounded by another concentric conducting shell B of radius r_b.

Due to electrostatic shielding, electric field within A is zero, i.e. $\vec{E} = 0$ for $r < r_a$

Also, Electric field outside B is zero, $\vec{E} = 0$ for $r > r_b$

So, electric field E exists betn the two spheres & is directed radially outwards.

Now, potential of A

$$V_A = \frac{Q}{4\pi\varepsilon_0 r_a} - \frac{Q}{4\pi\varepsilon_0 r_b}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_a} - \frac{1}{r_b}\right]$$

As B is earthed, $V_B = 0$

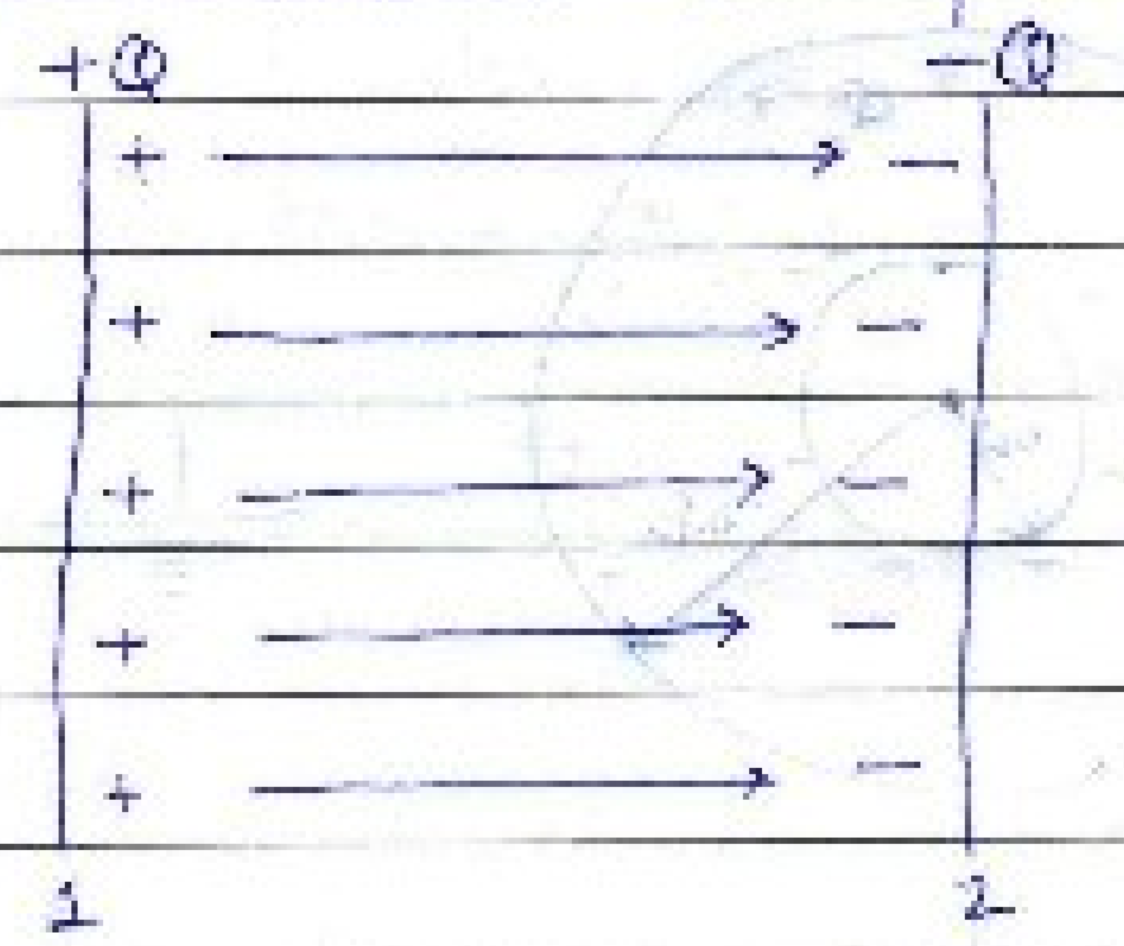
Potential difference betn A & B

$$V = V_A - V_B$$

$$= \frac{Q}{4\pi\varepsilon_0} \frac{(r_b - r_a)}{r_a r_b}$$

$$\boxed{C = \frac{Q}{V} = \frac{4\pi\varepsilon_0 r_a r_b}{(r_b - r_a)}}$$

Energy stored in a capacitor



Suppose the 2 conductor plates of a capacitor are uncharged initially.
Let positive charge be transferred from 2 to 1 in small installments of dq till 1 gets charge +Q.

At every stage of charging, conductor 1 is at a higher potential than 2, so work is done externally in transferring the charge.

Suppose the conductor is charged gradually & at any stage, the charge on conductor is q.

So, potential $V = \frac{q}{C}$

The small work done in giving an additional charge dq to the capacitor is

$$\therefore dW = \frac{q}{C}\,dq$$

Total work done is

$$W = \int_0^Q \frac{q}{C}$$

$$= \frac{1}{2}\frac{Q^2}{C}$$

As electrostatic force is conservative, this work is stored in the form of potential energy of capacitor

$$U = \frac{Q^2}{2C}$$

$$= \frac{1}{2C}(CV)^2$$

$$\boxed{U = \frac{1}{2}CV^2} \quad \& \quad \boxed{U = \frac{1}{2}QV} \quad [CV = Q]$$

Energy density of a parallel plate capacitor

It is defined as the total energy stored per unit volume of capacitor.

$$u = \frac{U}{V}$$

$$= \frac{\frac{1}{2}CV^2}{Ad}$$

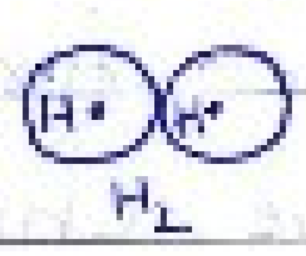
$$= \frac{1}{2}\frac{\varepsilon_0 A}{d}\frac{E^2d^2}{Ad} \quad [E = V/d]$$

$$\boxed{u = \frac{1}{2}\varepsilon_0 E^2}$$

Polar & Non-polar dielectrics

Non-polar dielectrics

→ Molecules in which the centre of positive charge coincides with the centre of negative charge in the molecule.



→ Molecules are symmetric in shape

→ Each molecule has zero dipole moment in normal state.

→ Example: H_2, N_2, O_2, CO_2, Benzene, CH_4

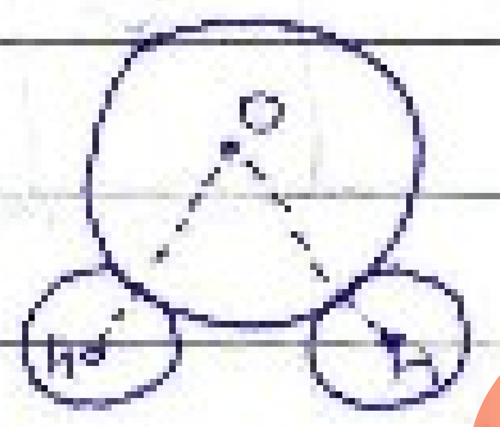
Polar dielectrics

→ Molecules in which the centres of positive & negative charges do not coincide.

→ Molecules are asymmetric in shape.

→ Each molecule has some (intrinsic) dipole moment.

eg H_2O, NH_3, alchol, HCl



Dielectric polarization

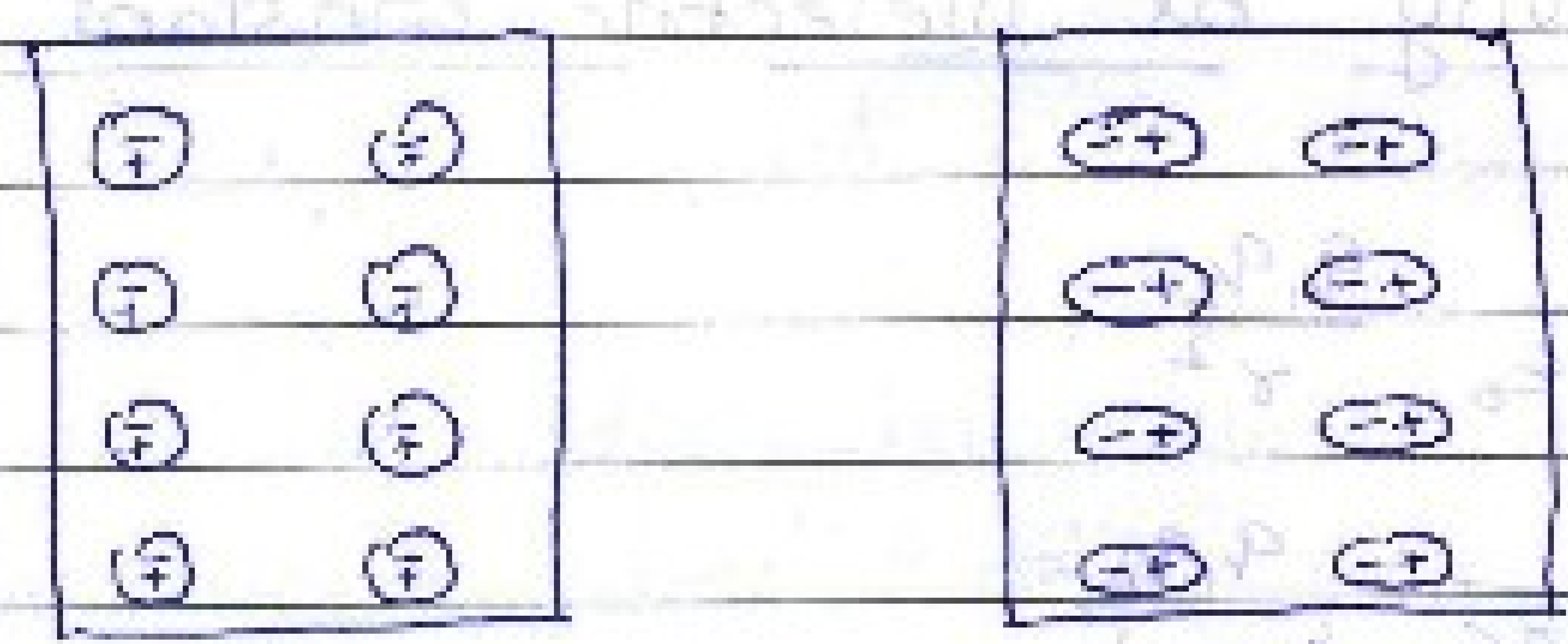
(a) Non-polar dielectric

(i) When a non-polar dielectric is held in external electric field $\vec{E}_0$, the centre of positive charge in each molecule is pulled in the direction of $\vec{E}$ (towards −ve plate) & the centre of −ve charge is pulled in a direction opposite of $\vec{E}$

(ii) So, the 2 centres of positive & negative charges in the molecule are separated.

(iii) The molecule gets distorted & is said to be polarised as a tiny dipole moment is imparted to each molecule.

(iv) Force due to electric (pulling the 2 centres apart) & the force of mutual attraction (betn centres of +ve and −ve charges) reach an equilibrium & the molecule gets polarised.

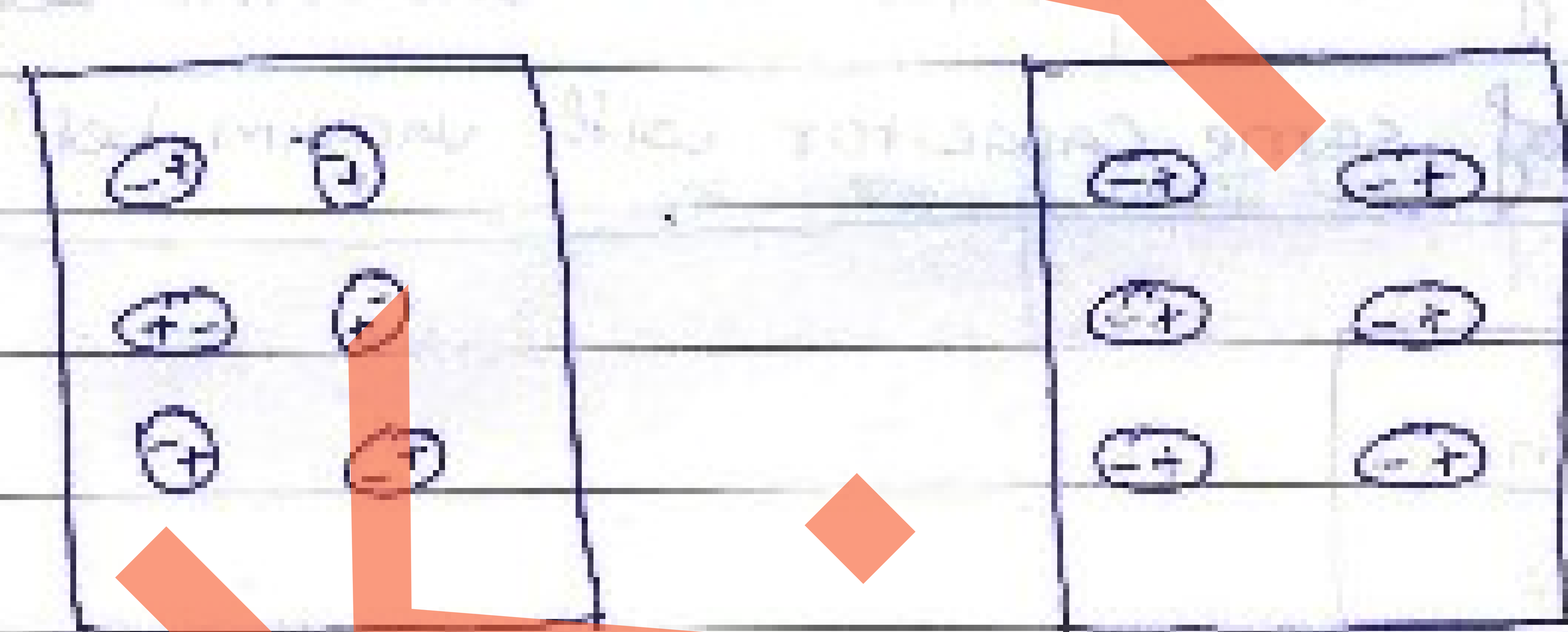


$\vec{E} = 0$ $\quad$ $\vec{E} \rightarrow$

(b) Polar dielectric

(i) When no external field is applied, the different dipole moments of the dielectric are oriented randomly so total dipole moment is zero.

(ii) When an external electric field is applied, the individual dipole moments tend to align with the field & get some net dipole moment in the direction of the field. i.e. Dielectric is polarised.



$\vec{E} = 0$ $\quad$ $\vec{E} \rightarrow$

(iii) The extent of polarisation depends on the relative strength of 2 mutually opposing forces

(a) Applied external electric field (tending to align the dipoles with field)

(b) thermal energy (tending to disrupt the alignment).

Dielectric strength

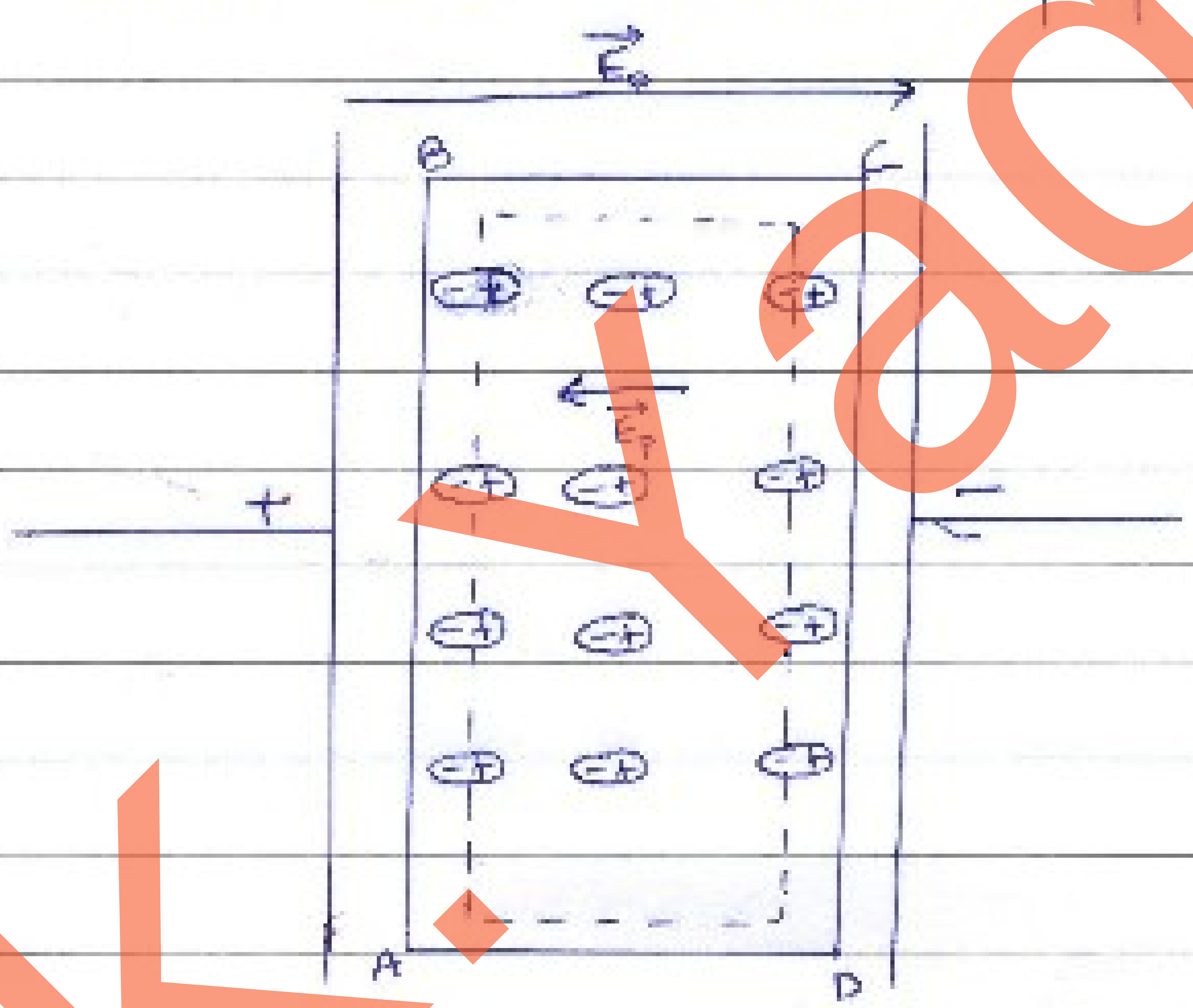
The max. electric field that a dielectric medium can withstand without breaking its insulating property is called its dielectric strength.

Dielectric Polarisation (P)

- A dielectric develops a net dipole moment in the presence of an external field for both polar and non-polar molecules.
- The dipole moment per unit volume is called polarisation for linear isotropic dielectrics

$$\boxed{P = \chi_e E} \quad \text{or} \quad \boxed{P = \varepsilon \chi_e E}$$

where χ_e - electric susceptibility of dielectric medium



Consider a non-polar dielectric slab ABCD placed in an electric field E_0

Effective electric field in the polarized dielectric is

$$E = E_0 - E_p$$

$$= E_0 - \frac{P}{\varepsilon} \qquad \left[\because E_p = \frac{P}{\varepsilon}\right]$$

$$= E_0 - \frac{\varepsilon \chi_e E}{\varepsilon}$$

$$E = E_0 - \chi_e E$$

$$\boxed{K = \frac{E}{E_0} = 1 + \chi_e}$$

Relative permittivity or Dielectric constant (K)

$$F_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

$$F_m = \frac{1}{4\pi\varepsilon}\frac{q_1 q_2}{r^2}$$

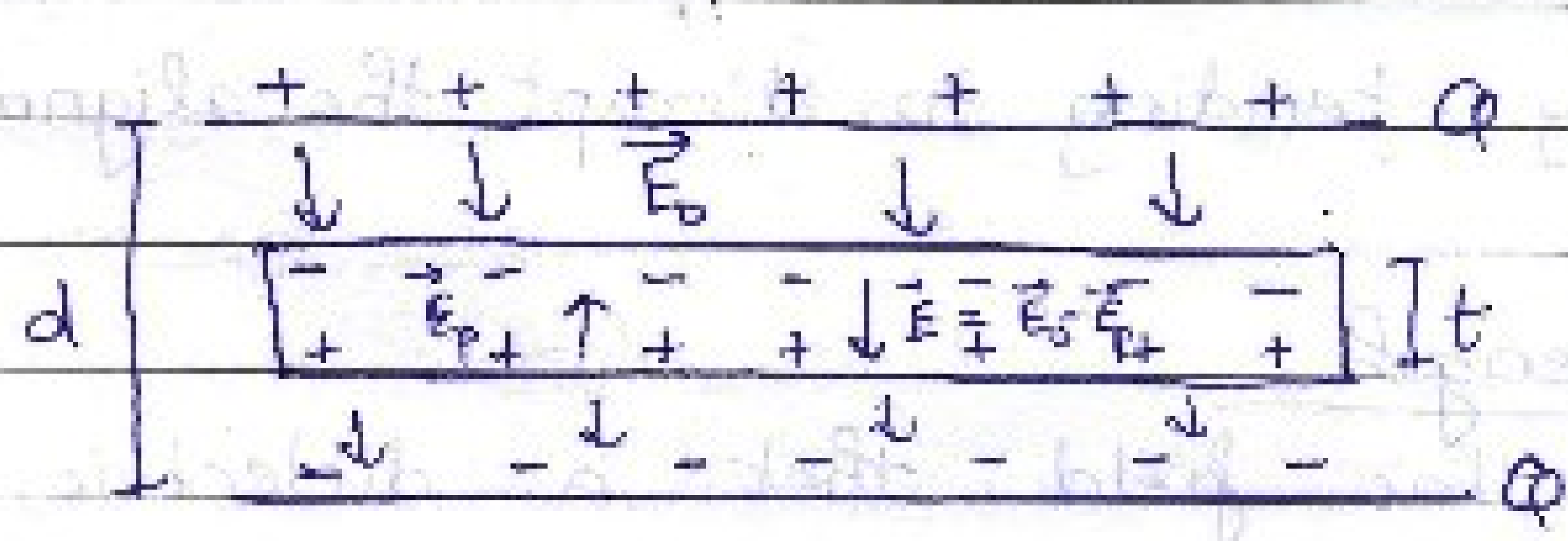
$$\frac{F_0}{F_m} = \frac{\varepsilon}{\varepsilon_0} = K$$

Faraday observed that when an insulating material is occupying the space betn charged plates of a capacitor its capacitance increases.
The factor by which capacitance is multiplied depends upon dielectric constant (K) of material.

$$\boxed{K = \frac{\text{Capacitance of a capacitor with dielectric bet}^n\text{ plates}}{\text{Capacitance of same capacitor with vacuum bet}^n\text{ plates}}}$$

$$\boxed{K = \frac{C_m}{C_0}}$$

Capacitance of a parallel plate capacitor with a dielectric slab



Capacitance of a parallel plate capacitor of area A & plate separation d with vacuum is

$$C_0 = \frac{\varepsilon_0 A}{d}$$

When a dielectric slab of thickness $t<d$ is introduced betⁿ the plates, the molecules in the slab get polarized in the direction of E_0. The polarization vector P induces an electric field E_p opposite to E_0.

So, effective field inside dielectric $E = E_0 - E_p$
" " outside " $E = E_0$

Potential difference betⁿ the 2 plates

$$V = E_0(d-t) + Et \qquad \left[\because K = \frac{E_0}{E}\right]$$

$$= E_0(d-t) + \frac{E_0}{K}t$$

$$= E_0\left[d - t + \frac{t}{K}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left[d-t+\frac{t}{K}\right] \qquad \left[\because E_0 = \frac{\sigma}{\varepsilon_0}\right]$$

$$= \frac{Q}{A\varepsilon_0}\left[d-t+\frac{t}{K}\right] \qquad \left[\because \sigma = \frac{Q}{A}\right]$$

$$C = \frac{Q}{V}$$

$$\boxed{C = \frac{A\varepsilon_0}{d - t\left(1-\frac{1}{K}\right)}}$$

Clearly $C > C_0$
i.e. on the introduction of a dielectric slab inbetⁿ the plates of a capacitor, its capacitance increases.

Note

1. Dielectric slab introduced between the plates of a charged capacitor with battery connected across plates

(a) Potential → remains constant $(V = V_0)$

(b) Capacity → increases $(C = KC_0)$

(c) Charge → increases $(Q = CV)$

(d) Electric field → decreases $(E = \frac{E_0}{K})$

(e) Energy → increases $(U = \frac{1}{2}CV^2 = KU_0)$

2. Dielectric slab introduced between the plates of a charged capacitor with battery switched off

(a) Charge → remains constant $(Q = Q_0)$

(b) Capacity → increases $(C = KC_0)$

(c) Potential → decreases $(V = \frac{Q}{C} = \frac{V_0}{K})$

(d) Electric field → decreases $\left(E = \frac{V}{d} = \frac{V_0}{Kd}\right)$

(e) Energy → decreases $\left(U = \frac{Q^2}{2C} = \frac{U_0}{K}\right)$

3. Capacitance of a parallel plate capacitor with a conducting slab is $C = \frac{C_0}{\left(1 - \frac{t}{d}\right)}$

where C_0 - capacitance in vacuum

d - distance between the plates

t - thickness of conducting slab

As $C > C_0$ so the capacitance increases on introducing a conducting slab inbetween the plates.

Action of sharp points

→ When a spherical conductor of radius 'r' carries a charge 'q', the surface charge density is

$$\sigma = \frac{Q}{A} = \frac{q}{4\pi r^2}$$

for a pointed end, $r \rightarrow$ very-very small

so, $\sigma \rightarrow$ large

→ The particles of air coming in contact with pointed ends get similarly charged and are repelled

→ In this way an electric wind is set up which takes away the charge continuously from the pointed ends of the conductor

→ This process of spraying the charge is called corona discharge

* For this reason, conductors used for storing charge are always sphere of large radii

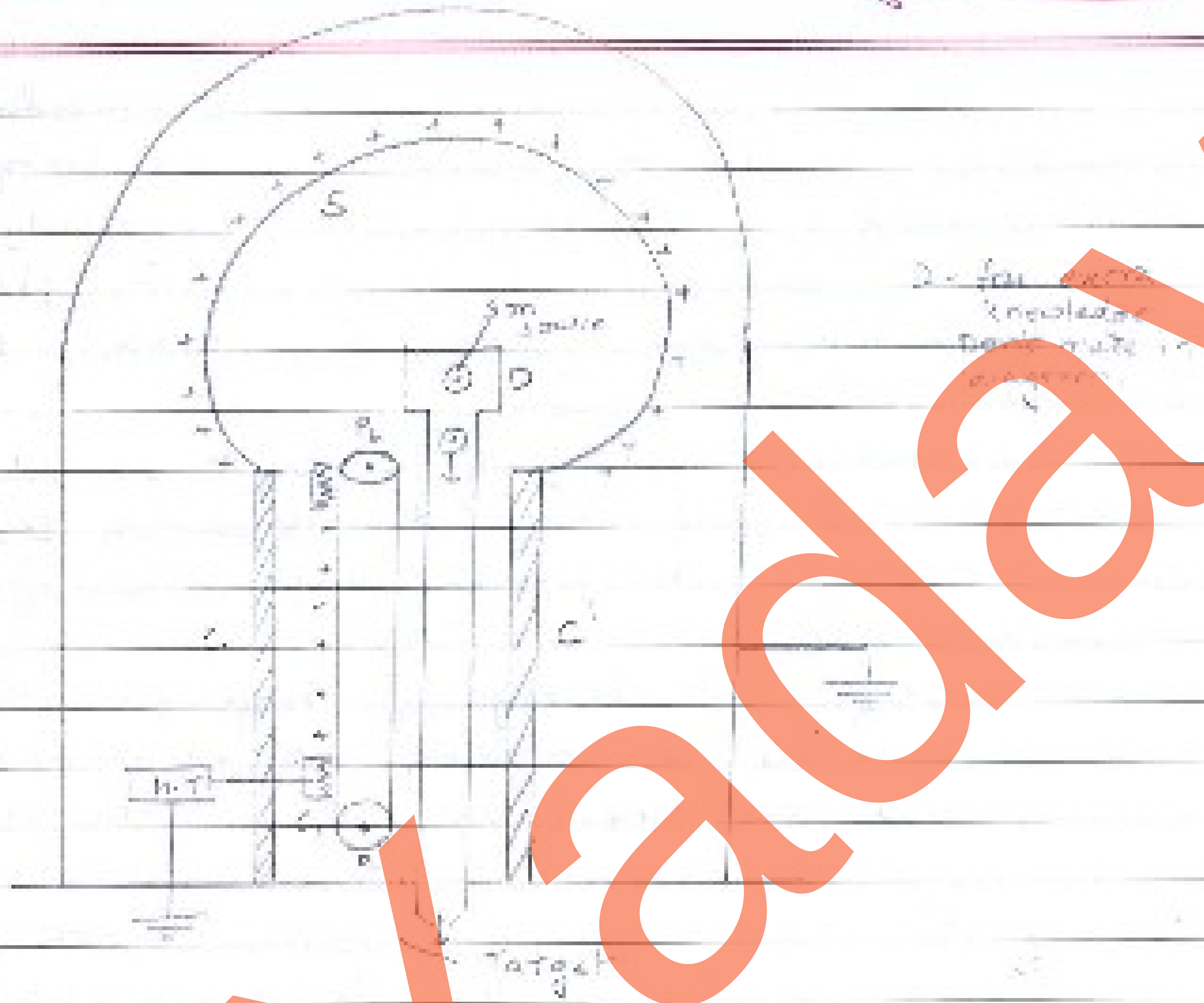
Van De Graaff Generator

It is a device used for building up high potential differences of the order of million volts

Principle

(i) action of sharp points (i.e. corona discharge)

(ii) If charge is given to a hollow conductor it is transferred to outer surface & is distributed uniformly over it



Construction – It consists of
- S – a large hollow sphere mounted on CC' (column)
- Belt of insulating material running bet^n pulleys P_1 & P_2 with the help of motor
- C_1 – spray comb, C_2 – collecting comb
  [One end touching the belt
  other end in contact of S]

Working
(i) Spray comb C_1 is given a positive potential 10^4 V w.r.t earth by high tension source H.T.
(ii) Due to discharging action of sharp points (corona discharge) positive charge is sprayed on the belt and is carried upward by moving belt
(iii) C_2 collects the positive charge & transfers it to the metallic sphere.
(iv) The charge transferred by C_2 immediately moves to the outer surface of S

(v) As the belt goes on moving, the accumulation of positive charge on C also keeps on taking place continuously & its potential rises considerably.

(vi) With the increase of charge on the sphere its leakage due to ionisation of surrounding air also becomes faster.

(vii) The max potential of the sphere is reached after rate of loss of charge due to leakage becomes equal to the rate at which charge is transferred to the sphere.

(viii) To prevent the leakage of charge from the sphere, the generator is completely enclosed inside an earth connected steel tank (filled with air under pressure).

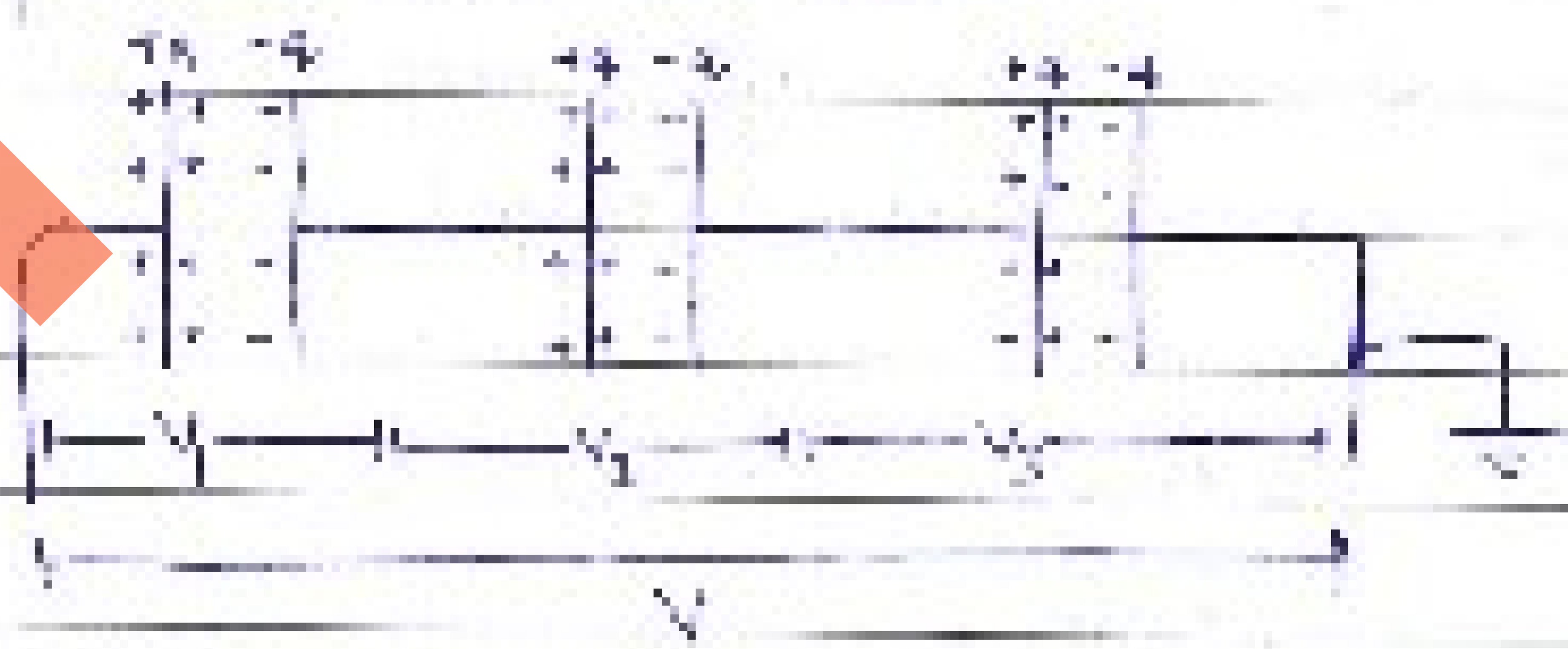
* If the projectiles like neutrons are now generated in discharge tube D, then they get accelerated in (downward direction (Fig. 44))

If q - charge of ion like neutron to be accelerated
V - p.d. developed across ends of discharge tube

So,
the ions hit the target with energy $E = qV$ & carry out nuclear disintegration.

Grouping of capacitors

① Capacitors in series

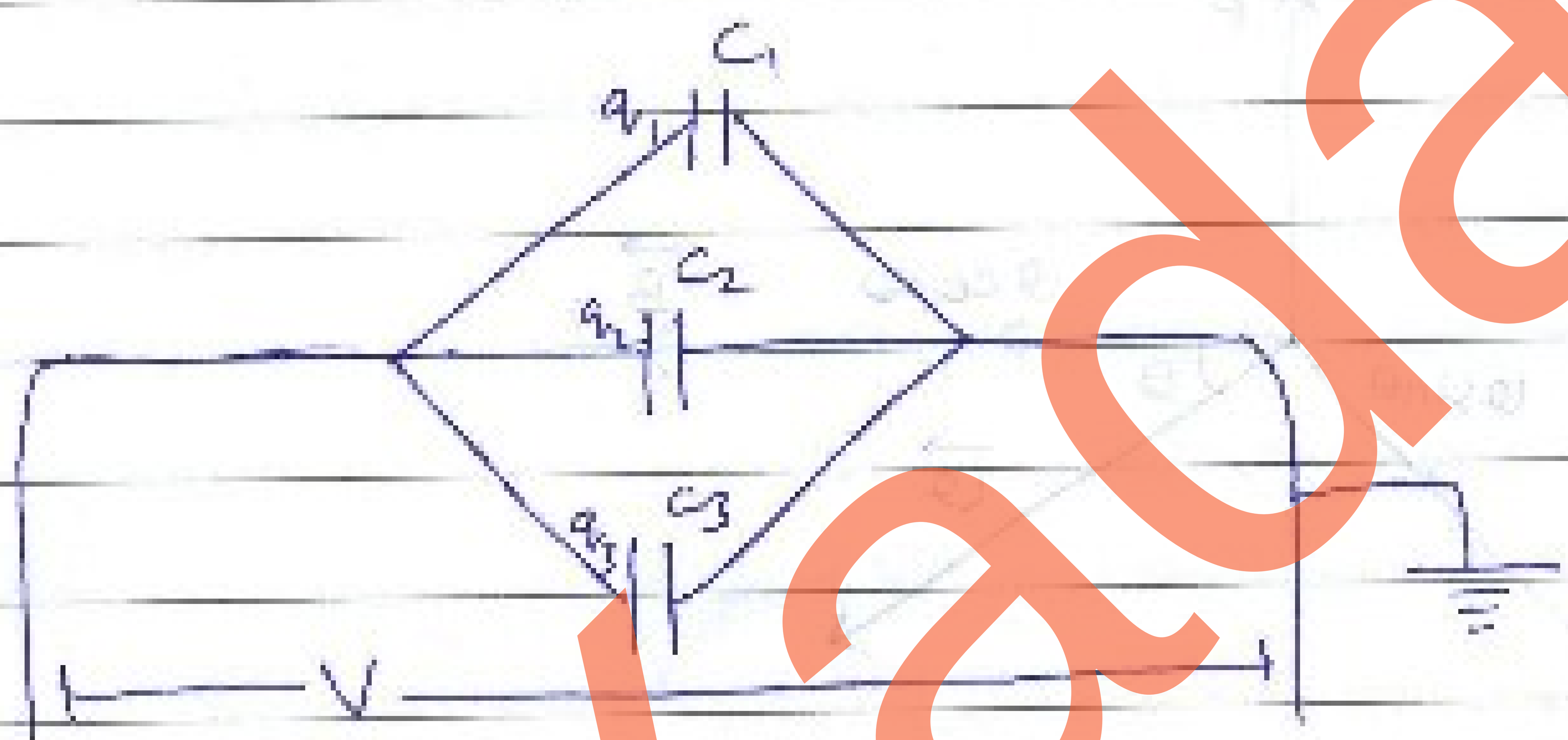


$$V = V_1 + V_2 + V_3$$

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

$$\boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Capacitors in parallel



$$q = q_1 + q_2 + q_3$$

$$CV = C_1V + C_2V + C_3V$$

$$\boxed{C = C_1 + C_2 + C_3}$$