Elasticity

Elasticity

The property of a body by virtue of which it tends to regain its original configuration, when deforming forces are removed is called elasticity.

Elastic body - A body which regains its original configuration on removal of deforming forces is called an elastic body.

Plastic body - Does not regain its original configuration on removal of deforming forces.

* No body is perfectly elastic or plastic.
* Perfectly elastic body (nearest) - Quartz
  " plastic " " - putty, paraffin wax

Stress

It is the internal restoring force acting per unit area of a deformed body.

$$\text{Stress} = \frac{\text{restoring force}}{\text{area}}$$

If there is no permanent change in the configuration of the body, the restoring force is equal & opposite to the external deforming force, so

$$\text{stress} = \frac{\text{external deforming force}}{\text{area}}$$

S.I. unit - Nm^{-2} or Pa

Dimensions - $[ML^{-1}T^{-2}]$

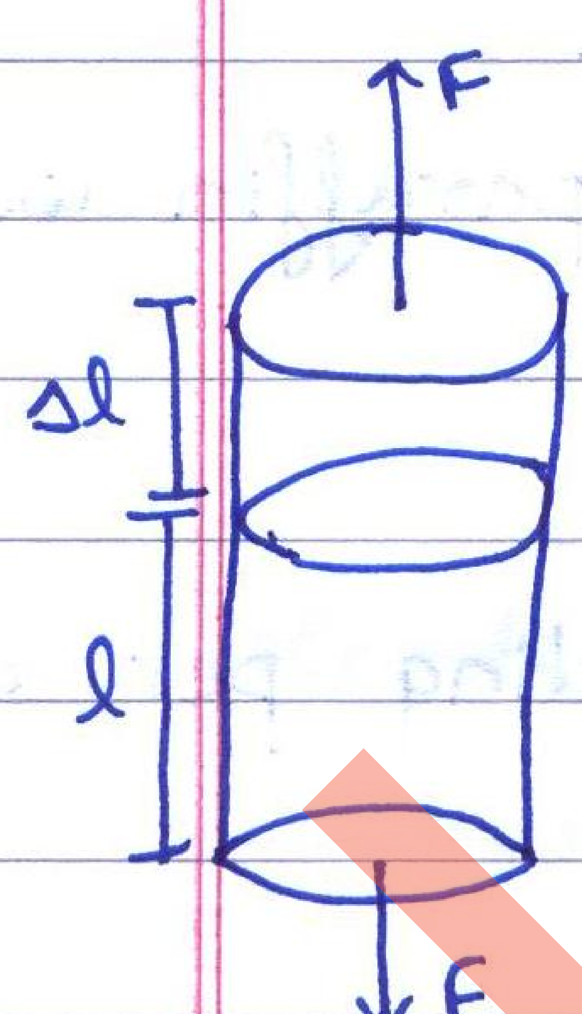
Types of stress

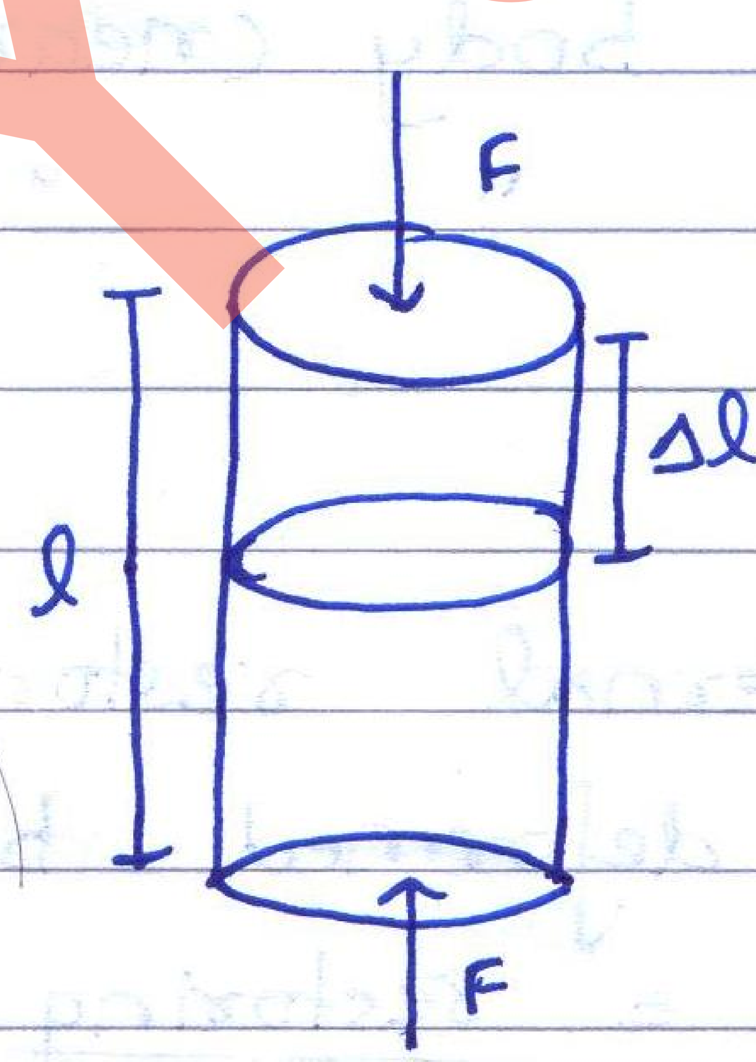
(a) Normal stress

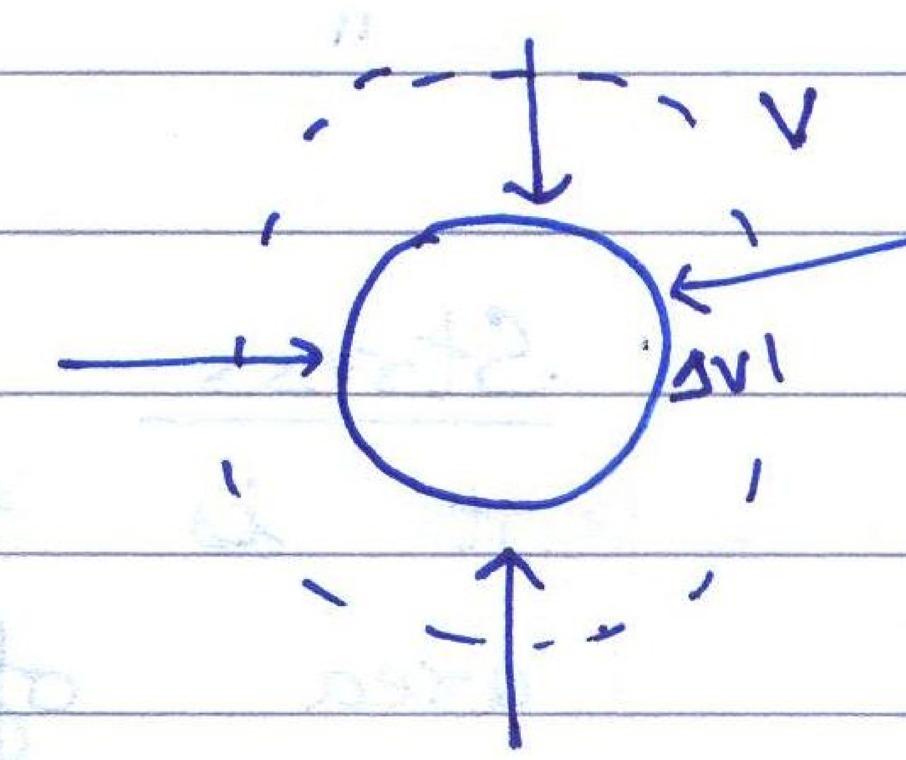
When a deforming force acts normally over an area of a body, then the restoring force set up per unit area is called normal stress

Normal stress

- **Tensile** — Increase in length of body in the direction of applied force
- **Compressive** — Decrease in length of body due to applied force
- **Hydraulic** — Change in volume of solid body without any change in shape on applying force.







(b) Tangential stress

When a deforming force, acting tangentially to the surface of a body produces a change in the shape of the body without any change in the volume, then the stress set up in the body is called tangential stress.

Strain

The ratio of change in dimension of the body to its original dimension is called strain.

$$\text{strain} = \frac{\text{change in configuration}}{\text{original configuration}}$$

(i) **Longitudinal strain**

It is defined as the increase in length per unit original length, when deformed by external force.

$$\text{Longitudinal strain} = \frac{\text{change in length } (\Delta l)}{\text{original length } (l)}$$

(ii) Volumetric strain $= \dfrac{\text{change in volume } (\Delta V)}{\text{original volume } (V)}$

(iii) **Shear strain (θ)**

It is defined as the angle θ (in radian), through which a line originally perpendicular to the fixed face gets turned on applying tangential deforming force.

or

It is the ratio of displacement of a surface under a tangential force to the $\perp^r$ distance of the displaced surface from the fixed surface.

$$\theta = \frac{\Delta L}{L}$$

Unit - no unit

Hooke's Law

Within elastic limit, the extension produced in a wire is directly proportional to the load attached to it.

i.e. within elastic limit

extension $\propto$ load

or stress $\propto$ strain

$$\boxed{\text{stress} = E \times \text{strain}}$$

$$\boxed{E = \frac{\text{stress}}{\text{strain}}}$$

modulus of elasticity

Types of modulus of elasticity

(a) **Young's modulus of elasticity (Y)**

It is defined as the ratio of normal stress to longitudinal strain.

$$Y = \frac{\text{normal stress}}{\text{longitudinal strain}}$$

$$= \frac{\frac{F}{a}}{\frac{\Delta l}{l}}$$

$$\boxed{Y = \frac{Fl}{a\Delta l} = \frac{Fl}{\pi r^2 \Delta l}}$$

* Higher is the value of Y of a material, larger is its elasticity, that's why steel is most elastic than iron.

(b) **Bulk modulus (K)**

$$K = \frac{\text{normal stress}}{\text{volumetric strain}}$$

$$= \frac{\frac{F}{a}}{-\frac{\Delta V}{V}}$$ [-ve sign as volume is decreasing]

$$= \frac{-FV}{a\,\Delta V}$$

$$\boxed{K = \frac{-pV}{\Delta V}}$$ $[\because p = \frac{F}{a}]$

S.I. unit - Nm^{-2}

Compressibility - reciprocal of bulk modulus of a material

$$\text{compressibility} = \frac{1}{K}$$

S.I. unit - $N^{-1}m^{2}$

* $K_{solid} > K_{liq} > K_{gas}$

(c) **Modulus of rigidity (G)**

$$G = \frac{\text{tangential stress}}{\text{tangential strain}}$$

$$= \frac{F/a}{\theta}$$

$$= \frac{FL}{a\,\Delta L}$$

* G is only for solids & $G < Y$.

Work done in stretching a wire

Consider a wire of length 'L' & area 'a' suspended from a rigid support.
Let a force 'F' is applied at its free end & its length increases by l.

$$Y = \frac{FL}{a\,l}$$

$$F = \frac{Y a\, l}{L}$$

Let the length of the wire is increased by a very small amount 'dl' under the action of constant force 'F'.
The small work done is

$$dW = F\,dl$$

$$= \frac{Y a\, l}{L}\,dl$$

$\therefore$ Total work done in stretching the wire by length l is

$$W = \int_0^l \frac{Y a\, l}{L}\,dl$$

$$= \frac{Ya}{L}\int_0^l l\,dl$$

$$= \frac{Ya}{L}\left[\frac{l^2}{2}\right]_0^l$$

$$= \frac{1}{2}\,\frac{Y a\, l^2}{L}$$

$$= \frac{1}{2} \frac{Yal}{L} \times l$$

$$= \frac{1}{2} F \times l$$

$$\boxed{\text{Elastic potential energy} = \frac{1}{2} \times \text{stretching force} \times \text{increase in length}}$$

Work done per unit volume of the wire

$$= \frac{W}{a \times L}$$

$$= \frac{1}{2} \frac{F \times l}{a \times L}$$

$$= \frac{1}{2} \times \text{stress} \times \text{longitudinal strain}$$

Applications of elasticity

1. Any metallic part of a machinery is never subjected to a stress beyond the elastic limit of the material as it will get permanently deformed & will not work properly.

2. Max. height of a mountain on earth

Let h - max. possible height of mountain
ρ - density of material forming the mountain

Pressure at the base of mountain

$$P = \rho g h$$

For the mountain to exist

$$P < \text{elastic limit of earth}$$

$$\rho g h < 3 \times 10^8$$

$$h < \frac{3 \times 10^8}{3 \times 10^3 \times 10}$$

$$\boxed{h < 10^4 \text{ m}}$$

③ The thickness of the metallic rope used in the crane in order to lift a given load is decided from the knowledge of elastic limit of the material of the rope & the factor of safety.

Consider a crane having steel ropes to lift a heavy load upto 10^4 kg.

Elastic limit of steel = ultimate stress on the rope = $3 \times 10^8 \text{ Nm}^{-2}$

$$\text{Ultimate stress} = \frac{F}{a}$$

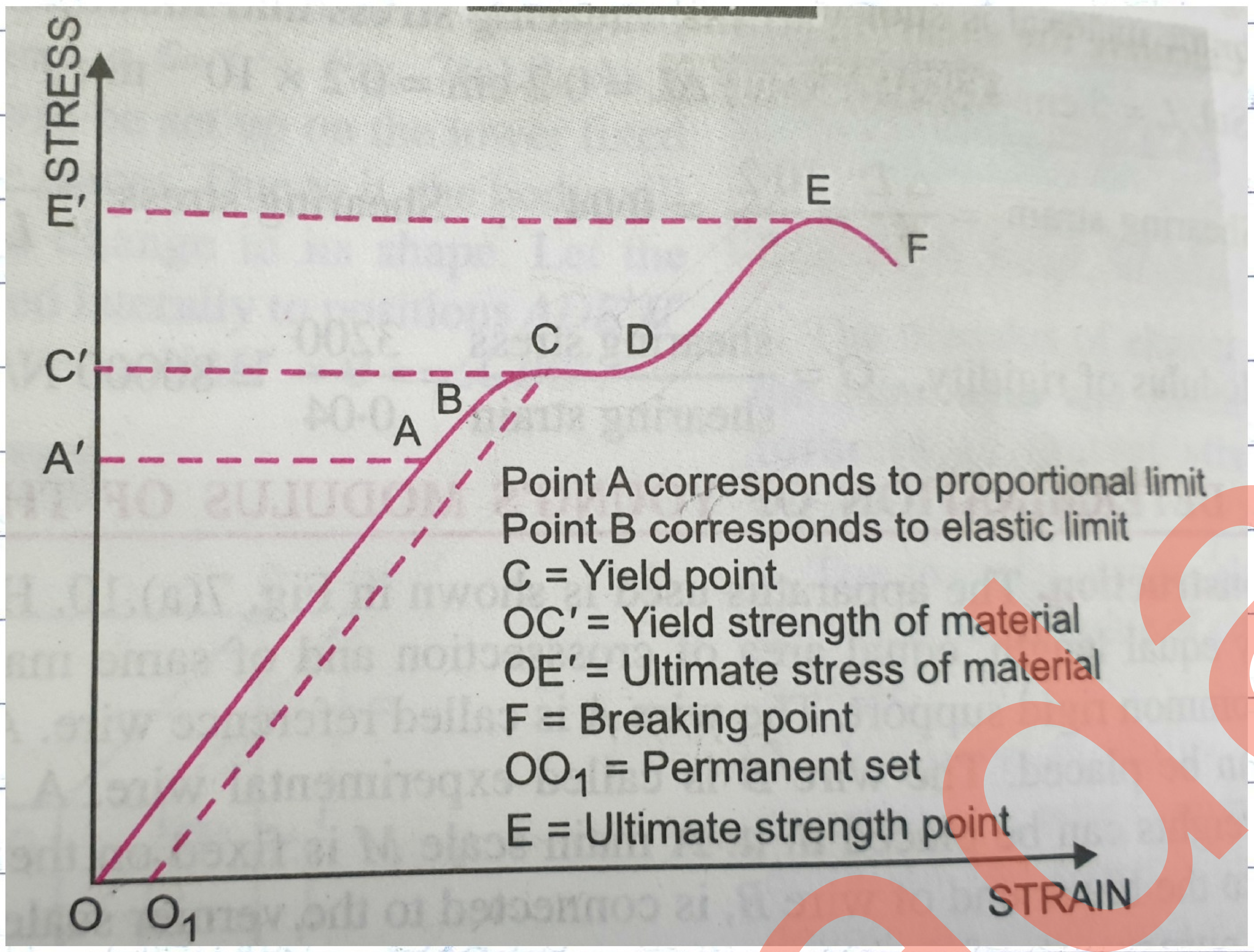
$$= \frac{mg}{\pi r^2}$$

$$3 \times 10^8 = \frac{10^4 \times 10}{3.14 \times r^2}$$

$$\boxed{r = 0.0103 \text{ m} = 1.03 \text{ cm}}$$

* In order to impart flexibility to the rope, it is made of large no. of thin wires which are combined together to have a radius more than 1.03 cm.

Stress- strain graph



(i) portion OA
- graph straight line
- stress $\propto$ strain, Hooke's law obeyed
- A → pt. of proportional limit

(ii) portion AB
- not a straight line i.e. Hooke's law not obeyed.
- If wire is unloaded at B, then graph follows the reverse path BAO, then B is called elastic limit.
- OB → elastic region.

(iii) portion BC
- strain increases much more rapidly with stress.
- slope becomes small.
- If wire unloaded at C, graph traces curve CO_1 i.e. stress = 0, strain = CO_1
  It means permanent deformation (OO_1)
- extension in wire partially elastic & partly plastic

(iv) portion CD

- The wire begins showing increase in strain without any increase in stress.
- point C → yield point (point at which wire yields to applied stress and begins to flow down)
- behaviour of wire → perfectly plastic

→ stress corresponding to C is yield strength of wire

(v) portion DEF

- Beyond D, even if the wire is unloaded a little the thinning of wire starts & necks & waists are developed at few weaker portions of the wire & finally the wire breaks (pt. F)
- breaking stress - stress corresponding to E
  " point - pt. at which wire breaks.

Classification of material

① Ductile material

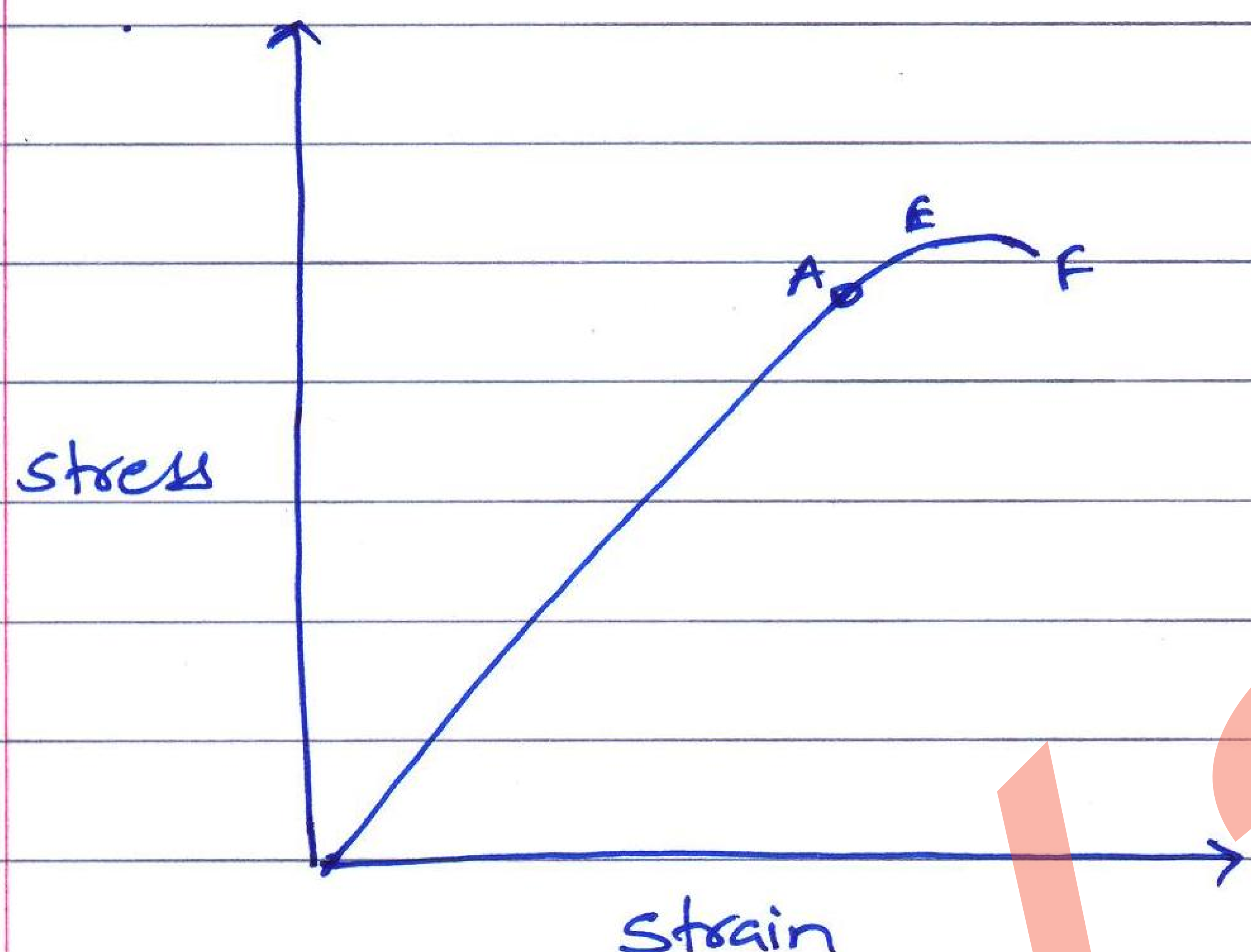
- material which show large plastic range beyond elastic limit.
- breaking pt. is widely separated from point of elastic limit.
- used for making springs & sheets.
- eg → Cu, Ag, Fe

② Brittle material

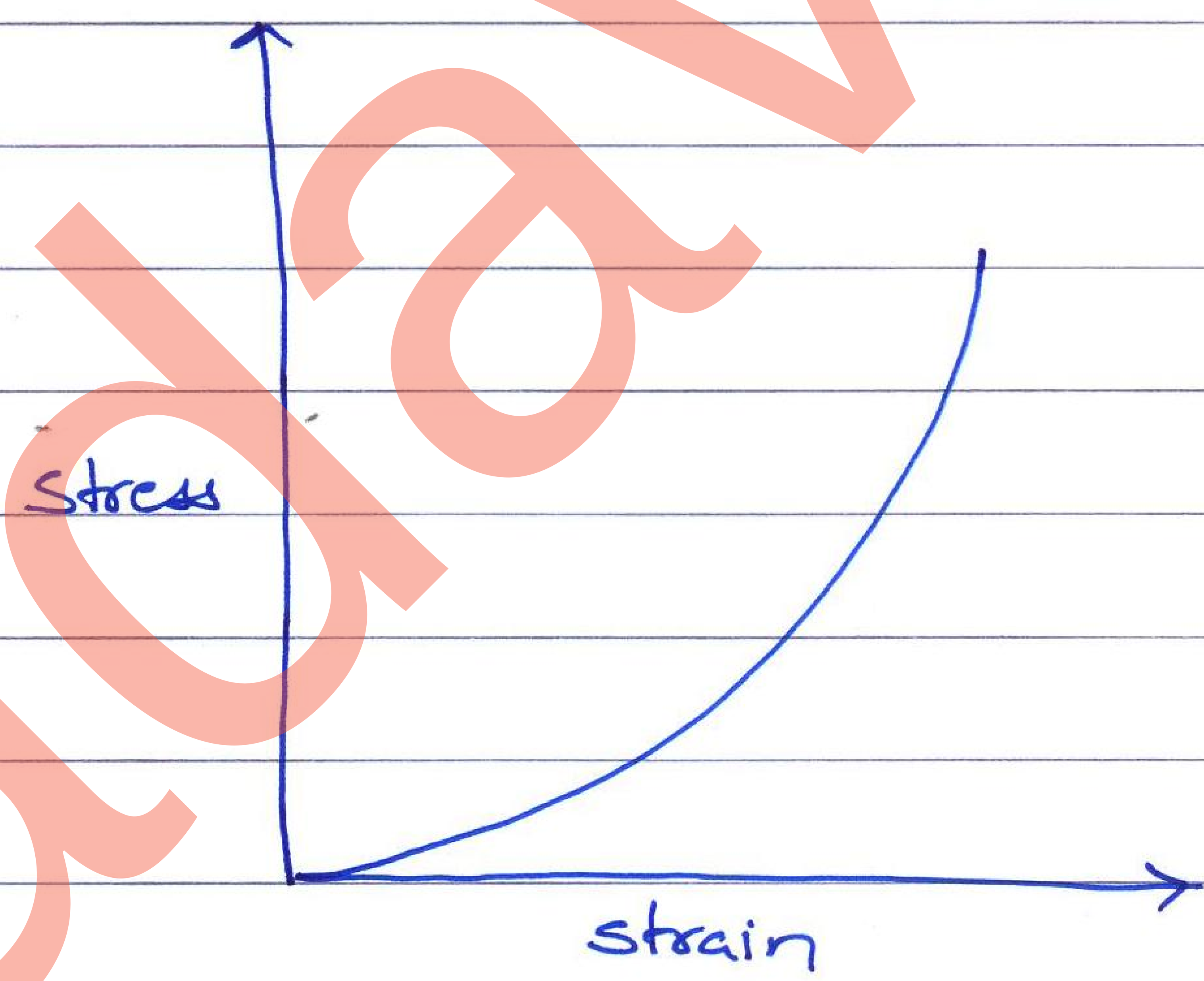
- material which show very small plastic range beyond elastic limit.
- breaking pt. lies close to elastic limit.
- eg → glass, cast iron

③ Elastomers

- material for which stress-strain variation is not a straight line within elastic limit.
- very large elastic region
- no plastic range
- eg → rubber, blood vessels



(Brittle)



(Elastomer)