Derivation of formulae (By Dimensional Analysis)

STEPS (METHOD)

1. Write all the factors (given in the question) affecting the given physical quantity with the proportionality sign and put the power a, b and so on the factors.
   "The centripetal force F acting on a particle moving uniformly in a circle may depend upon mass (m), velocity (v) and radius (r) of the circle."
   $$F \propto m^a v^b r^c$$
2. Remove the sign of proportionality and put the constant 'k'
   $$F = k\, m^a v^b r^c$$
3. Put the dimensions of all the physical quantities
   $$[MLT^{-2}] = k\, [M]^a [LT^{-1}]^b [L]^c$$
   $$[MLT^{-2}] = k\, [M]^a [L]^{b+c} [T]^{-b}$$
4. Equate the powers of M,L and T on both sides of equation and find the value of a,b,c etc.
   $a = 1$ (for M)
   $b+c = 1$ (for L)
   $-b = -2$ (for T)
   So, $a = 1$, $b = 2$, $c = -1$
5. Put the value of a,b,c in 2nd equation
   $$F = k\, m^1 v^2 r^{-1} \quad \text{or } F = mv^2/r$$

Problems for practice

1. The period of oscillation of a simple pendulum depends upon its length (l), mass of the bob (m) and acceleration due to gravity (g). Derive the expression for its time period using method of dimensions. $[t = k\sqrt{\frac{l}{g}}]$
2. If force, velocity and time are taken as the fundamental quantities, what would be the dimensions of work? [Fvt]
3. The wavelength (λ) of matter waves may depend upon Planck's constant (h), mass (m) and velocity (v) of the particle. Use the method of dimensions to find the formula. [$\lambda = h/mv$]
4. Using the method of dimensions, derive an expression for rate of flow (v) of a liquid through a pipe of radius (r) under a pressure gradient (P/l). Given that v also depends on coefficient of viscosity (η) of the liquid. [$v = k\, Pr^4/l\eta$]