Test 8

1. What is conserved in Bernoulli's theorem? 1

Ans. total energy of liquid at all points

2. If the rate of flow of liquid through a horizontal pipe of length l and radius R is Q. What is rate of flow of liquid if length and radius of tube is doubled? 1

Ans.
$$V=\frac{\pi}{8}\frac{pr^4}{\eta l}$$
$$r'=2r$$
$$l'=2l$$
$$V'=\frac{\pi}{8}\frac{p\left(r'\right)^4}{\eta l'}$$
$$=\frac{\pi}{8}\frac{p(2r)^4}{\eta(2l)}$$
$$=8\left(\frac{\pi}{8}\frac{pr^4}{\eta l}\right)$$
$$=8V$$

3. Water is coming out of a hole made in the wall of tank filled with fresh water. If the size of the hole is increased, will the velocity of efflux change? 1

Ans.
$$V=\sqrt{2gh}$$
Since the velocity of efflux is independent of area of hole, it will remain the same.

4. The accumulation of snow on an aero plane wing may reduce the lift. Explain? 1

Ans. Due to the accumulation of snow on the wings of the aero plane, the structure of wings no larger remains as that of aerofoil. As a result, the net upward force (i.e. lift) is decreased.

5. Two pipes P and Q having diameters 2×10^{-2} m and 4×10^{-2} m respectively are joined in Series with the main supply line of water. What is the velocity of water flowing in pipe P? 2

Ans.
$$D_P=2\times10^{-2}m$$
$$r_P=\frac{D_P}{2}=\frac{2\times10^{-2}}{2}=10^{-2}m$$
$$a_P=\pi r_P^2=3.14\times\left(10^{-2}\right)^2m^2=3.14\times10^{-4}m^2$$
$$D_Q=4\times10^{-2}m$$
$$r_Q=\frac{D_Q}{2}=\frac{4\times10^{-2}}{2}=2\times10^{-2}m$$
$$a_Q=\pi r_Q^2=3.14\times\left(2\times10^{-2}\right)^2m^2=3.14\times4\times10^{-4}m^2$$
Acc. to equation of continuity
$$a_Pv_P=a_Qv_Q$$
$$3.14\times10^{-4}\times v_P=3.14\times4\times10^{-4}v_Q$$
$$v_P=4v_Q$$

6. A horizontal pipe of diameter 20 cm has a constriction of diameter 4 cm. The velocity of water in the pipe is 2m/s and pressure is 10^7 N/m^2. Calculate the velocity and pressure at the constriction? 2

Ans. $r_1 = 20cm = 20\times10^{-2}m$

$r_1 = \frac{D_1}{2} = \frac{20\times10^{-2}}{2} = 10^{-1}m$

$a_1 = \pi r_1^2 = 3.14\times\left(10^{-1}\right)^2 m^2 = 3.14\times10^{-2}m^2$

$D_2 = 4cm = 4\times10^{-2}m$

$r_2 = \frac{D_2}{2} = \frac{4\times10^{-2}}{2} = 2\times10^{-2}m$

$a_2 = \pi r_2^2 = 3.14\times\left(2\times10^{-2}\right)^2 m^2 = 3.14\times4\times10^{-4}m^2$

Acc. to equation of continuity

$a_1v_1 = a_2v_2$

$3.14\times10^{-2}\times2 = 3.14\times4\times10^{-4}v_2$

$v_2 = 50ms^{-1}$

Acc. to Bernoullie's theorem (for a horizontal pipe)

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$P_1 - P_2 = \frac{1}{2}\rho\left[v_2^2 - v_1^2\right]$

$10^7 - P_2 = \frac{1}{2}\times10^3\left[50^2 - 2^2\right]$

$= \frac{1}{2}\times10^3\left[2500-4\right]$

$= 12.48\times10^5$

$P_2 = 10^7 - 12.48\times10^5 = 8.75\times10^6 Nm^{-2}$

7. The reading of a pressure metre attached to a closed is 2.5×10^5 N/m^2. On opening the valve of pipe, the reading of the pressure metre reduces to 2.0×10^5 N/m^2. Calculate the speed of water flowing through the pipe? 2

Ans. *Given:* $P_1 = 2.5\times10^5 Nm^{-2}$ $P_2 = 2\times10^5 Nm^{-2}$

$\rho = 1000Kgm^{-3}$ $v_1 = 0ms^{-1}$

To find: $v_2 = ?$

Sol^n: Acc. to Bernoullie's theorem (for a horizontal pipe)

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$P_1 - P_2 = \frac{1}{2}\rho\left[v_2^2 - v_1^2\right]$

$2.5\times10^5 - 2\times10^5 = \frac{1}{2}\times10^3\left[v_2^2 - 0^2\right]$

$5\times10^4 = \frac{1}{2}\times10^3\times v_2^2$

$v_2^2 = 100$

$v_2 = 10ms^{-1}$

8. A large bottle is fitted with a siphon made of capillary glass tubing. Compare the co-efficient of viscosity of water and petrol if the time taken to empty the bottle in the two cases is in the ratio 2:5. Given specific gravity of petrol = 0.8 2

Ans.
$$\frac{V}{t_1} = \frac{\pi}{8}\frac{p_1 r^4}{\eta_1 l} \qquad \rightarrow (1)$$

$$\frac{V}{t_2} = \frac{\pi}{8}\frac{p_2 r^4}{\eta_2 l} \qquad \rightarrow (2)$$

$$(1) \div (2)$$

$$\frac{\frac{V}{t_1}}{\frac{V}{t_2}} = \frac{\frac{\pi}{8}\frac{p_1 r^4}{\eta_1 l}}{\frac{\pi}{8}\frac{p_2 r^4}{\eta_2 l}}$$

$$\frac{t_2}{t_1} = \frac{p_1}{p_2} \times \frac{\eta_2}{\eta_1}$$

$$= \frac{\rho_1 gh}{\rho_2 gh} \times \frac{\eta_2}{\eta_1}$$

$$= \frac{\rho_1}{\rho_2} \times \frac{\eta_2}{\eta_1}$$

$$\frac{5}{2} = \frac{10^3}{0.8 \times 10^3} \times \frac{\eta_2}{\eta_1}$$

$$\frac{\eta_1}{\eta_2} = \frac{1}{2}$$

9. Under a pressure head, the rate of flow of liquid through a pipe is Q. If the length of pipe is doubled and diameter of pipe is halved, what is the new rate of flow? 2

Ans.
$$Q = \frac{\pi}{8}\frac{pr^4}{\eta l}$$

$$d' = \frac{d}{2}s \quad so \quad r' = \frac{r}{2}$$

$$l' = 2l$$

$$Q' = \frac{\pi}{8}\frac{p(r')^4}{\eta l'}$$

$$= \frac{\pi}{8}\frac{p\left(\frac{r}{2}\right)^4}{\eta(2l)}$$

$$= \frac{1}{32}\left(\frac{\pi}{8}\frac{pr^4}{\eta l}\right)$$

$$= \frac{1}{32}Q$$

10. In a horizontal pipeline of uniform area of cross – section, the pressure falls by 5 N/m^2 between two points separated by a distance of 1 Km. What is the change in kinetic energy per Kg of oil flowing at these points? Given Density of oil = 800 Kg/m^3? 3

Ans. $Given: \quad P_1 - P_2 = 5 Nm^{-2}$

$\rho = 800 Kgm^{-3}$

$v_1 = 0 ms^{-1}$

$To\ find:\ Change\ in\ K.E\ per\ kg = ?$

$Sol^n:$

$Acc. to\ Bernoullie's\ theorem\ (for\ a\ horizontal\ pipe)$

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2}\rho\left[v_2^2 - v_1^2\right]$$

$$5 = \frac{1}{2} \times 800\left[v_2^2 - v_1^2\right]$$

$$\frac{1}{2}\left[v_2^2 - v_1^2\right] = \frac{5}{800}$$

$$\therefore Change\ in\ K.E\ per\ kg = \frac{5}{800} Jkg^{-1} = 6.25 \times 10^{-3} Jkg^{-1}$$

11. (a) Water flows steadily along a horizontal pipe at a rate of $8 \times 10^{-3} m^3/s$. If the area of cross – section of the pipe is $40 \times 10^{-4}\ m^2$, Calculate the flow velocity of water. 3

(b) Find the total pressure in the pipe if the static pressure in the horizontal pipe is 3×10^4 Pa. Density of water is 1000 Kg/m^3.

(c) What is the net flow velocity if the total pressure is 3.6×10^4 Pa?

Ans.

$$(a) Velocity\ of\ water = \frac{Rate\ of\ flow}{area\ of\ cross-\sec.} = \frac{8 \times 10^{-3}}{40 \times 10^{-4}} = 2 ms^{-1}$$

$$(b) Total\ pressure = Static\ pressure + \frac{1}{2}\rho v^2$$

$$= 3 \times 10^4 + \frac{1}{2} \times 1000 \times 4$$

$$= 3.2 \times 10^4 Pa$$

$$(c) Total\ pressure = Static\ pressure + \frac{1}{2}\rho v^2$$

$$3.6 \times 10^4 = 3 \times 10^4 + \frac{1}{2} \times 1000 \times v^2$$

$$500 v^2 = 0.6 \times 10^4 Pa$$

$$v^2 = 12$$

$$v = \sqrt{12} = 3.5 ms^{-1}$$