Acceleration

- It is defined as the rate of change in velocity of the object.

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

- vector quantity
- S.I. unit – ms^{-2}
- negative acceleration is called retardation.

Uniform acceleration

An object is said to be moving with a uniform acceleration if its velocity changes by equal amounts in equal intervals of time.

Variable acceleration

An – – – – – – – – – – – – – – – – – – a variable
– – – – – – – – – – – – – – – – by unequal
– – – – – – – – – – – – – – – time.

Average acceleration

It is the ratio of the total change in velocity of the object during motion to the total time taken.

$$\vec{a}_{av} = \frac{\text{total change in velocity}}{\text{total time taken}}$$

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous velocity

The velocity of an object at a given instant of time is called its instantaneous velocity.

OR

The instantaneous velocity of an object at an instant of time 't', is defined as the limit of average velocity as time interval Δt, around time 't' becomes infinitesimally small.

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

$$\boxed{v = \frac{dx}{dt}}$$

Instantaneous acceleration

The acceleration of the object at a given instant of time is called its instantaneous acceleration

OR

It is defined as the limiting value of the average acceleration in a small interval around the given instant, when time interval tends to zero.

$$\boxed{\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{dv}{dt}}$$

Numerical on instantaneous velocity and acceleration

Q1.	The position of an object moving along x- axis is given by $x = a + bt^2$, where a = 8.5m and b = 2.5 ms^{-2} and t is measured in seconds. What is the velocity at t = 0s and t = 2s? What is the average velocity between t = 2s and t = 4s?
Q2.	The distance x of a particle moving in one dimension, under the action of a constant force is related to time t by equation $t = \sqrt{x} + 3$ where x is in metres and t is in seconds. Find the displacement of the particle when its velocity is zero.
Q3.	A particle moves along a straight line such that its displacement 's' at any time 't' is given by $s = t^3 - 6t^2 + 3t + 4$ metres. Find the velocity when the acceleration is zero.
Q4.	If the velocity of a particle is given by $v = \sqrt{180 - 16x}$ m/s, what will be its acceleration?
Q5.	The relation between time 't' and distance 'x' is $t = \alpha x^2 + \beta x$ where α and β are constants. Show that retardation is $2\alpha v^3$, where v is the instantaneous velocity.

Date ___/___/___

Page ______

Ans-1

$$x = a + bt^2$$

$$v = \frac{dx}{dt}$$

$$= \frac{d}{dt}(a+bt^2)$$

$$= \frac{d}{dt}(a) + \frac{d}{dt}(bt^2)$$

$$= 0 + b(2t) \quad \left[\because \frac{d}{dt}t^2 = 2t\right]$$

$$= 2bt$$

At $t = 0\,s$, $v = 2\times 2.5 \times 0 = 0\ ms^{-1}$

$= 2\,s$, $v = 2 \times 2.5 \times 2 = 10\ ms^{-1}$

At $t = 2\,s$, $x_2 = a + b(2)^2 = a + 4b$

$= 4\,s$, $x_4 = a + b(4)^2 = a + 16b$

Displacement $= x_4 - x_2 = a + 16b - a - 4b = 12b$

Average velocity $= \frac{\text{disp.}}{\text{time}} = \frac{12b}{2} = 6 \times 2.5 = 15\ ms^{-1}$

Ans-2

$$t = \sqrt{x} + 3$$

$$t - 3 = \sqrt{x}$$

$$(t-3)^2 = x$$

$$t^2 + 9 - 6t = x$$

$$v = \frac{dx}{dt} = \frac{d}{dt}(t^2 - 6t + 9)$$

$$= \frac{d}{dt}t^2 - \frac{d}{dt}(6t) + \frac{d}{dt}(9)$$

$$= 2t - 6 + 0$$

$$v = 2t - 6$$

A.T.Q $\quad v = 0$

$2t - 6 = 0$

$t = 3s$

$\therefore$ Displacement at $t = 3$

$x = (3)^2 - 6(3) + 9 = 0$

Ans-3 $\quad s = t^3 - 6t^2 + 3t + 4$

$$v = \frac{ds}{dt}$$

$$= \frac{d}{dt}(t^3 - 6t^2 + 3t + 4)$$

$$= \frac{d}{dt}(t^3) - \frac{d}{dt}(6t^2) + \frac{d}{dt}(3t) + \frac{d}{dt}(4)$$

$$= 3t^2 - 6(2t) + 3(1) + 0$$

$$v = 3t^2 - 12t + 3$$

$$a = \frac{dv}{dt}$$

$$= \frac{d}{dt}(3t^2 - 12t + 3)$$

$$= \frac{d}{dt}(3t^2) - \frac{d}{dt}(12t) + \frac{d}{dt}(3)$$

$$= 3(2t) - 12(1) + 0$$

$$a = 6t - 12$$

When $a = 0$

$6t - 12 = 0$

$t = 2$ sec.

At $t = 2s$, $\quad v = 3(2)^2 - 12(2) + 3$

$= 12 - 24 + 3 = -9\ ms^{-1}$

Date __/__/__

Page ____

Ans-4

$$v = \sqrt{180-16x}$$
$$= (180-16x)^{1/2}$$

$$a = \frac{dv}{dt}$$
$$= \frac{d}{dt}(180-16x)^{1/2}$$
$$= \frac{1}{2}(180-16x)^{\frac{1}{2}-1} \frac{d}{dt}(180-16x)$$
$$= \frac{1}{2(180-16x)^{1/2}}\left[\frac{d}{dt}(180) - \frac{d}{dt}(16x)\right]$$
$$= \frac{1}{2(180-16x)^{1/2}}\left[0 - 16\frac{dx}{dt}\right]$$
$$= \frac{1}{2v}(-16v)$$
$$= -8\ ms^{-2}$$

Ans-5

$$t = \alpha x^2 + \beta x$$

Differentiating both sides w.r.t time

$$\frac{d}{dt}t = \frac{d}{dt}\alpha x^2 + \frac{d}{dt}\beta x$$
$$1 = 2\alpha x\frac{dx}{dt} + \beta\frac{dx}{dt}$$
$$1 = 2\alpha x v + \beta v$$
$$v = \frac{1}{2\alpha x+\beta} = (2\alpha x+\beta)^{-1}$$

Differentiating both sides w.r.t time

$$\frac{dv}{dt} = \frac{d}{dt}(2\alpha x+\beta)^{-1}$$
$$a = -1(2\alpha x+\beta)^{-2}\frac{d}{dt}(2\alpha x+\beta)$$

$$a = \frac{-1}{(2\alpha x+\beta)^2}\left[\frac{d}{dt}(2\alpha x) + \frac{d}{dt}(\beta)\right]$$

$$= \frac{-1}{(2\alpha x+\beta)^2}\left[2\alpha \frac{dx}{dt} + 0\right]$$

$$= \frac{-1}{(2\alpha x+\beta)^2} \, 2\alpha v$$

$$= -v^2 \times 2\alpha v$$

$$= -2\alpha v^3$$

Equations of motion [Calculus method]

Consider an object moving in a straight line with uniform acceleration 'a'.

(1)

$$a = \frac{dv}{dt}$$

$$dv = a.dt$$

when $t = 0$, $v = u$
$= t$, $v = v$

Integrating both sides

$$\int_u^v dv = \int_0^t a.dt$$

$$\int_u^v dv = a\int_0^t dt$$

$$[v]_u^v = a[t]_0^t$$

$$v - u = a(t-0)$$

$$\boxed{v = u + at}$$

Date ___/___/___

Page _______

(2) $$v = \frac{dx}{dt}$$

$$dx = v.dt$$
$$= (u+at)dt$$
$$dx = u.dt + a.t.dt$$

when $t = 0$, $x = x_0$

$= t$ $x = x$

Integrating both sides

$$\int_{x_0}^{x} dx = \int_0^t u.dt + \int_0^t a.t.dt$$

$$\int_{x_0}^{x} dx = u\int_0^t dt + a\int_0^t t.dt$$

$$[x]_{x_0}^{x} = u[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t \qquad \left[\int dx^n = \frac{x^{n+1}}{n+1}\right]$$

$$[x - x_0] = u(t-0) + a\left[\frac{t^2}{2} - 0\right]$$

$$\boxed{s = ut + \frac{1}{2}at^2}$$

(3) $$a = \frac{dv}{dt}$$

$$= \frac{dv}{dt} \times \frac{dx}{dx}$$

$$= \frac{dx}{dt} \cdot \frac{dv}{dx}$$

$$a = v.\frac{dv}{dx}$$

$$v \cdot dv = a\,dx$$

when $x = x_0$ $\quad v = u$

$= x$ $\quad v = v$

Integrating both sides

$$\int_u^v v \cdot dv = \int_{x_0}^{x} a\,dx$$

$$\left[\frac{v^2}{2}\right]_u^v = a\left[x\right]_{x_0}^{x} \qquad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1}\right]$$

$$\frac{v^2}{2} - \frac{u^2}{2} = a\left[x - x_0\right]$$

$$\frac{v^2 - u^2}{2} = as$$

$$\boxed{v^2 - u^2 = 2as}$$

Distance travelled in n^{th} second

Distance travelled by a body in n seconds

$$S_n = un + \frac{1}{2}an^2$$

Distance travelled by the body in $(n-1)$ seconds

$$S_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2$$

So, distance travelled in n^{th} second

$$D_n = S_n - S_{n-1}$$

Date ___/___/___

Page ________

$$D_n = u\cancel{n} + \frac{1}{2}\cancel{a}n^2 - \cancel{un} + u - \frac{1}{2}\cancel{a}n^2 + an - \frac{a}{2}$$

$$= u + a\left(n - \frac{1}{2}\right)$$

$$\boxed{D_n = u + \frac{a}{2}(2n-1)}$$

Numerical on equations of motion	
Q1.	A car moving a speed of $50 kmh^{-1}$ can be stopped by brakes after at least 6m.What will be the minimum stopping distance, if the same car is moving with a speed of $100 kmh^{-1}$? [24.1m]
Q2.	A bullet fired into a fixed target loses half of its velocity after penetrating 3cm.How much further it will penetrate before coming to rest? [1cm]
Q3.	On a foggy day, 2 car drivers spot each other, when they are just 80m apart. They are travelling at $72 kmh^{-1}$ and $60 kmh^{-1}$ respectively. Both of them simultaneously apply brakes, which retard both the cars at the rate of $5 ms^{-2}$.Determine whether they avert the collision or not? [Yes]
Q4.	Two trains A and B of length 400m each moving on 2 parallel tracks with a uniform speed of $72 kmh^{-1}$ in the same direction with A ahead of B. The driver of B decides to overtake A and accelerates by $1 ms^{-2}$. If after 50s, the guard of B just brushes past the driver of A, what was the original distance between them? [450 m]
Q5.	A body covers 4m in 3rd second and 12m in 5th second. If the motion is uniformly accelerated, how far will it travel in the next 3 seconds? [60m]
Q6.	An automobile travelling with a speed of $60 kmh^{-1}$ can brake to stop within a distance of 20m. If the car is going twice as fast i.e. $120 kmh^{-1}$, find the stopping distance. [80m]
Q7.	A body travels a distance 2m in 2s and 2.8 m in next 4 s. What will be speed of the body at the end of 10th second from the start? [$0.1 ms^{-1}$]
Q8.	A body moving with a uniform acceleration travels distances of 24m and 64m during the first 2 equal consecutive intervals of time, each of duration 4 s. Determine the initial velocity and acceleration of the moving body. [$1 ms^{-1}$, $2.5 ms^{-2}$]
Q9.	A stone is thrown vertically upwards with an initial velocity of $14 ms^{-1}$. Find the maximum height reached and the time of ascent. [10m, 1.429 s]
Q10.	A balloon is moving upwards with a speed of $5 ms^{-1}$. When it is at a height of 98m, a packet is dropped from it. What is the velocity of the packet, when it strikes the ground? [$44.1 ms^{-1}$]
Q11.	From the top of a tower 100m in height, a ball is dropped and at the same time another ball is projected vertically upwards from the ground with a velocity of $25 ms^{-1}$. Find when and where the 2 balls will meet? [4s, 78.4 (from the top)]
Q12.	Two balls are thrown simultaneously. A vertically upwards with a speed of $20 ms^{-1}$ from the ground and B, vertically downwards from a height of 40 m with the same speed and along the same line of motion. At what points the 2 collide? [15.1 m (above the ground)]

Ans-1 Case 1

$u = 50\ kmh^{-1} = 50 \times \frac{5}{18}\ ms^{-1}$

$v = 0\ ms^{-1}$

$s = 6m$

$v^2 - u^2 = 2as$

$0^2 - \frac{250 \times 250}{18 \times 18} = 2 \times a \times 6$

$a = -\frac{250 \times 250}{18 \times 18 \times 12}\ ms^{-2}$

Case 2

$u = 100\ kmh^{-1} = 100 \times \frac{5}{18}\ ms^{-1}$

$v = 0\ ms^{-1}$

$s = ?$

$v^2 - u^2 = 2as$

$0^2 - \frac{500 \times 500}{18 \times 18} = 2\left(\frac{-250 \times 250}{18 \times 18 \times 12}\right)s$

$+\frac{\cancel{500}^{2} \times 500}{\cancel{18} \times \cancel{18}} = +\frac{250 \times \cancel{250} \times \cancel{2}}{\cancel{18} \times \cancel{18} \times 12} \times s$

$\boxed{s = 24m}$

Ans-2 Case 1

$u_1 = u$, $v_1 = \frac{u}{2}$, $s_1 = 3cm$

Date ___/___/___
Page ______

$$v_1^2 - u_1^2 = 2as_1$$

$$\frac{u^2}{4} - u^2 = 2\times a\times 3$$

$$\frac{u^2 - 4u^2}{4} = 6a$$

$$\frac{-3u^2}{4} = 6a$$

$$a = \frac{-u^2}{8}\ ms^{-2}$$

<u>Case 2</u>

$$u_2 = \frac{u}{2}, \quad v_2 = 0, \quad a = \frac{-u^2}{8}$$

$$v_2^2 - u_2^2 = 2as_2$$

$$0^2 - \frac{u^2}{4} = 2\times\left(\frac{-u^2}{8}\right)\times s_2$$

$$-\frac{u^2}{4} = \frac{-2u^2}{8}\times s_2$$

$$\boxed{s_2 = 1\,cm}$$

Ans-3 <u>For car A</u>

$$u_1 = 72\,kmh^{-1} = 72\times\frac{5}{18}\,ms^{-1} = 20ms^{-1}$$

$$v_1 = 0ms^{-1}$$

$$a = -5\,ms^{-2}$$

$$v_1^2 - u_1^2 = 2as_1$$

$$0^2 - 20^2 = 2(-5)s_1$$

$$-400 = -10s_1$$

$$s_1 = 40\,m$$

Date / /
Page

For car B

$u_2 = 60\,kmh^{-1} = 60 \times \frac{5}{18}\,ms^{-1} = \frac{50}{3}\,ms^{-1}$

$v_2 = 0\,ms^{-1}$

$a = -5\,ms^{-2}$

$v_2^2 - u_2^2 = 2as_2$

$0^2 - \left(\frac{50}{3}\right)^2 = 2 \times (-5)\, s_2$

$+\frac{2500}{9} = +10\, s_2$

$s_2 = +\frac{250}{9} = 27.8\,m$

Total distance covered by both the cars

$s = s_1 + s_2 = 40 + 27.8 = 67.8\,m$

As this distance is less than 80m so the cars will avert collision.

Ans-4 For train A

$u_A = 72\,kmh^{-1} = 72 \times \frac{5}{18}\,ms^{-1} = 20\,ms^{-1}$

$a = 0$

$t = 50\,s$

$s_A = u_A t + \frac{1}{2} a t^2$

$= 20 \times 50 + \frac{1}{2} \times 0 \times 2500$

$= 1000\,m$

For train B

$u_B = 72\ kmh^{-1} = 20\ ms^{-1}$

$a = 1\ ms^{-2}$

$t = 50\ s$

$$S_B = u_B t + \frac{1}{2} a t^2$$

$$= 20 \times 50 + \frac{1}{2} \times 1 \times 2500$$

$$= 2250\ m$$

Now, $S_B - S_A = S + l_A + l_B$

$$2250 - 1000 = S + 400 + 400$$

$$1250 = S + 800$$

$$\boxed{S = 450\ m}$$

Ans-5

$$D_n = u + \frac{a}{2}(2n-1)$$

$$D_3 = u + \frac{a}{2}(2 \times 3 - 1)$$

$$4 = u + \frac{5}{2} a \quad \text{—— (1)}$$

$$D_5 = u + \frac{a}{2}(2 \times 5 - 1)$$

$$12 = u + \frac{9}{2} a \quad \text{—— (2)}$$

(2) − (1)

$$8 = 2a$$

$$a = 4\ ms^{-2}$$

from (1) $u = 4 - \frac{5}{2} \times 4 = -6\ ms^{-1}$

Now, $S_5 = -6 \times 5 + \frac{1}{2} \times 4 \times 25$

$= -30 + 50$

$= 20m$

$S_8 = -6 \times 8 + \frac{1}{2} \times 4 \times 64$

$= -48 + 128$

$= 80$

Now, $S = S_8 - S_5$

$= 80 - 20$

$\boxed{S = 60m}$

Ans-6 Case 1

$u_1 = 60\ kmh^{-1} = 60 \times \frac{5}{18}\ ms^{-1} = \frac{50}{3}\ ms^{-1}$

$v_1 = 0\ ms^{-1}$

$s_1 = 20m$

$v_1^2 - u_1^2 = 2as_1$

$0^2 - \left(\frac{50}{3}\right)^2 = 2 \times a \times 20$

$-\frac{2500}{9} = 40a$

$a = -\frac{125}{18}\ ms^{-2}$

Case 2

$u_2 = 120\ kmh^{-1} = 120 \times \frac{5}{18}\ ms^{-1} = \frac{100}{3}\ ms^{-1}$

$v_2 = 0\ ms^{-1}$, $a = -\frac{125}{18}\ ms^{-2}$

$S_2 = ?$

$v_2^2 - u_2^2 = 2as_2$

$0^2 - \left(\frac{100}{3}\right)^2 = 2 \times \left(-\frac{125}{18}\right) \times s_2$

$+\frac{\cancel{100}^{4} \times 10\cancel{0}}{9} = +\frac{\cancel{250}}{\cancel{18}_{2}} \times s_2$

$\boxed{s_2 = 80m}$

Ans-7 Case 1 $t = 2\,sec$

$s = ut + \frac{1}{2}at^2$

$2 = u(2) + \frac{1}{2} \times a \times (2)^2$

$2 = 2u + \frac{1}{\cancel{2}} \times a \times \cancel{4}\,2$

$u + a = 1 \quad \text{---} (1)$

Case 2 $t = 6s\,(2+4)$; $s = 2 + 2.8 = 4.8m$

$s = ut + \frac{1}{2}at^2$

$4.8 = u(6) + \frac{1}{2} \times a \times (6)^2$

$4.8 = 6u + \frac{1}{\cancel{2}} \times a \times \cancel{36}\,18$

$4.8 = 6u + 18a$

$u + 3a = 0.8 \quad \text{---} (2)$

(2) - (1)

$2a = -0.2$

$a = -0.1\ ms^{-2}$

from (1) $u + (-0.1) = 1$

$u = 1.1\ ms^{-1}$

Now, $v = u + at$

$= 1.1 + (-0.1)10$

$= 1.1 - 1$

$$\boxed{v = 0.1 \ ms^{-1}}$$

Ans-8 Case 1

$$s = ut + \frac{1}{2}at^2$$

$$24 = u(4) + \frac{1}{2} \times a(4)^2$$

$$24 = 4u + \frac{1}{2} \times a \times \cancel{16}\,8$$

$$24 = 4u + 8a$$

$$u + 2a = 6 \quad \text{—— (1)}$$

Case 2 $t = 4+4 = 8$ sec. , $s = 24+64 = 88m$

$$s = ut + \frac{1}{2}at^2$$

$$88 = u(8) + \frac{1}{2} \times a \times (8)^2$$

$$88 = 8u + \frac{1}{2} \times a \times \cancel{64}\,32$$

$$88 = 8u + 32a$$

$$u + 4a = 11 \quad \text{—— (2)}$$

(2) − (1) $2a = 5$

$$\boxed{a = 2.5 \ ms^{-2}}$$

from (1) $u + 2(2.5) = 6$

$u + 5 = 6$

$$\boxed{u = 1 \ ms^{-1}}$$

Date ___/___/___

Page ______

Ans-9

$u = 14\ ms^{-1}$

$a = -9.8\ ms^{-2}$

$v = 0\ ms^{-1}$ (At max. height)

$s = ?$

$t = ?$

$v^2 - u^2 = 2as$

$0^2 - 14^2 = 2 \times (-9.8)\ s$

$+196 = +19.6 \times s$

$\boxed{s = \frac{196}{19.6} = 10\ m}$

$v = u + at$

$0 = 14 - 9.8 \times t$

$-14 = -9.8 \times t$

$t = \frac{14}{9.8}$

$\boxed{t = 1.429\ s}$

Ans-10

$u = -5\ ms^{-1}$

$s = 98\ m$

$a = 9.8\ ms^{-2}$

$v^2 - u^2 = 2as$

$v^2 - (-5)^2 = 2 \times 9.8 \times 98$

$v^2 - 25 = 1920.8$

$v^2 = 1920.8 + 25$

$v = \sqrt{1945.8}$

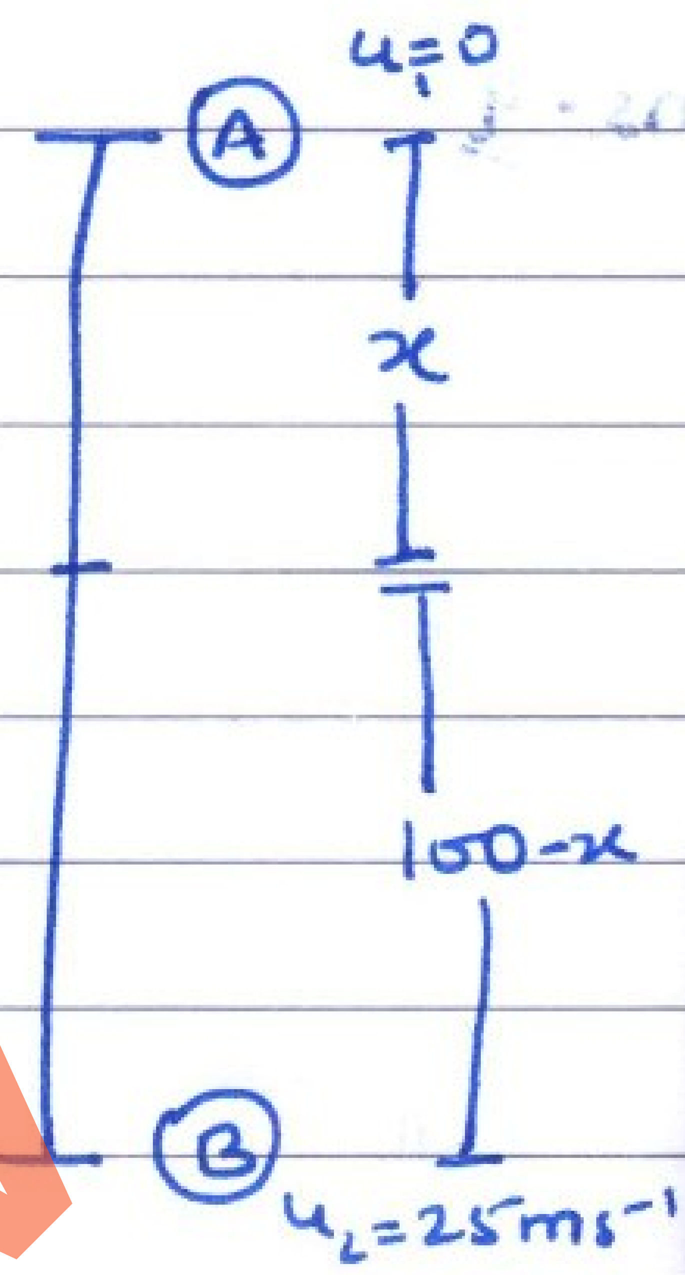
$\boxed{v = 44.1\ ms^{-1}}$

Date ___/___/___

Page ______

Ans-11

Let the 2 balls meet in time 't'.



For ball A

$u = 0$

$s = x$

$a = 9.8\ ms^{-2}$

$$s = ut + \frac{1}{2}at^2$$

$$x = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$$

$$x = 4.9t^2 \quad \text{—— (1)}$$

For ball B

$u = 25\ ms^{-1}$

$s = 100 - x$

$a = -9.8\ ms^{-2}$

$$s = ut + \frac{1}{2}at^2$$

$$100 - x = 25t + \frac{1}{2}(-9.8)t^2$$

$$100 - x = 25t - 4.9t^2$$

$$100 - x = 25t - x$$

$$100 = 25t$$

$$t = \frac{100}{25}$$

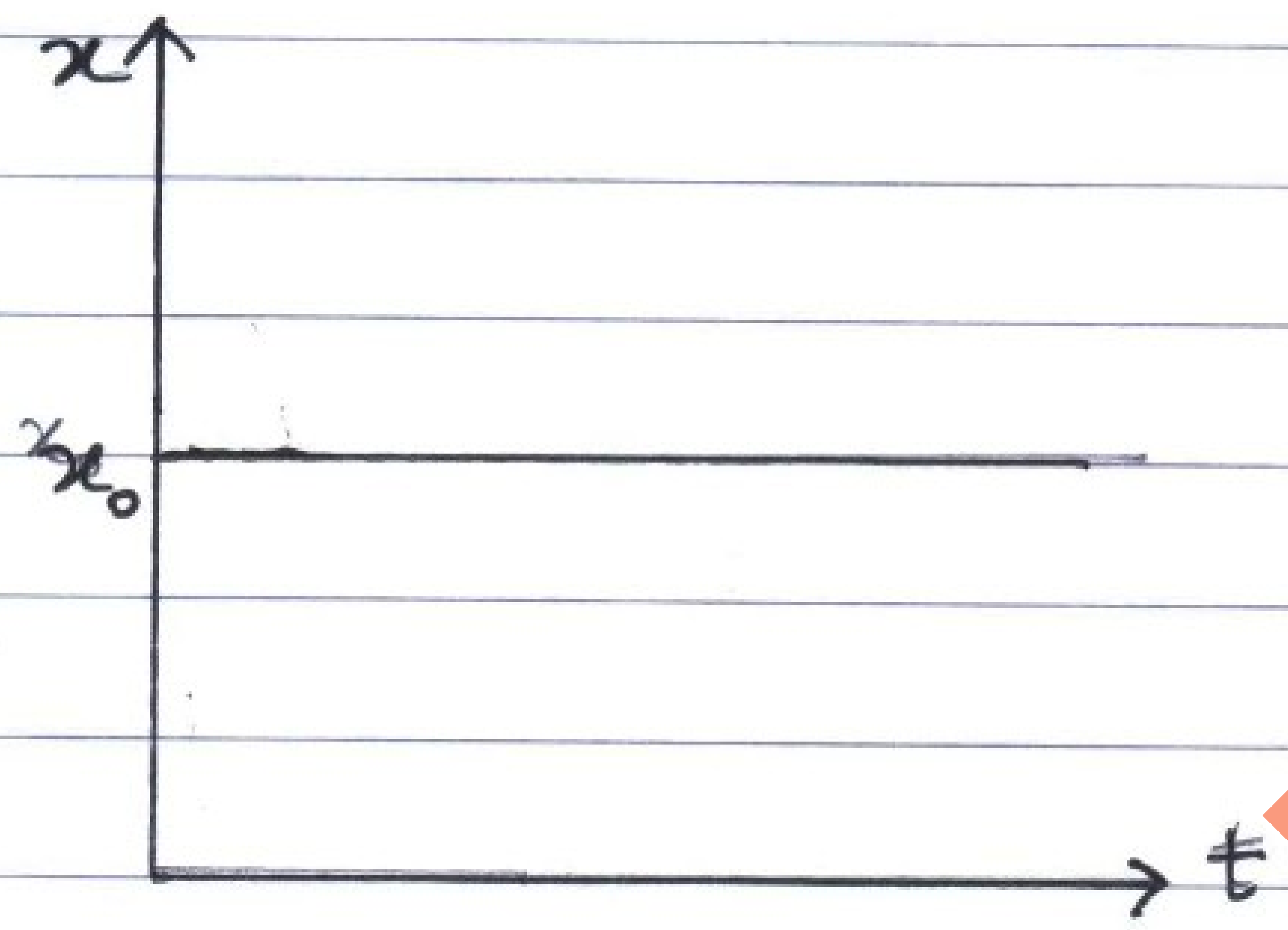
$$\boxed{t = 4\ s}$$

from (1) $\quad x = 4.9t^2 = 4.9 \times 4^2 = 4.9 \times 16$

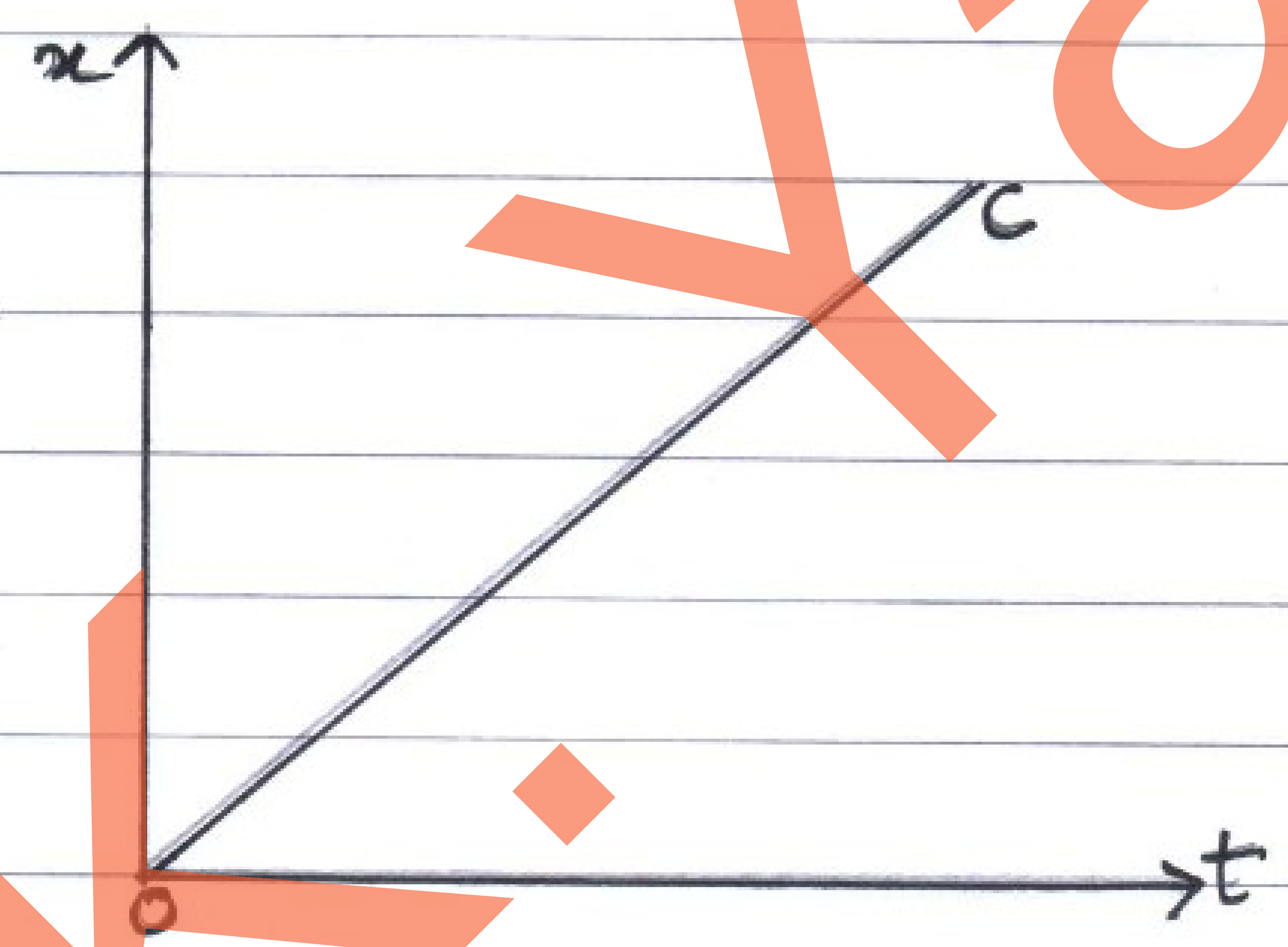
$$\boxed{x = 78.4\ m}$$

Position-time graph for a moving object

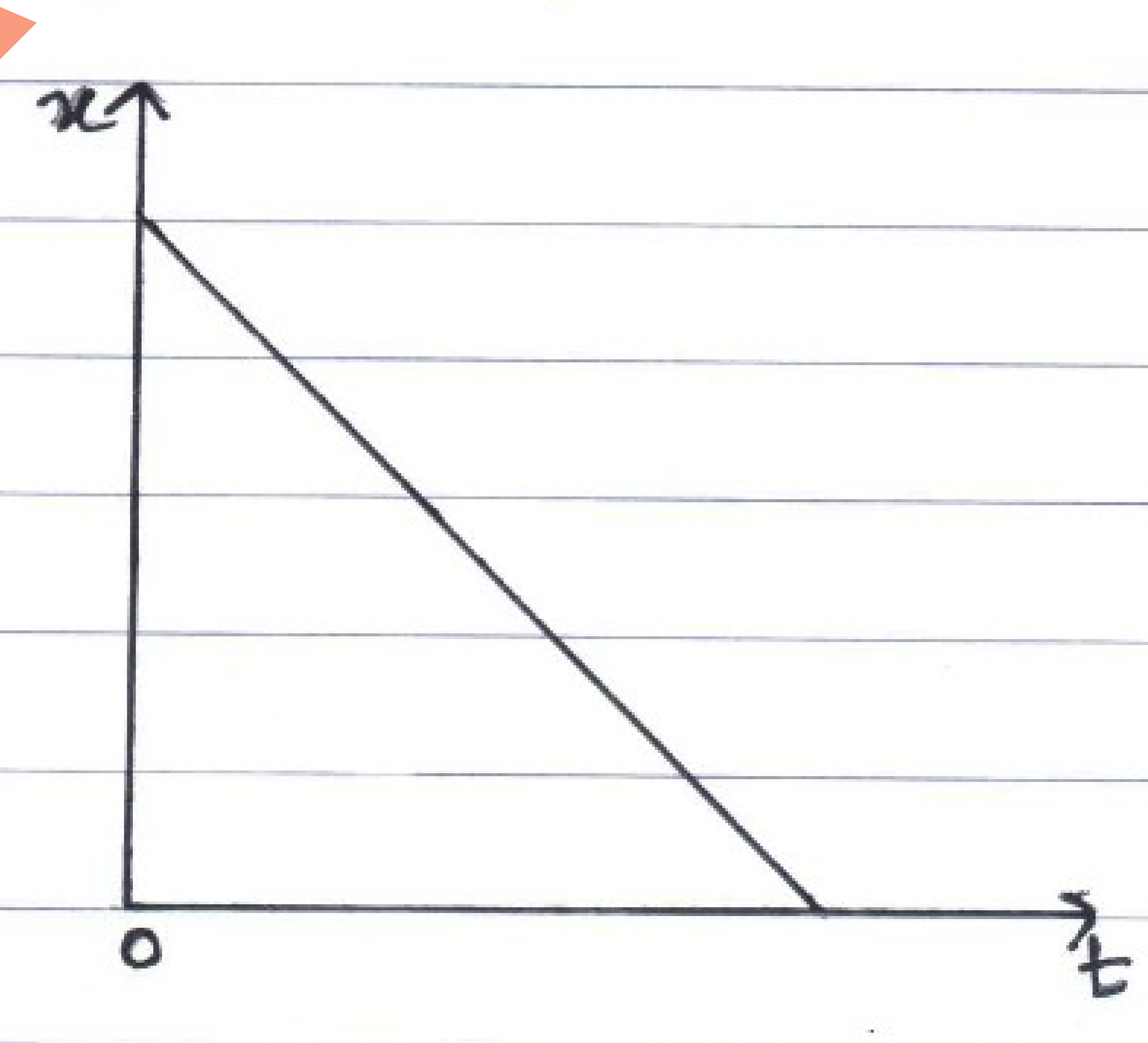
① Object at rest



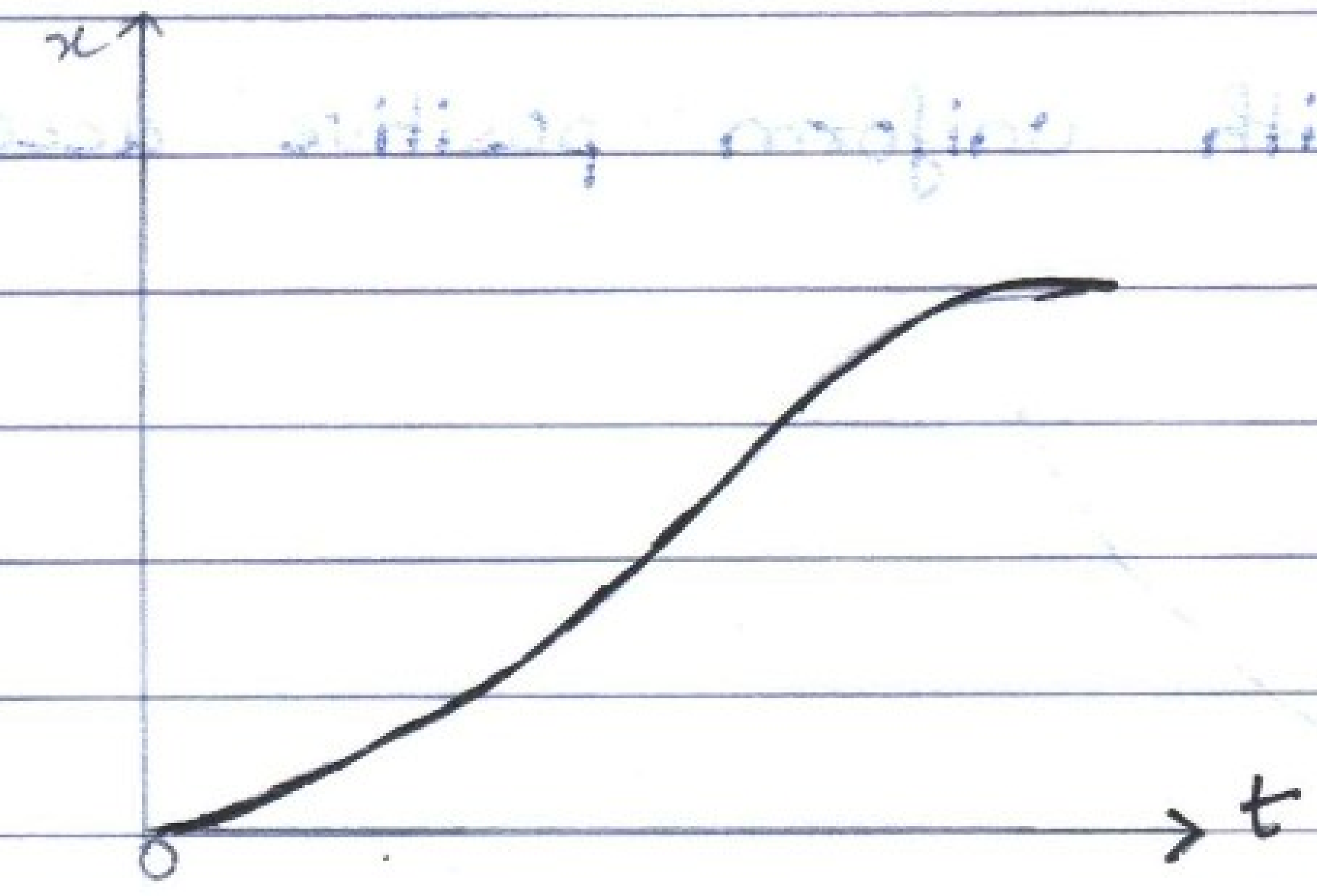
② Object in uniform motion (along a straight line) starting from origin.



③ Object moving with constant negative velocity starting from positive position

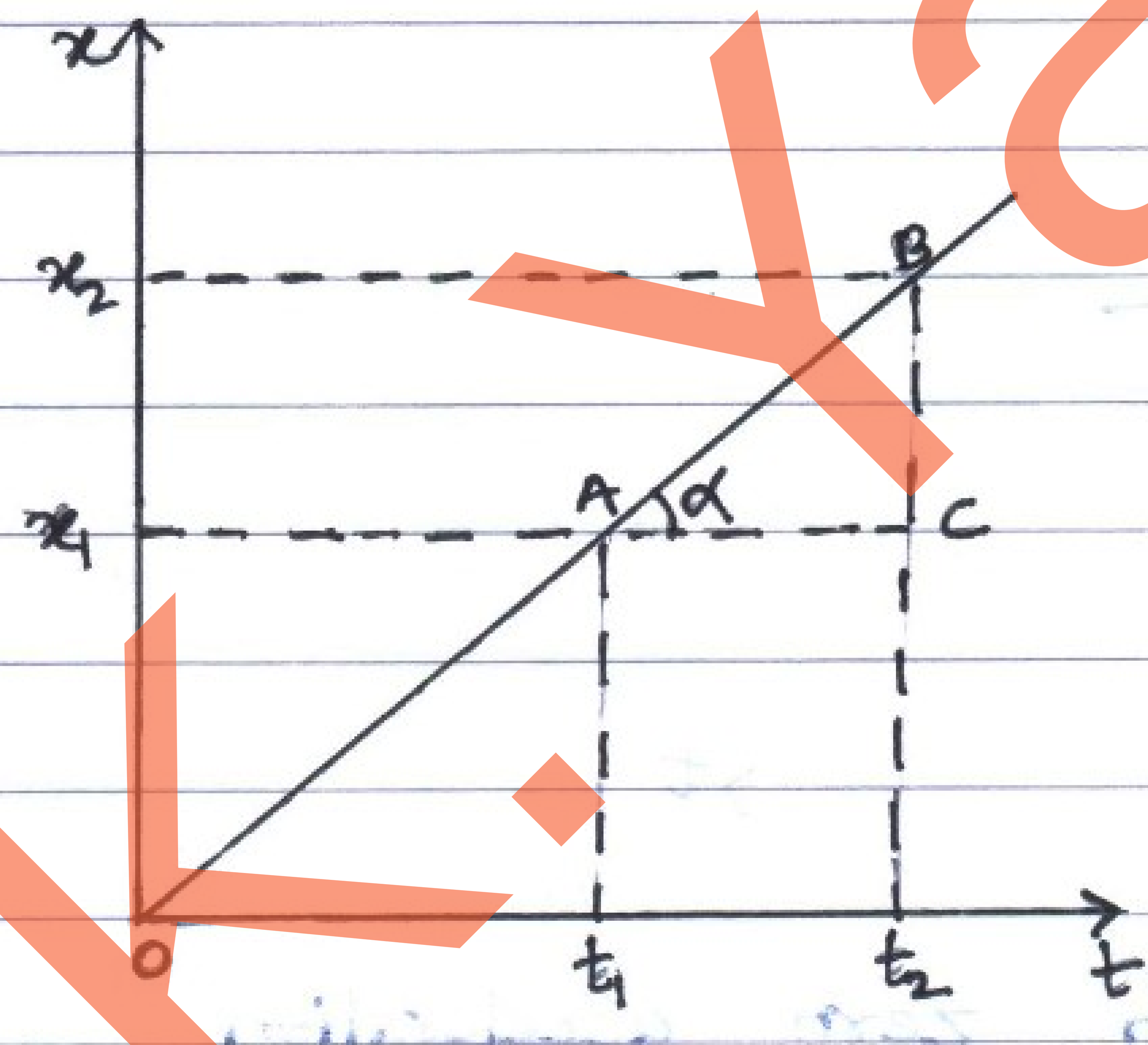


(4) Object in non-uniform motion along a straight line



Important points

(i) Velocity of an object in uniform motion is equal to the slope of position-time graph ~~t~~ with time axis.



$$\text{velocity} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{BC}{AC} = \tan\alpha = \text{slope of } x\text{-}t \text{ graph}$$

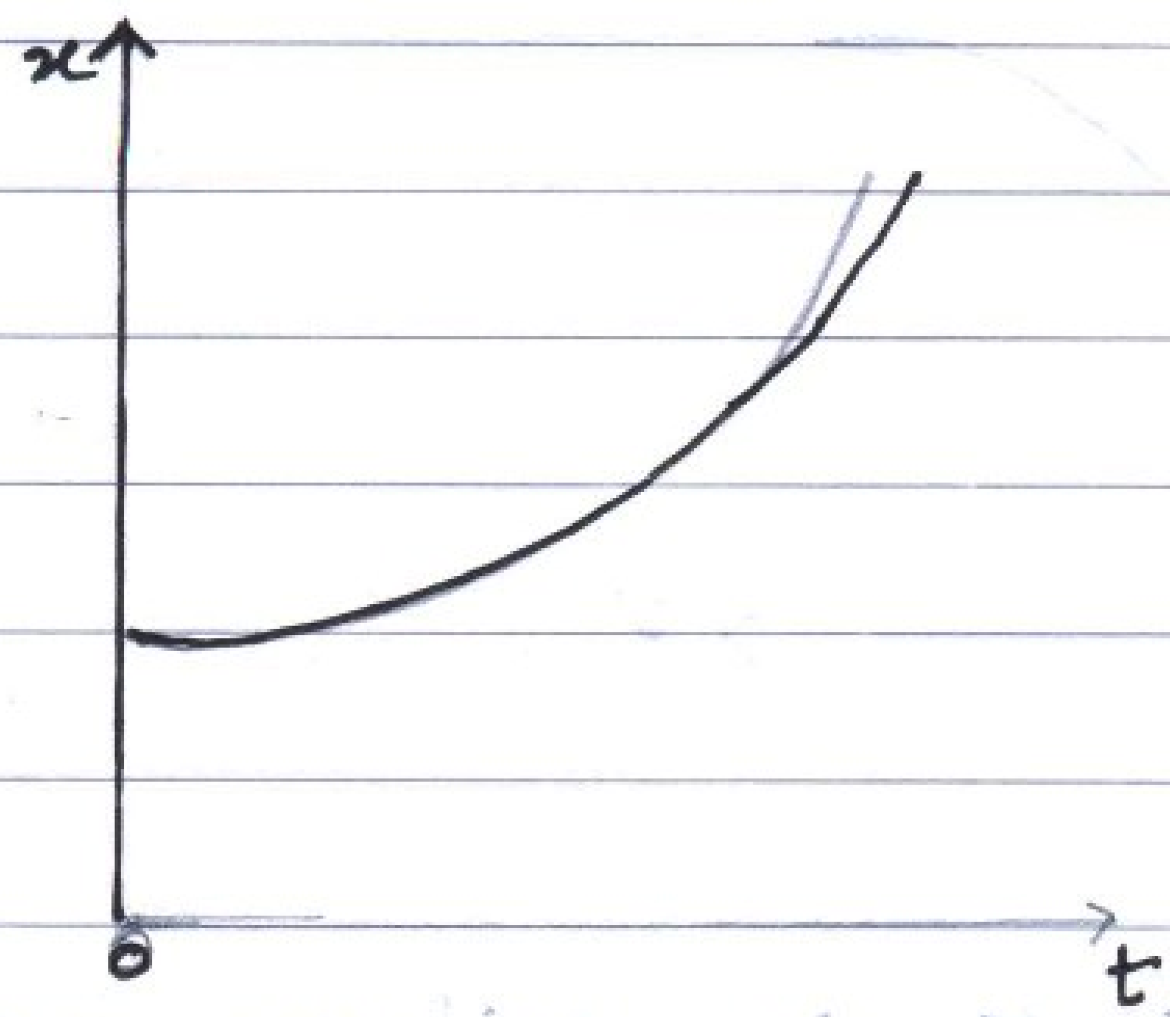
(ii) Position time graph cannot be a straight line parallel to position axis, as it will indicate infinite velocity.

(iii) Velocity - time graph in uniform motion

Distance/Displacement = area under v-t graph

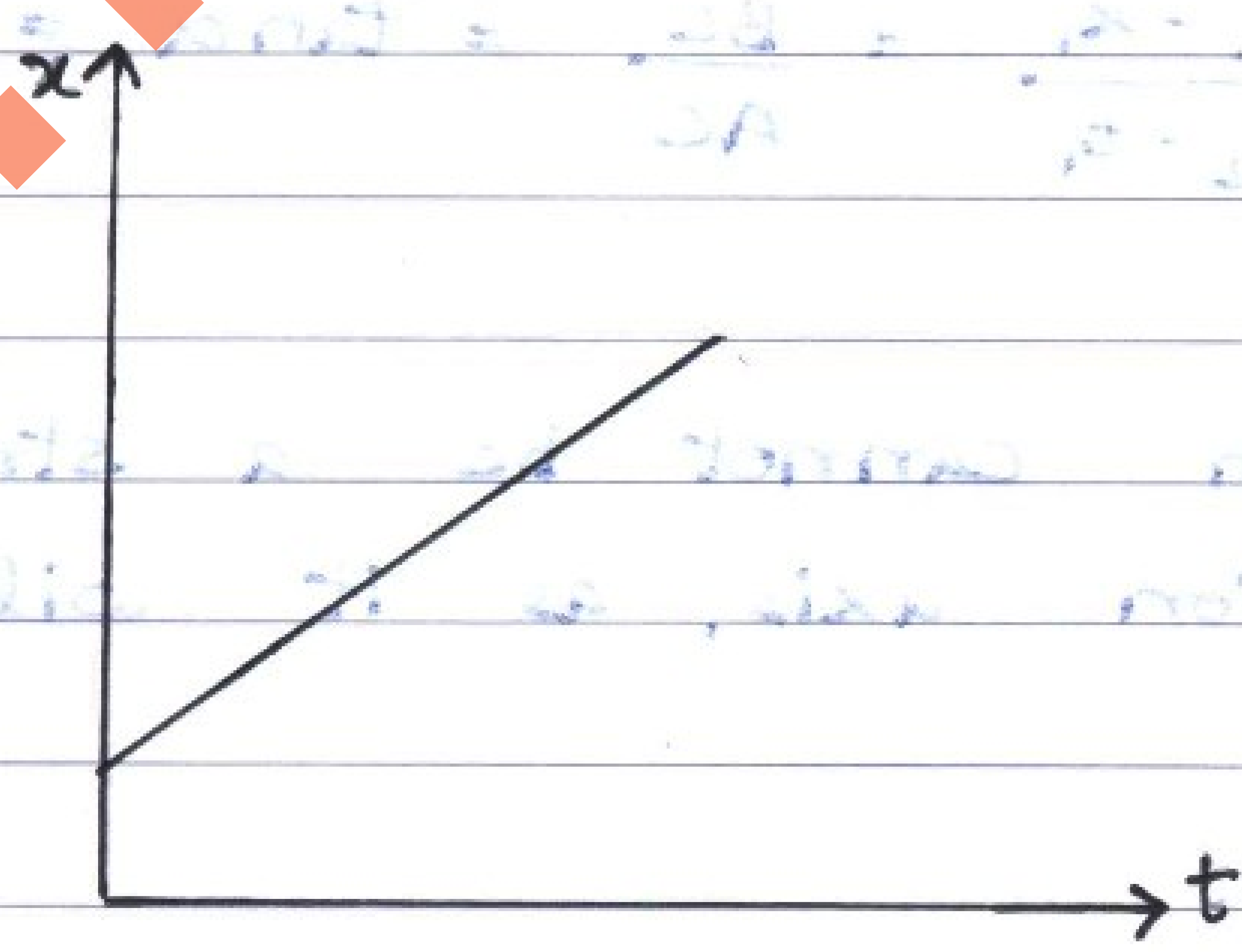
Position - time graph for accelerated motion

① Object moving with uniform positive acceleration



② Object moving with negative acceleration

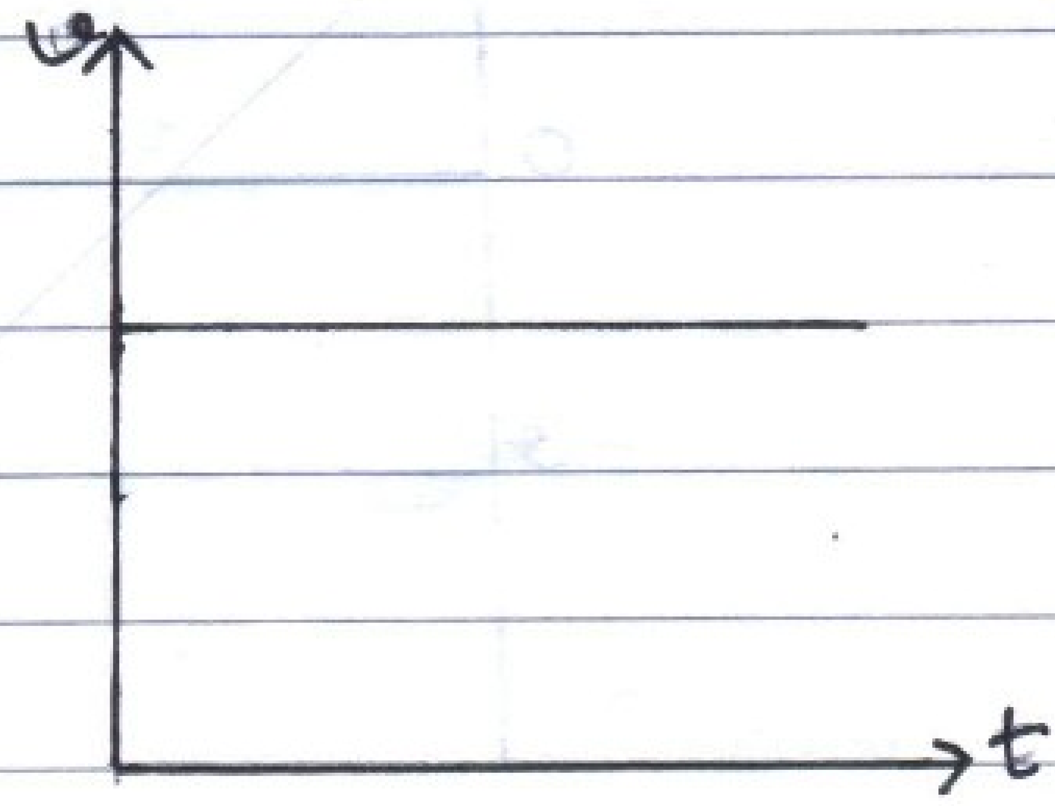
③ Object moving with zero acceleration



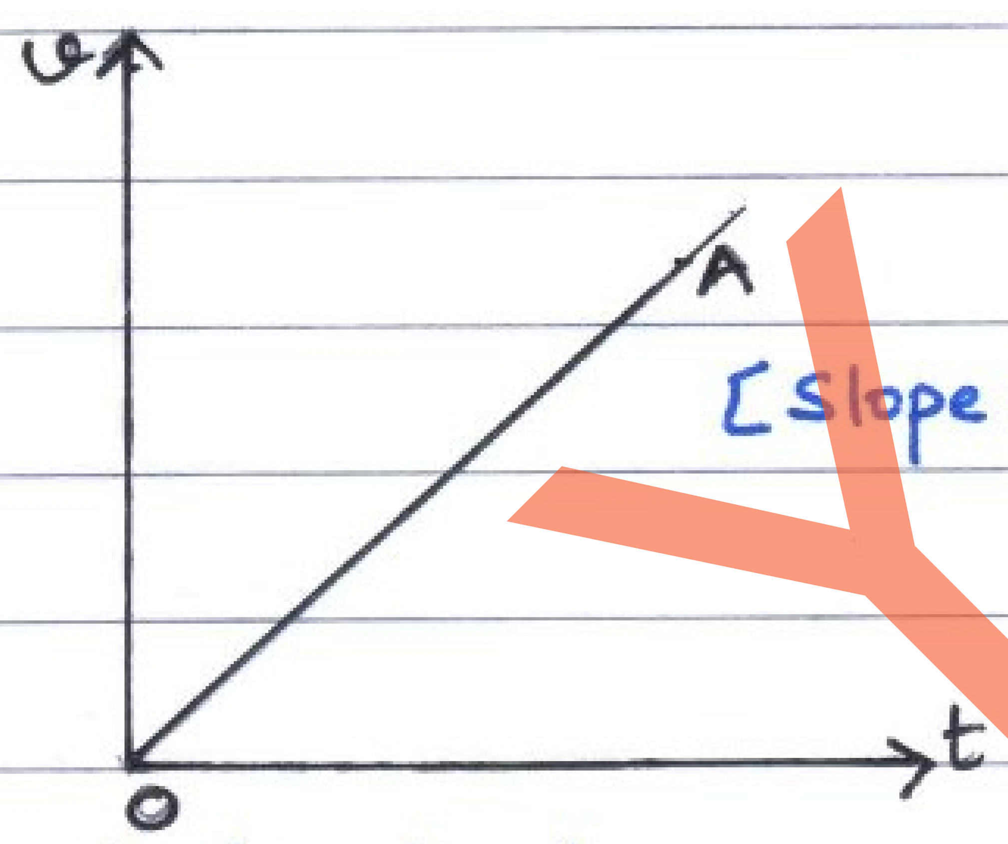
* Slope of x-t graph at any instant of time gives the instantaneous velocity of the object at that instant of time.

Velocity - time graph of an accelerated motion

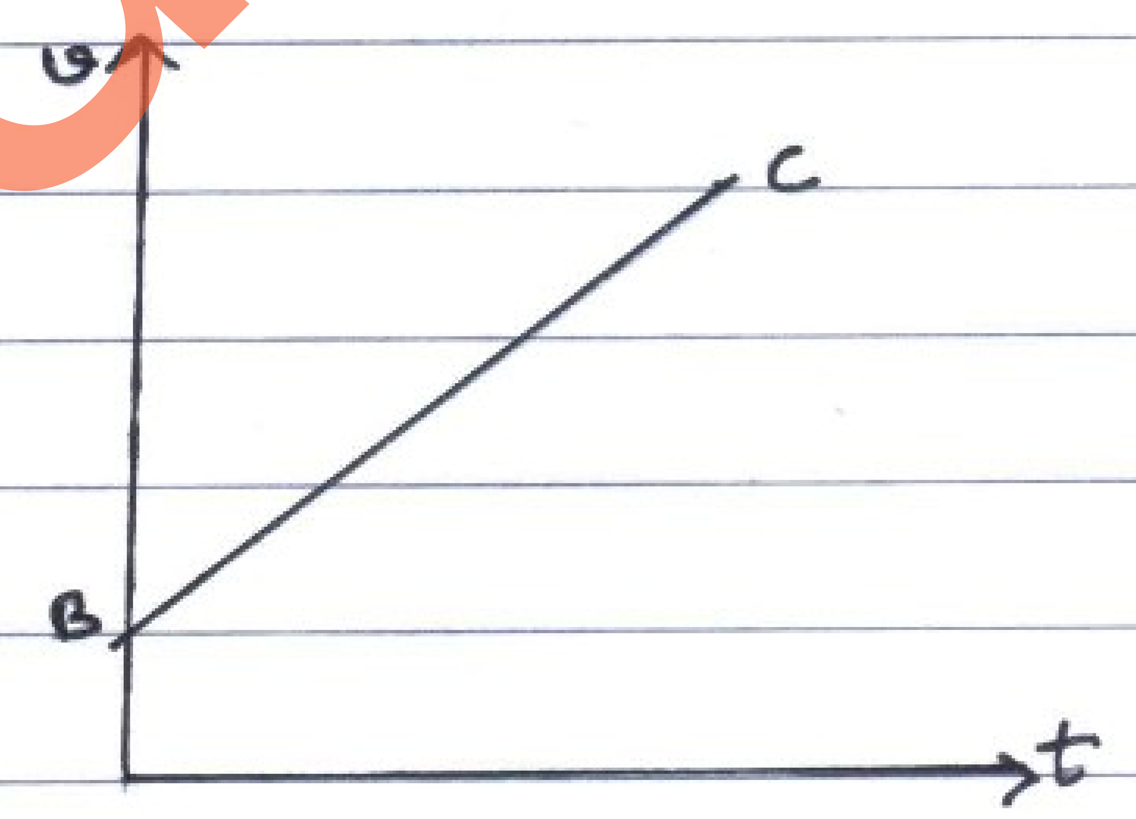
① Object moving with zero acceleration



② Object moving with constant positive acceleration

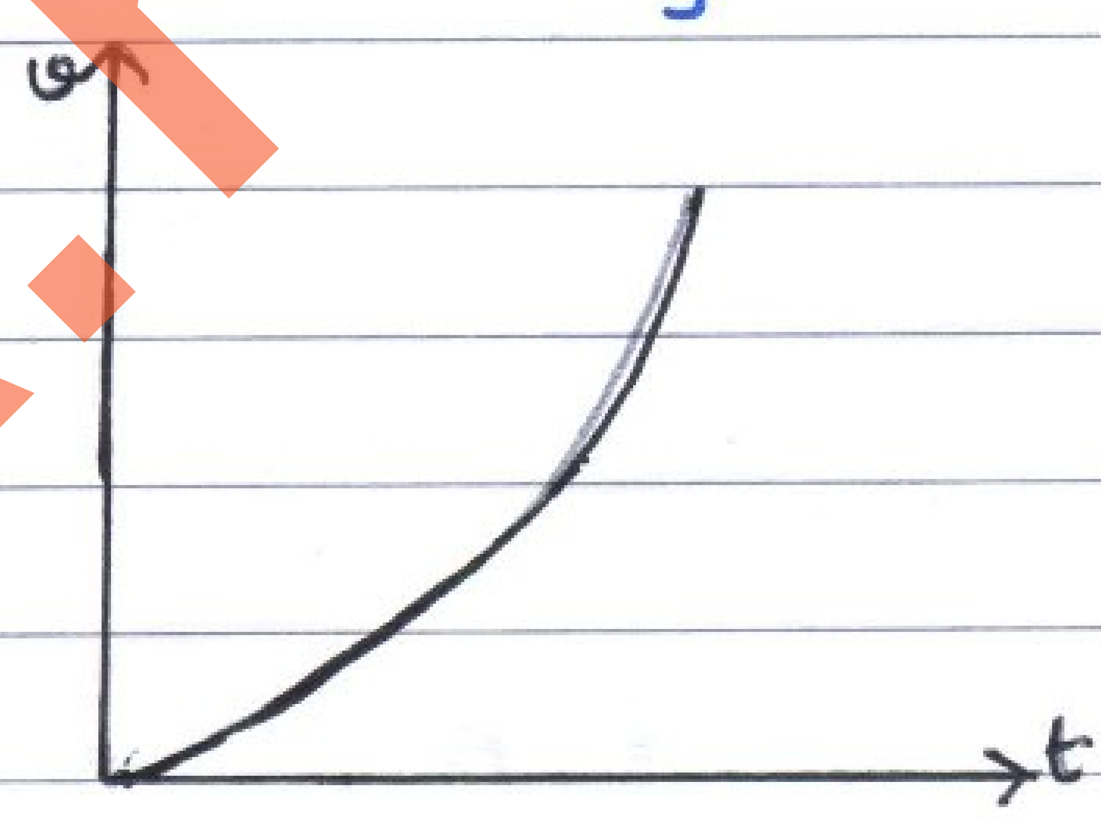


(a) initial velocity zero

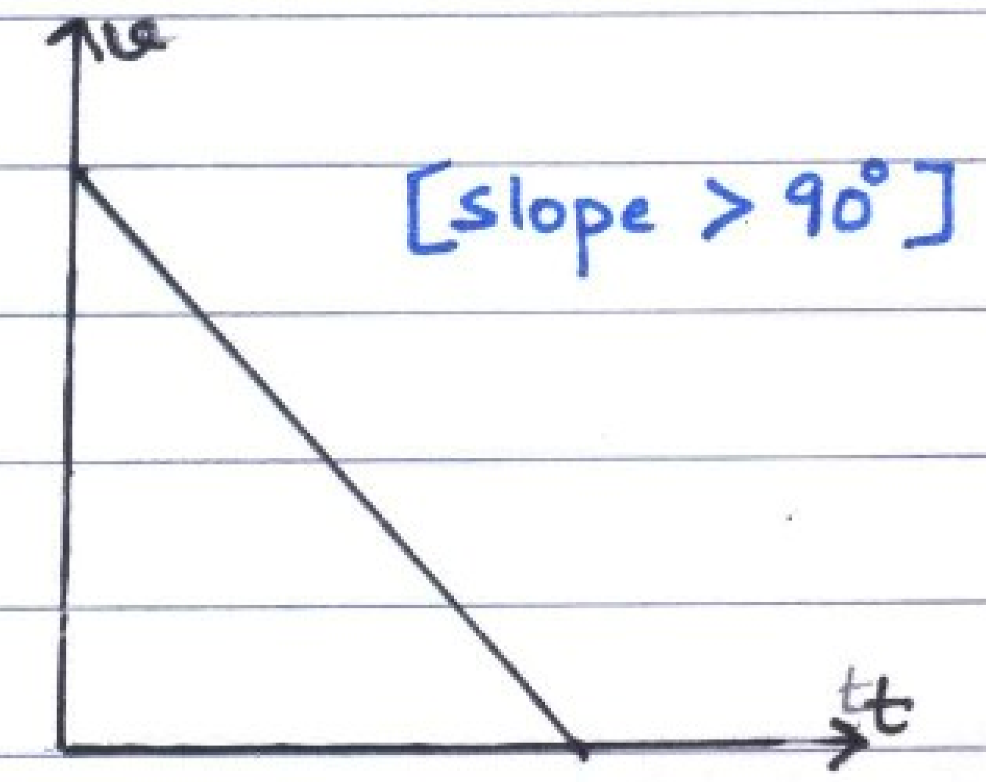


(b) some initial velocity

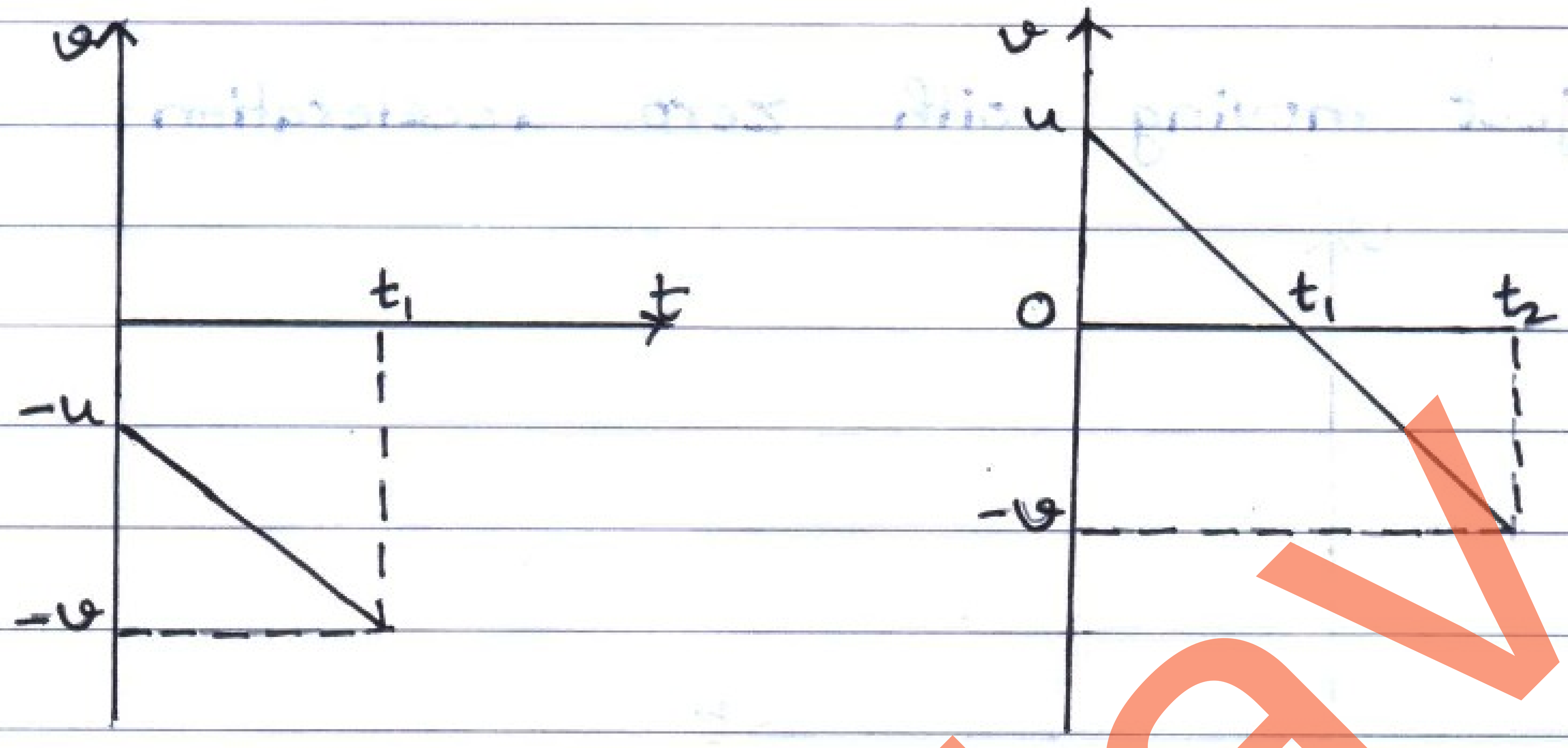
③ Object moving with increasing acceleration, having zero initial velocity



④ Object moving with constant negative acceleration, having positive initial velocity.



⑤ Object moving with uniform negative acceleration



(a) with −ve initial velocity

(b) with +ve initial velocity