

Hydrostatics

Pressure of liquid

The normal force exerted by a liquid at rest per unit area of the surface in contact with it is called hydrostatic pressure or pressure of liquid.

$$P = \frac{F}{A}$$

- Unit - Pa (Nm^{-2})
- Scalar quantity

Applications of pressure

① Bags and suitcases are provided with broad handles.

Reason Broad handle - more area - less pressure.

② Pins and nails have pointed ends.

Reason Less area \rightarrow high pressure - easily inserted.

③ Railway tracks are laid on large sized wooden sleepers.

Density and Relative Density

Density - It is defined as the mass per unit volume of the substance.

$$\rho = \frac{M}{V}$$

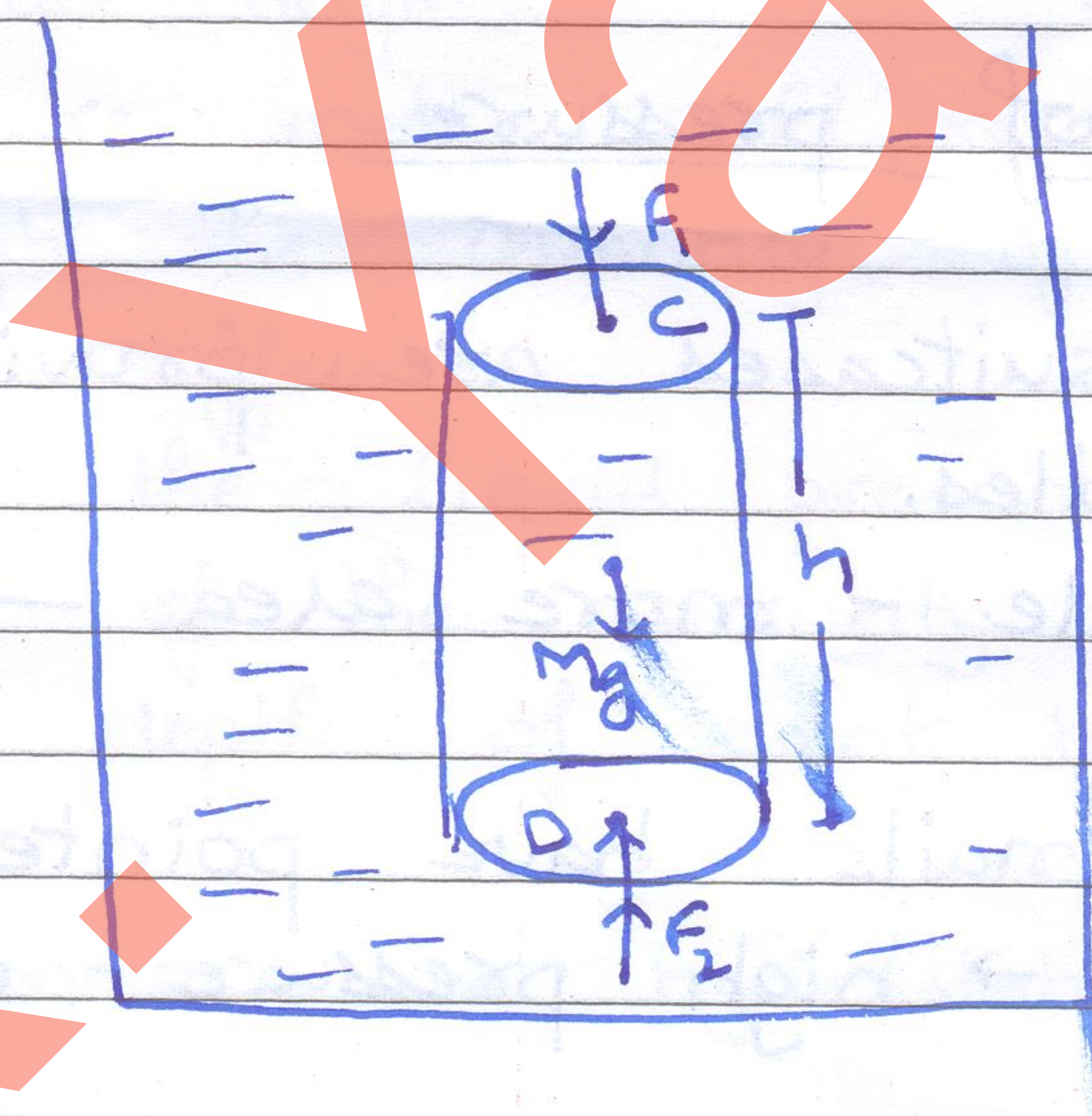
Unit $\rightarrow \text{Kgm}^{-3}$

Relative Density - It is defined as the ratio of density of a substance to the density of water at 4°C

$$\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water at } 4^{\circ}\text{C}}$$

- It has no units.
- scalar quantity

Variation of pressure with depth



Consider a liquid of density ρ contained in a vessel in equilibrium of rest.

Imagine a cylinder of liquid with axis CD area A & height h .

Mass of liquid in imaginary cylinder is

$$\begin{aligned} M &= \rho \times V \\ &= \rho \times Ah \\ &= Ah\rho \end{aligned}$$

Let P_1 - pressure of liquid at C,
 P_2 - " " " " " D

Force acting at C, $F_1 = P_1 A$ (downwards)
 D, $F_2 = P_2 A$ (upwards)

Weight of liquid cylinder = $Mg = Ah\rho g$

As the liquid is in equilibrium of rest

$$F_1 + Mg = F_2$$

$$P_1 A + Ah\rho g = P_2 A$$

$$\boxed{P_2 - P_1 = \rho g h}$$

If C & D ^{are} at same level, $h = 0$

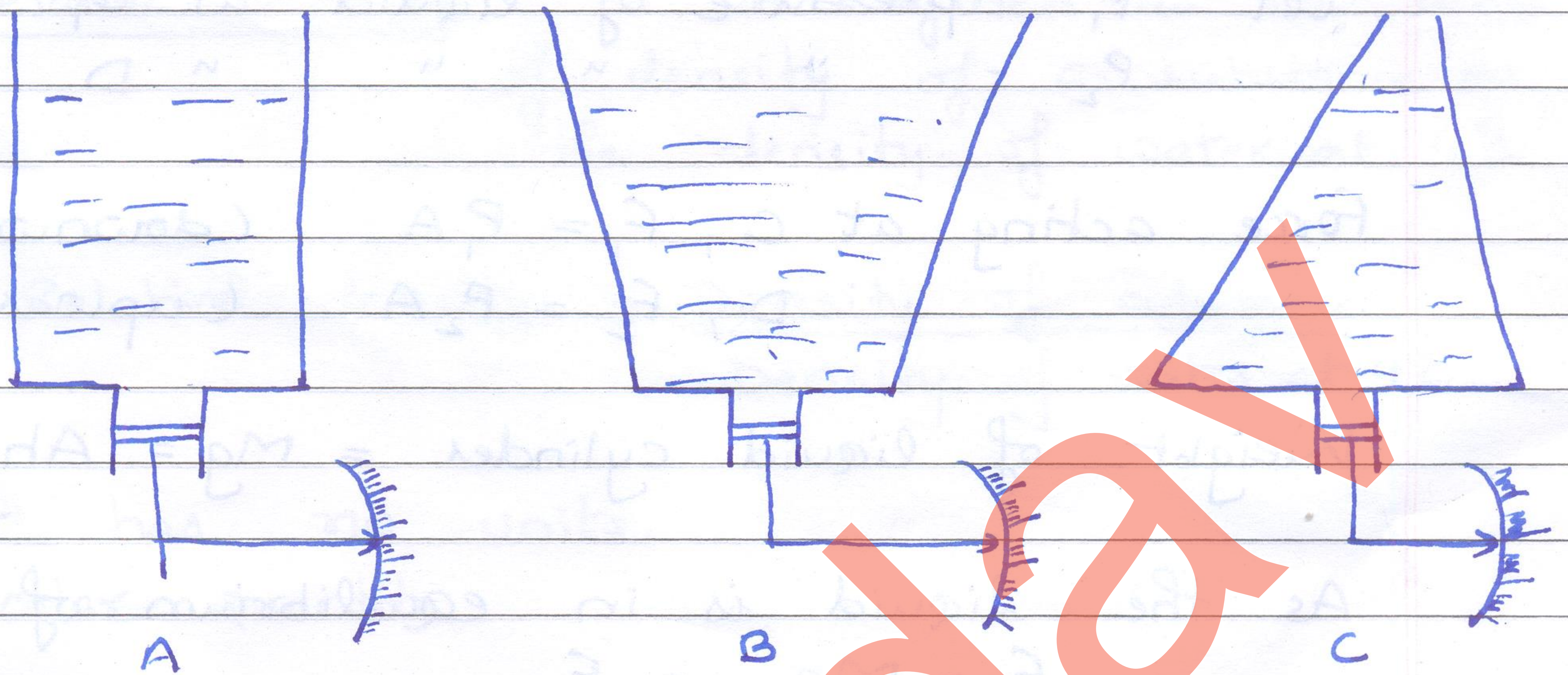
$$P_2 - P_1 = 0$$

$$\boxed{P_1 = P_2}$$

i.e. Pressure is same at all points inside the liquid lying at the same depth in a horizontal plane.

Hydrostatic paradox

Pressure exerted by a liquid depends only on the height of liquid column and is independent of the shape of the containing vessel.



- Fill the 3 vessels with the same liquid upto same vertical height.
- Each pressure meter will record the same pressure even though the quantity of liquid in different vessels is different.
- It means the liquid pressure at a point is independent of the quantity of liquid but depends upon the depth of point below the liquid surface.
- This is known as hydrostatic paradox.

Pascal's law

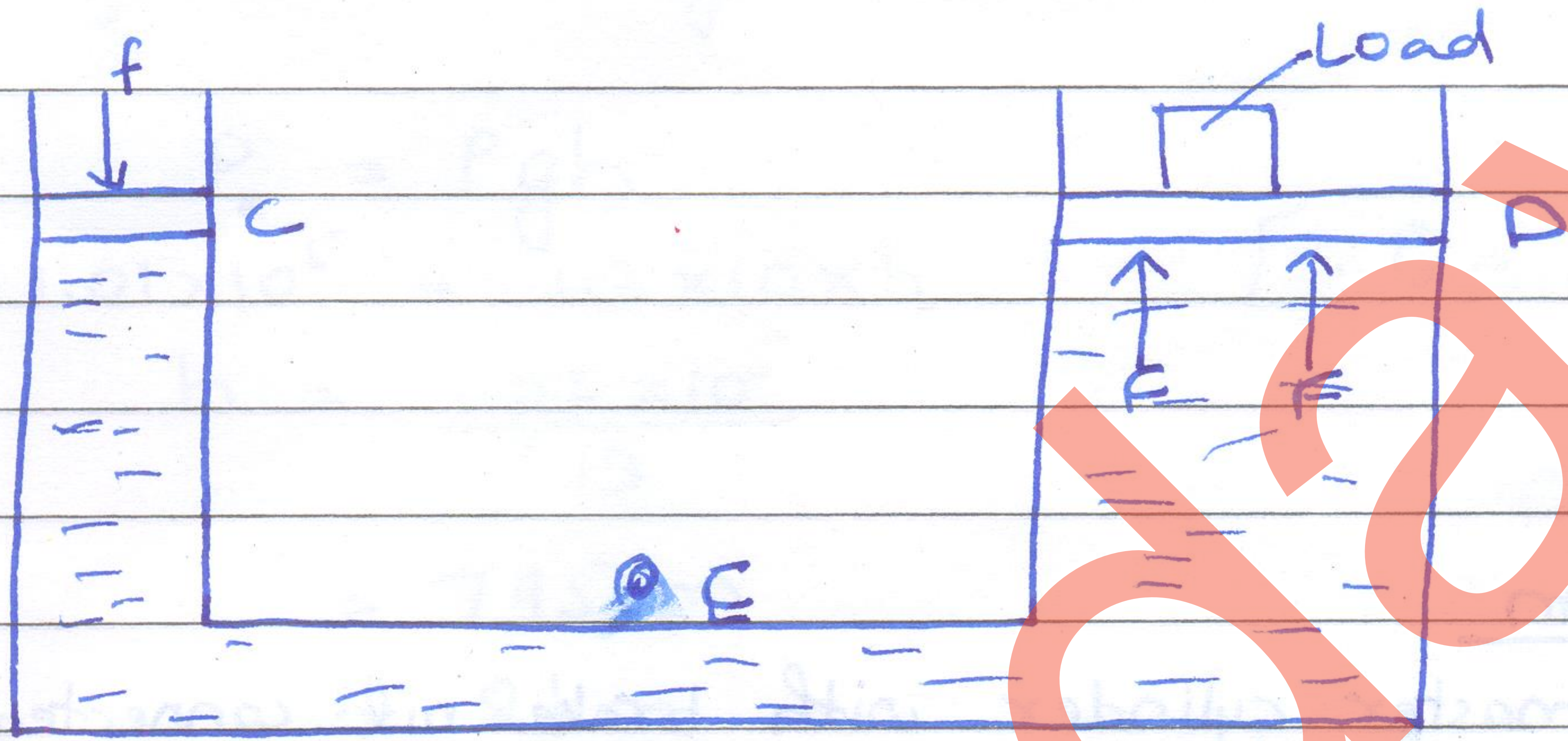
It states that if gravity effect is neglected, the pressure at every point of liquid in equilibrium of rest is the same.

OR

It states that the increase in pressure at one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of the liquid and also to the walls of the container, provided the effect of gravity is neglected.

Hydraulic Lift

- It is used to lift heavy loads.
- It's working is based on Pascal's law.



Let a - area of cross-section of piston C
 A - " " " " " " " " D
 f - force applied on C
 F - " " " " " " D

When force 'f' is applied on C, the pressure exerted on the liquid is

$$P = \frac{f}{a}$$

Acc. to Pascal's law, this pressure is transmitted equally to piston of cylinder D.

∴ Upward force acting on D is

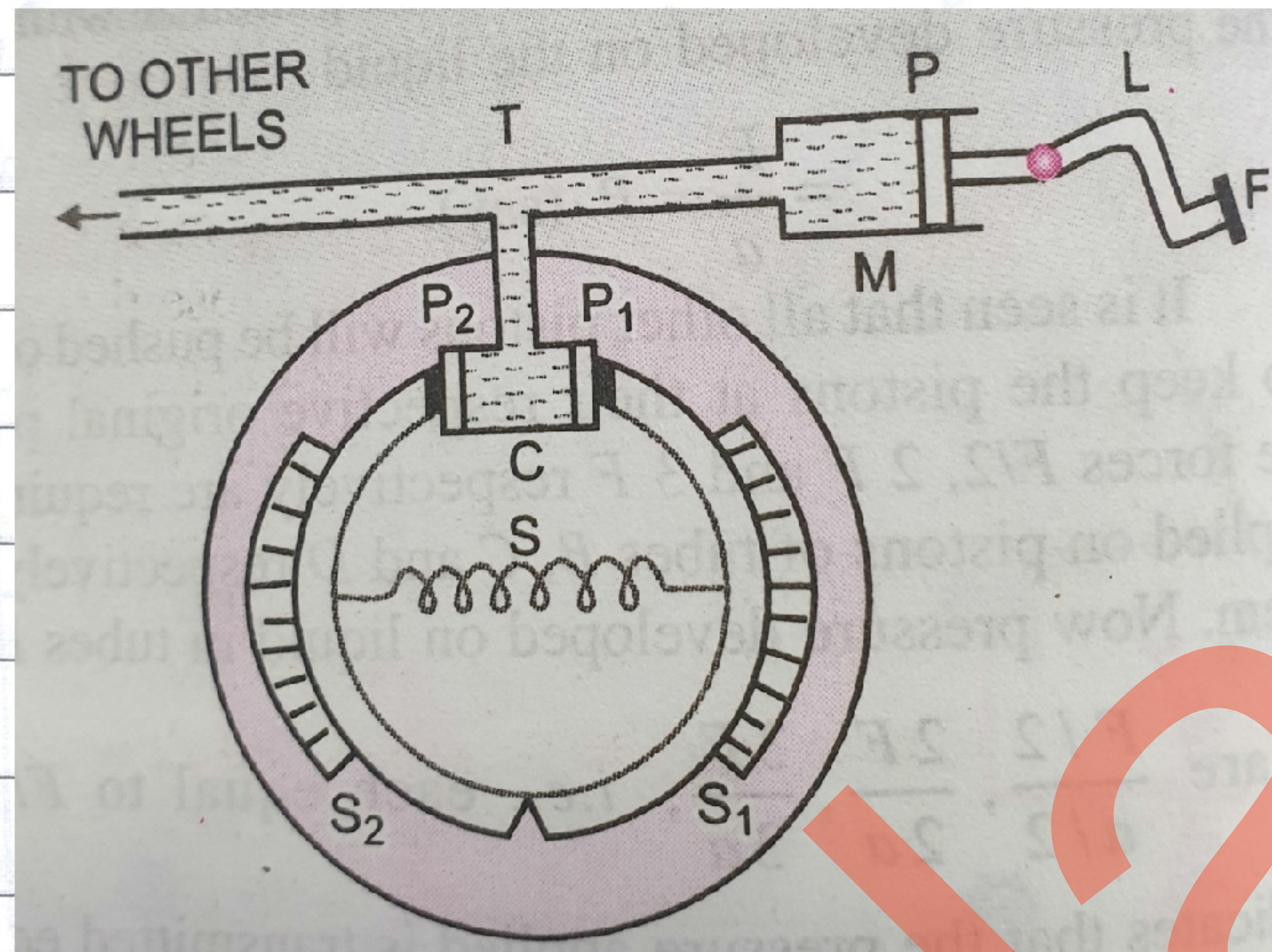
$$F = PA = \frac{f}{a} A = f \frac{A}{a}$$

As $A \gg a$

$$\text{so } \boxed{F \gg f}$$

As a result, a heavy load placed on the larger piston is easily lifted upwards.

Hydraulic Brakes



Construction

M - master cylinder with brake oil connected with P.

P - air tight frictionless piston P.

P_1, P_2 - pistons of wheel cylinder

S_1, S_2 - brake shoes

C - wheel cylinder

Working

- (i) When the brake pedal is pressed, piston P of master cylinder is pushed inwards.
- (ii) The increased pressure on liquid at P, is equally transmitted to P_1 & P_2 & so P_1 & P_2 move outwards.
- (iii) They force the brake shoes to move away from each other which in turn press against the inner rim of the wheel & hence retard the motion of the wheel.

Atmospheric pressure

The atmospheric pressure at any point is equal to the weight of a vertical column of air of unit area extending from that point to the top of the earth's atmosphere.

Height of atmosphere

Consider the atmospheric air to be of uniform density $\rho = 1.3 \text{ kgm}^{-3}$.

$$P_a = \rho gh$$

$$1.01 \times 10^5 = 1.3 \times 10 \times h \quad [\because P_a = 1.01 \times 10^5 \text{ Pa}]$$

$$h = \frac{1.01 \times 10^5}{13}$$

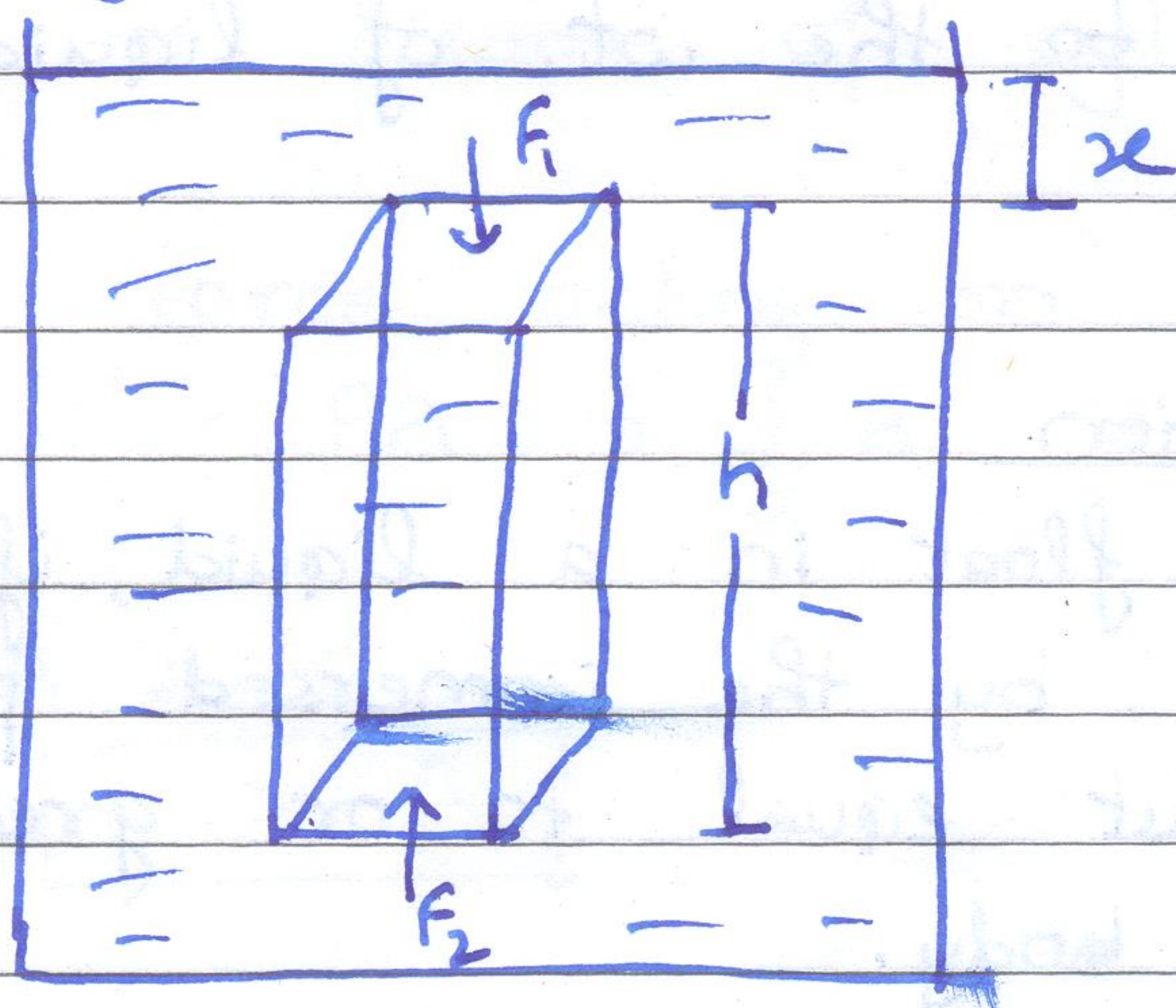
$$= 7951 \text{ m}$$

$$\approx 8 \text{ km}$$

- * This is not the true height of atmosphere as the density is not uniform.
- * 1 torr = 1 mm of Hg column
- 1 bar = 10^5 Pa

Archimede's Principle

It states that when a body is wholly or partly immersed in a liquid at rest, it loses some of its weight equal to the weight of the liquid displaced by the immersed part of the body.



Volume of liquid displaced, $V = axh$

Mass of liquid displaced, $m = V\rho = ah\rho$

Pressure on the top face, $P_1 = x\rho g$
 " " " bottom " , $P_2 = (x+h)\rho g$

Vertical downward thrust on top face, $F_1 = x\rho g a$
 " upward " " bottom " , $F_2 = (x+h)\rho g a$

Net upward thrust acting on the body

$$\begin{aligned} F &= F_2 - F_1 \\ &= (x+h)\rho g a - x\rho g a \\ &= h\rho g a \\ &= mg \\ &= \text{weight of liquid displaced.} \end{aligned}$$

True weight of body, $W = Mg$
 Upward thrust , $F = mg$

\therefore Observed weight of body in liquid = $W - F$
 $= Mg - mg$

i.e. Observed wt. is less than true wt. by an amount equal to the wt. of liquid displaced by body.

Law of floatation

A body will float in a liquid, if weight of the liquid displaced by the immersed part of the body is atleast equal to or greater than the weight of the body.