SOLUTIONS

SAMPLE QUESTION PAPER - 6

Self Assessment

Time : 3 Hours **Maximum Marks : 70**

1. Yes, it will be in equilibrium if the vector sum of the forces acting upon the body is zero. **1**
2. The stress at which the specimen breaks or ruptures ultimately is called ultimate or tensile strength. **1**
3. When no exchange of heat energy is possible between the system and surrounding, the process is adiabatic. Such processes are carried in (i) non-conducting cylinders, and (ii) at a fast pace. **1**
4. Elastomers are those substances which can be stretched to cause large strain. Substances like tissue of aorta, rubber etc. are elastomers. **1**
5. The necessary and sufficient condition for motion to be simple harmonic is that the restoring force must be linear, *i.e.*, F = $-kx$ or torque $\tau = -c\theta$. **1**
6. Joule is the S.I. unit of work.

   Using the relation,

$$\begin{aligned} \text{work} &= \text{force} \times \text{displacement} \\ &= \text{mass} \times \text{acceleration} \times \text{displacement} \\ &= \text{mass} \times \frac{\text{velocity}}{\text{time}} \times \text{displacement} \\ &= \text{mass} \times \frac{\text{displacement}}{\text{time} \times \text{time}} \times \text{displacement} \\ &= \text{mass} \times \text{displacement}^2 \times \text{time}^{-2} \end{aligned}$$ **1**

   Unit of work,
$$\begin{aligned} J &= kg \times m^2 \times s^{-2} \\ &= kgm^2\, s^{-2}. \end{aligned}$$ **1**

7. If a ball is thrown up, the direction of motion of the body is the same as the direction of its velocity whereas the acceleration due to gravity acts on it in the downward direction. **1**

   Thus, the direction in which an object moves is given by the direction of velocity and not by the direction of acceleration. **1**

8. Using,
$$P = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$
   and
$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$ **½**

   (a) $R = P + Q$ and $\alpha = 0^\circ$ **½**

   (b) $R = P - Q$ and $\alpha = 0^\circ$ or 180° depends on $P > Q$ or $Q > P$. **½**

S O L U T I O N S

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Unit of work, $J = kg \times m^2 \times s^{-2}$

$= kgm^2\, s^{-2}$. **1**

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8. Using, $$P = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

and $$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$ **½**

(a) $R = P + Q$ and $\alpha = 0^\circ$ **½**

(b) $R = P - Q$ and $\alpha = 0°$ or $180°$ depends on $P > Q$ or $Q > P$. **½**

(c) $R = \sqrt{P^2 + Q^2}$ and $\alpha = \tan^{-1}\dfrac{Q}{P}$ ½

Or

Component of force along horizontal,

$$F_x = F\cos 60°$$
$$= 72 \times \frac{1}{2} = 36 \text{ dyne} \qquad \mathbf{1}$$

Using $\qquad F_x = ma_x$

We get $\qquad a_x = \dfrac{F_x}{m} = \dfrac{36}{9}$

$$= 4 \text{ cms}^{-2}. \qquad \mathbf{1}$$

9. If there is only one propeller, the helicopter will start rotaing in a direction opposite to that of the rotation of the propeller so as to conserve angular momentum. **2**

10. Sound waves are mechanial waves whose velocity is given by :

$$v = \sqrt{\gamma\, RT/M} \qquad \mathbf{1}$$

Light waves are non-mechanical waves or electromagnetic waves for which $c = 1/\sqrt{\mu_0\varepsilon_0}$, where μ_0 is the absolute magnetic permeability of free space and ε_0 is the absolute electrical permittivity of the free space. Therefore, v depends upon T, but c does not. **1**

11. Here $Y = \dfrac{mgl^3}{4bd^3\delta}$, g is constant.

∴ Maximum relative error in Y is given by :

$$\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{3\Delta l}{l} + \frac{\Delta b}{b} + \frac{3\Delta d}{d} + \frac{\Delta\delta}{\delta} \qquad \mathbf{2}$$

Thus clearly m, l, b, d and δ introduce the maximum error in the measurement of Y. **1**

12. **(a)** A lives closer to the school than B because B has to cover higher distance [OP < OQ].

(b) A starts from the school earlier than B because $t = 0$ for A but B has some finite time.

(c) B walks faster than A because it covers more distance in less duration of time [slope of B is greater than that of A].

(d) A and B reach home at the same time.

(e) B overtakes A on the road once (at X, *i.e.*, the point of intersection). **3**

13. The position vector $(\vec{r})$ of the particle is

$$\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k} \text{ m} \qquad \text{....(i)}$$

(a) velocity $\vec{v}\,(t)$ of the particle is given by :

$$\vec{v}\,(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}\,(\vec{r})$$
$$= \frac{d}{dt}\,(3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k})$$
$$= 3\,\hat{i} - 4t\,\hat{j} + 0 \qquad \text{....(ii)} \quad \mathbf{1}$$

Also, acceleration $\vec{a}\,(t)$ of the particle is given by :

$$\vec{a}\,(t) = \frac{d\vec{v}(t)}{dt} = \frac{d}{dt}\,\vec{v}\,(t)$$
$$= \frac{d}{dt}\,(3\,\hat{i} - 4t\,\hat{j}) \qquad \text{[by using (ii)]}$$

$$= 0 - 4\hat{j}$$

$$\vec{a}\ (t) = -4\hat{j} \qquad ....(iii)\ ½$$

(b) At time t, the veocity of the particle is given by using equation (ii).

$$\vec{v}\ (t) = 3\hat{i} - 4t\hat{j}$$

∴ At $t = 2$ s, $v = 3.0\hat{i} - 4 \times 2\hat{j}$

$$= 3.0\hat{i} - 8.0\hat{j} \qquad ½$$

∴ Its magnitude is :

$$v = \sqrt{3^2 + (-8)^2} = \sqrt{9 + 64}$$

$$= \sqrt{73} = 8{\cdot}544\ \text{ms}^{-1} \qquad ½$$

and, direction of v is given by,

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-8}{3}\right)$$

$= 70°$ with x-axis. ½

14. Speed of train, $v = 54$ km/hr

$$= \frac{54 \times 1000}{60 \times 60} = 15\ \text{m/s} \qquad ½$$

Mass of train $= 10^6$ kg

Angle of banking $\theta = ?$ ½

The centripetal force is provided by the lateral thrust by the outer rail. According to Newton's third law of motion, the train exerts (*i.e.* causes) an equal and opposite thrust on the outer rail causing its wear and tear.

Centripetal force $= mv^2/r$

$$= \frac{10^6 \times 15^2}{30} = 75 \times 10^5\text{N} \qquad \mathbf{1}$$

Angle of banking, $\theta = \tan^{-1}(v^2/rg)$

$$= \tan^{-1}\frac{15^2}{30 \times 9.8}$$

$$\theta = 37^{\circ} \qquad \mathbf{1}$$

15. The ratio of relative velocity of separation after collision to the relative velocity of approach before collision is called coefficient of restitution. ½

Coefficient of restitution, $e = \dfrac{v_2 - v_1}{u_1 - u_2}$ ½

where u_1 and u_2 are initial velocities of the two colliding bodies and v_1, v_2 are their final velocities after collision. ½

(i) For elastic collision, velocity of separation is equal to the velocity of approach.

∴ $e = 1$ ½

(ii) For inelastic collision, velocity of separation is not zero but always less than the velocity of approach.

∴ $0 < e < 1$ ½

(iii) For perfectly inelastic collision, the colliding bodies do not separate out but move with same velocity.

$e = 0$ ½

16. Using Newton's second law of motion for a system of N particles, total force,

$$\vec{F} = \sum_{i=1}^{i=N} \frac{d}{dt}(m_i \vec{v}_i)$$

$$= \frac{d}{dt}(m_1 \vec{v}_1 + m_2 \vec{v}_2 + ... + m_n \vec{v}_n)$$ **1**

Internal forces acting on the particles cancel out in pairs. Taking external force also to be zero.

i.e., $\vec{F} = 0$ **1**

We get $\frac{d}{dt}(m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_N\vec{v}_N) = 0$

or $(m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_n\vec{v}_n) =$ constant.

The above expression is called principle of conservation of linear momentum. **1**

17. **(a)** No, from the formula $v_e \sqrt{\frac{2GM}{R}} =$, it is clear that escape velocity does not depend on the mass of the body. **1**

(b) The escape velocity depends upon the value of gravitational potential at the point from where the body is projected. The gravitational potential energy of body $E = -\sqrt{\frac{2Gm}{R}}$ is slightly different at different points.

($\therefore$ the earth is not a perfect sphere and hence R is different at different points). Because of this escape velocity depend slightly on the latitude of the place from where the body is projected. **1**

(c) The escape velocity of a body does not depend upon its direction of projection. **½**

(d) Since the gravitational potential energy at a point at the height h from the earth surface is $\frac{GMm}{(R+h)}$, the escape velocity will be different for different value of h. **½**

18. **(a)** Here at $t = 0$, OP makes an angle with x-axis. As motion is clockwise, so $\phi = \frac{+\pi}{2}$ radian. So the x-projection of OP at time t will give us the equation of S.H.M. given by,

$$x = A\cos\left(\frac{2\pi t}{T} + \phi\right)$$ **½**

$$= 3\cos\left(\frac{2\pi t}{T} + \frac{\pi}{2}\right),$$ **($\because$ A = 3 cm, T = 2 s)**

$$\Rightarrow \quad x = 3\cos\left(\pi t + \frac{\pi}{2}\right)$$

$$= -3\sin \pi t$$ **(x is in cm)**

$$\therefore \quad x = -3 \sin \pi t \text{ cm.}$$ **1**

(b) T = 4 s, A = 2 m

At $t = 0$, OP makes an angle π with the positive direction of x-axis, *i.e.*, $\phi = + \pi$.

Hence, the x-projection of OP at time t will give us the equation of S.H.M.

As, $$x = A\cos\left(\frac{2\pi t}{T} + \phi\right)$$ **½**

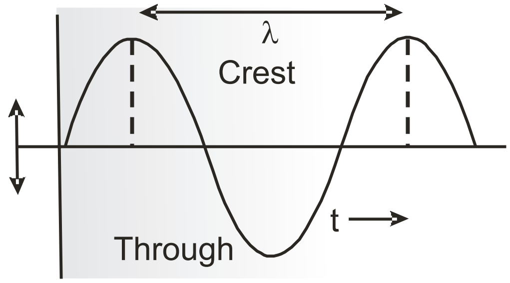
$$= 2\cos\left(\frac{2\pi}{T} + \pi\right)$$

$$= +2\cos\left(\frac{\pi}{2}t + \pi\right)$$

Or $$x = -2\cos\left(\frac{\pi}{2}t\right) \text{ m.}$$ **1**

Or

If the particles of a medium vibrate in a direction normal to the direction of wave, the motion is called transverse wave motion, *e.g.* streched string of violin, guitar, sitar sonometer etc. Electromagnetic waves are also transverse in nature. **1 + 1**



Crests and trough are formed when transverse wave propagates. Distance between consecutive troughs or crests is called wavelength. **1**

19. **(a)** While deriving Bernoulli's equation, we say that

Descrease in pressure energy per second = increase in K.E. /sec + increase in P.E. /sec ½

Consider that viscous forces are absent. Thus as the fluid flows from lower to upper edge there is a fall of pressure energy due to the fall of pressure. If dissipating force are present, then a part of this pressure energy will be used in overcoming these forces during the flow of fluid. Hence there shall be greater drop of pressure as the fluid moves along the tube. **1**

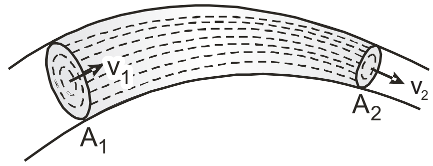
(b) Yes, the dissipative forces become more important as the fluid velocity increases. ½

From the Newton's law of viscous drag, we know that :

$$F = \eta A \frac{dv}{dx}$$

Clearly as v increases, velocity gradient increases and hence, viscous drag *i.e.* dissipative force also increases. **1**

20. **Equation of continuity :** Consider a non-viscous liquid in srteam line flow through a tube AB of varying cross-section. Let a_1, a_2 = areas of cross-sections of the tube at A and B respectively.



v_1, v_2 = velocities of flow of liquid at A and B respectively.
ρ_1, ρ_2 = density of liquid at A and B respectively.
$\therefore$ Volume of liquid entering per second at A $= a_1 v_1$ ½
Mass of liquid entering per second at A $= a_1 v_1 \rho_1$ ½
Similarly, mass of liquid leaving per second at B

$$= a_2 v_2 \rho_2$$

If there is no loss of liquid in the tube and the flow is steady, then
mass of the liquid entering per second at A = mass of the liquid leaving per second at B
Or $a_1 v_1 \rho_1 = a_2 v_2 \rho_2$ ....(1) ½
If the liquid is incompressible, then

$$\rho_1 = \rho_2$$

From (1), $a_1 v_1 = a_2 v_2$
Or av = constant ....(2) ½
This is known as equation of continuity.

From (2), $v \propto \frac{1}{a}$. It means the larger is the area of cross-section, the smaller will be the velocity of liquid flow and vice-versa. It is due to this reason that (a) deep water runs slow or slow water runs deep, (b) the jet of falling water becomes narrow as it goes down. **1**

21. Refer Ans 15 Sample Question Paper 4.

22. (i) At the triple point, *i.e.*, temperature = – 56·6°C and pressure = 5·11 atm, the vapour, liquid and the solid phase of CO_2 exist in equilibrium. ½

(ii) If the pressure decreases, both fusion point and boiling point of CO_2 decreases. ½

(iii) The critical temperature and pressure of CO_2 are 31·1°C and 73·0 atm respectively. If the temperature of CO_2 is more than 31·1°C, it cannot be liquified, how soever large pressure we may apply to it. ½

(iv) (a) CO_2 will be a vapour at – 70°C at a pressure of 1 atm. ½

(b) CO_2 will be a solid at – 60°C at a pressure of 10 atm. ½

(c) It will be a liquid at 15°C at a pressure of 56 atm. ½

23. (a) Navigator, he is a responsible citizen, he is duty minded, having presence of mind. **2**

(b) Apparent frequency received by an enemy submarine,

$$v' = \{(v + v_0)/v\}$$

$$v = \{(1450 + 100)/1450\} \times 40 \times 10^3 \text{ Hz}$$

$$= 4{,}276 \times 10^4 \text{ Hz.}$$

This frequency is reflected by the enemy submarine (source) and is observed by SONAR (now observer)

In this case apparent frequency, $v'' = \{v/(v - v_s)\} \times v$

$$= [1450/1450 - 100)] \times 4.276 \times 10^4 \text{ Hz} = 45.9 \text{ kHz.}$$ **2**

24. (i) $v = u + at$

Derivation : By definition of acceleration, we know that

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

Or $$v_2 - v_1 = a(t_2 - t_1)$$

Or $$v_2 = v_1 + a(t_2 - t_1) \quad \text{....(i)}$$

where v_1 and v_2 are the velocities of an object at time t_1 and t_2 respectively.

If $v_1 = u$ (initial velocity of the object) at $t_1 = 0$

$v_2 = v$ (final velocity of the object) at $t_2 = t$.

Then (i) reduces to $v = u + at$ **1**

(ii) $v^2 = u^2 + 2as$

Derivation : We know that acceleration is given by,

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$, where v_1 and v_2, t_1 and t_2 are as in (i),

Or $$t_2 - t_1 = \frac{v_2 - v_1}{a} \quad \text{....(i)}$$

Also we know that

$$x_2 - x_1 = v_1(t_1 - t_2) + \frac{1}{2} a(t_2 - t_1)^2 \quad \text{....(ii)}$$

From (i) and (ii), we get

$$x_2 - x_1 = v_1 \frac{v_2 - v_1}{a} + \frac{1}{2}a\left[\frac{v_2 - v_1}{a}\right]^2$$

$$= \frac{v_1 v_2 - v_1^2}{a} + \frac{v_2^2 + v_1^2 - 2v_1 v_2}{2a}$$

$$= \frac{2v_1 v_2 - 2v_1^2 + v_1^2 + v_2^2 - 2v_1 v}{2a}$$

$$= \frac{v_2^2 - v_1^2}{2a}$$

Or $$v_2^2 - v_1^2 = 2a\,(x_2 - x_1) \quad \text{....(iii)}$$

Now if
$$v_1 = u \text{ at } t_1 = 0$$
$$v_2 = v \text{ at } t_2 = t \quad \text{....(iv)}$$
$$x_2 - x_1 = s$$

Then from (iii) and (iv), we get
$$v^2 - u^2 = 2as \quad \text{....(v)} \quad \textbf{2}$$

Or $$v^2 = u^2 + 2as$$

(iii) $$s = ut + \frac{1}{2}at^2$$

Derivation : Let

x_1, v_1 = position and velocity of the object at time t_1

x_2, v_2 = position and velocity of the object at time t_2

a = uniform acceleration of the object

Also let v_{av} = average velocity in $t_2 - t_1$ interval.

∴ By definition,
$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1}$$

Or $$x_2 - x_1 = v_{av}\,(t_2 - t_1) \quad \text{....(i)}$$

Also we know that $$v_{av} = \frac{v_2 - v_1}{2} \quad \text{....(ii)}$$

∴ From eqns. (i) and (ii), we get
$$x_2 - x_1 = \frac{v_2 - v_1}{2}\,(t_2 - t_1) \quad \text{....(iii)}$$

Also we know that $v_2 = v_1 + a\,(t_2 - t_1)$ ....(iv)

∴ From eqns. (iii) and (iv), we get
$$v_2 - x_1 = \frac{1}{2}\,[v_1 + v_1 + a\,(t_2 - t_1)]\,(t_2 - t_1)]$$
$$= v_1\,(t_2 - t_1) + \frac{1}{2}\,a\,(t_2 - t_1)^2 \quad \text{....(v)}$$

Now, if
$$x_1 = x_0 \text{ at } t_1 = 0$$
$$x_2 = x \text{ at } t_2 = \text{t}$$
$$v_1 = u \text{ at } t_1 = 0 \quad \text{....(iv)}$$
$$v_2 = v \text{ at } t_2 = t$$

∴ From eqns. (v) and (vi), we get
$$x - x_0 = ut + \frac{1}{2}at^2$$

If $x - x_0 = s$, then
$$s = ut + \frac{1}{2}at^2. \quad \textbf{2}$$

Or

(a) Let v_1 and v_2 = finishing velocities of car A and car B
and t_1 and t_2 = finishing time intervals for car A and car B
$$v = v_1 - v_2 \quad \text{....(i)}$$
$$t = t_2 - t_1 \quad \text{....(ii)}$$

Let d = distance covered during race by each car.

Using eqn. $$s = \left(\frac{u+v}{2}\right)t$$

For car A, $$d = \left(\frac{0+v_1}{2}\right)t_1$$
$$= \frac{v_1 t_1}{2}, \quad (\because \text{u} = 0) \quad \text{....(iii)}$$

For car B, $d = \left(\frac{0+v_2}{2}\right)t_2$

$= \frac{v_2 t_2}{2}$ ($\because$ u = 0) ....(iv) ½

From (iii) and (iv), we get

$$d = \frac{v_1 t_1}{2} = \frac{v_2 t_2}{2}$$

$$v_1 = \frac{2d}{t_1}$$

and $v_2 = \frac{2d}{t_2}$ ....(v) ½

Also using eqn.

$$s = ut + \frac{1}{2}at^2$$

$$d = \frac{1}{2}a_1 t_1^2 = \frac{1}{2}a_2 t_2^2$$ ($\because$ u = 0)

$$a_1 = \frac{2d}{t_1^2}$$

and $a_2 = \frac{2d}{t_2^2}$ ....(vi) ½

Since, $a = \frac{v}{t} = \frac{v_1 - v_2}{t_2 - t_1} = \frac{\frac{2d}{t_1} - \frac{2d}{t_2}}{t_2 - t_1}$

$$= \frac{2d(t_2 - t_1)}{t_1 t_2 (t_2 - t_1)} = \frac{2d}{t_1 t_2}$$

$$= \sqrt{\left(\frac{2d}{t_2 t_1}\right)^2}$$

Or $\frac{v}{t} = \sqrt{\frac{2d}{t_1^2} \times \frac{2d}{t_2^2}}$

$$= \sqrt{a_1 \times a_2}$$

$v = t\sqrt{a_1 . a_2}$. **1**

(b) Given, $t = \sqrt{x} + 3$

Or $\sqrt{x} = t - 3$ ....(i)

Squaring on both sides of equation (i), we get

$x = (t-3)^2$ ....(ii)

$= t^2 + 9 - 6t$

If v be the velocity of the particle, then

$v = \frac{dx}{dt} = \frac{d}{dt}(x)$ ½

$$= \frac{d}{dt}\,(t^2 - 6t + a)$$

$$= 2t - 6$$

When $v = 0$, $2t - 6 = 0$

Or $t = 3$ sec. ....(iii) **1**

$\therefore$ From equations (ii) and (iii), we get

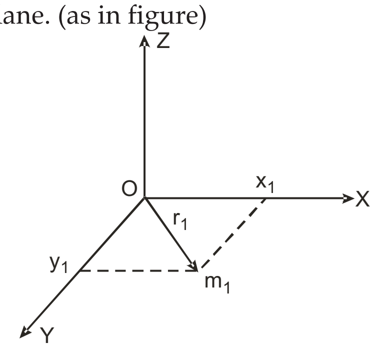
$$x = 3^2 + 9 - 6 \times 3$$

$$= 18 - 18$$

$$x = 0.$$ **1**

25. **Theorem of perpendicular axes :** According to this theorem, the moment of inertia of a plane lamina about any axis OZ $\perp$ to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about any two mutually $\perp$ axes OX and OY in the plane of the lamina, meeting at a point where the given axis OX passes through the lamina. ½

Suppose, the lamina is in XY plane. (as in figure)



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I_x = moment of inertia of the lamina about OX

I_y = moment of inertia of the lamina about OY

I_z = moment of inertia of the lamina about OZ

According to this theorem,

$$I_z = I_x + I_y$$ **1**

Proof : Suppose the lamina consists of n particles of masses $m_1, m_2, ..., m_n$ at $\perp$ distances $r_1, r_2, r_3, ..., r_n$ respectively from the axis OZ.

Suppose the corresponding $\perp$ distances of these particles from the axis OY are $x_1, x_2, x_3, ..., x_n$ and from the axis OX are $y_1, y_2, y_3, y_4, ..., y_n$ respectively.

$\therefore$
$$I_x = m_1 y_1^2 + m_2 y_2^2 + ... + m_n y_n^2$$

$$= \sum_{i=1}^{i=n} m_i y_i^2 \qquad ....(1)$$

$$I_y = m_1 x_1^2 + m_2 x_2^2 + ... + m_n x_n^2$$

$$= \sum_{i=1}^{i=n} m_i x_i^2 \qquad ....(2)$$

$$I_z = m_1 r_1^2 + m_2 r_2^2 + ... + m_n r_n^2$$

$$= \sum_{i=1}^{i=n} m_i r_i^2 \qquad ....(3)$$

Adding (1) and (2), we get

$$I_x + I_y = \sum_{i=1}^{i=n} m_i y_i^2 + \sum_{i=1}^{i=n} m_i x_i^2$$

$$= \sum_{i=1}^{i=n} m_i (y_i^2 + x_i^2)$$ ½

As clear from fig.,

$$r_i^2 = x_i^2 + y_i^2$$

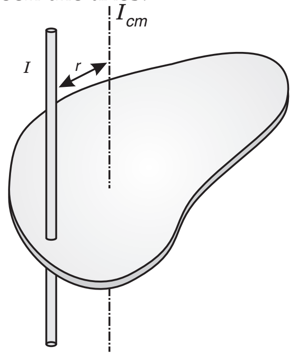
$\therefore$
$$I_x + I_y = \sum_{i=1}^{i=n} m_i r_i^2 = I_z$$

$\therefore$
$$I_x + I_y = I_z$$

Now
$$I_z = I_x + I_y.$$ ½

Or

Theorem of parallel axes : The theorem determines the moment of inertia of a rigid body about any given axis, given that moment of intertia about the parallel axis through the center of mass of an object and the perpendicular distance between the axes.



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The moment of inertia about Z-axis can be represented as :

$$I_z = I_{c.m.} + mr^2$$

Where $I_{c.m.}$ is the moment of inertia of the object about its centre of mass, m is the mass of the object and r is the perpendicular distance between the two axes.

Proof :

Assume that the perpendicular distance between the axes lies along the x-axis and the centre of mass lies at the origin. The moment of inertia relative to z-axis that passes through the centre of mass, is represented as

$$I_{c.m.} = \int (x^2 + y^2) dm$$ **1**

Moment of inertia relative to the new axis with its perpendicular distance r along the x-axis, is represented as :

$$I_z = \int ((x-r)^2 + y^2) dm$$ **1**

We get,

$$I_z = \int (x^2 + y^2) dm + r^2 \int dm - 2r \int x dm$$ **1**

The first term is $I_{c.m.}$ the second term is mr^2 and the final term is zero as the origin lies at the centre of mass. Finally,

$$I_z = I_{c.m.} + mr^2$$ **1**

26. **(a)** Let,

W = water equivalent of calorimeter and stirrer
t_1 = initial temperature of water and calorimeter
m_1 = mass of water
m_2 = mass of substance
C = specific heat of the substance
t_2 = temperature of the substance

Rise in temperature of water and calorimeter ½

$$= (t - t_1)$$

Fall in temperature of the substance = $(t_2 - t)$ ½

Heat gained by water and calorimeter = $(m_1 + w)(t - t_1)$ ....(i) ½

Heat lost by the substance = $C.m_1 (t_2 - t)$ ....(ii) ½

If we assume that there is no stray loss of heat, then

$$\text{Heat lost} = \text{Heat gained}$$

$$Cm_2.(t_2 - t) = (m_1 + W)(t - t_1)$$

$$C = \frac{(m_1 + W)(t - t_1)}{m_2(t_2 - t)}$$ ½

(b) Consider a cube of side x and area of each face 'A'. The opposite faces of the cube are maintained at temperatures, θ_1 and θ_2, where $\theta_1 > \theta_2$. Heat gets conducted in the direction of the fall of temperature.

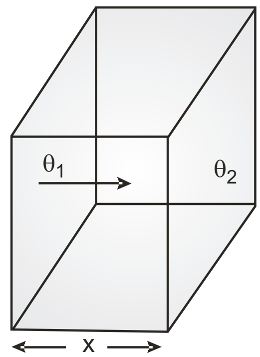
$$Q \propto A$$ ½

$$Q \propto (\theta_1 - \theta_2)$$ ½

$$Q \propto \tau$$

$$Q \propto \frac{1}{x} \propto \frac{A(\theta_1 - \theta_2)t}{x}$$

$$Q = \frac{KA(\theta_1 - \theta_2)t}{x}$$ ½



Here K is a constant called the coefficient of thermal conductivity of the material of the cube and t stands for time interval.

We can also write as

$$H = KA\left[\frac{\Delta\theta}{\Delta x}\right]$$ ½

Where, H = Heat flow per second

$\frac{\Delta\theta}{\Delta x}$ = temperature gradient

$T = \theta$ = temperature

If, $(\theta_1 - \theta_2) = 1°C$

$t = 1$ s

$x = 1$ cm

then $\theta = K.$ ½

Or

(a) The quantity $\frac{\theta_1 - \theta_2}{x}$ or $\frac{d\theta}{dx}$ respresents the rate of fall w.r. to distance.

The quantity $\frac{d\theta}{dx}$ represents the rate of change of temperature w. r. to distance and is called temperature gradient.

$$Q = -KA\left[\frac{d\theta}{dx}\right]t$$ ½

Dimensions of K.

Q represents energy and its dimensions are

$$[Q] = [ML^2T^{-2}]$$

$$[dx] = [L]\ [A] = [L^2]$$

$$[d\theta] = [\theta]\ [t] = [T]$$

$$[K] = \frac{[ML^2T^{-2}][L]}{[L^2][\theta][T]}$$

$$= [MLT^{-3}\ \theta^{-1}]$$ ½

(b) Consider a compound wall (or a slab) made of two materials A and B of thickness d_1 and d_2. Let K_1 and K_2 be the coefficients of thermal conductivity and θ_1 and θ_2 are the temperature of the end faces $(\theta_1 > \theta_2)$ and θ is the temperature of the surface in contact. ½

For material A :

$$Q_1 = \frac{K_1A_1(\theta_1 - \theta)}{d_1} \quad \text{....(i) ½}$$

For the material B :

$$Q_2 = \frac{K_2A_2(\theta - \theta_2)}{d_2} \quad \text{....(ii) } \mathbf{1}$$

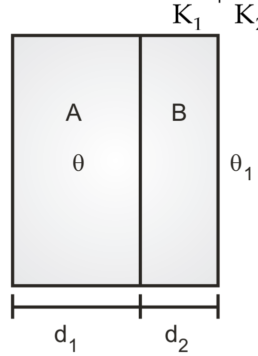
From eqns. (i) and (ii), we get

$$\frac{K_1A_1(\theta_1 - \theta)}{d_1} = \frac{K_2A_2(\theta - \theta_2)}{d_2} \quad \text{½}$$

$$= \frac{\dfrac{K_1\theta_1}{d_1} + \dfrac{K_2\theta_2}{d_2}}{\dfrac{K_1}{d_1} + \dfrac{K_2}{d_2}} \quad \text{½}$$

Putting the value of θ in eqn. (i),

$$\theta = \frac{A(\theta_1 - \theta_2)}{\dfrac{d_1}{K_1} + \dfrac{d_2}{K_2}}$$



½

In general for any number of walls,

$$\theta = \frac{A_1(\theta_1 - \theta_2)}{\Sigma\left(\dfrac{d}{K}\right)} . \quad \text{½}$$

❑❑❑

S O L U T I O N S

SAMPLE
QUESTION PAPER - 7

Self Assessment

Time : 3 Hours **Maximum Marks : 70**

1. It is called so because this law holds good irrespective of the nature of interacting bodies at all places and at all times. **1**

2. It is the ratio of change in dimension of a body to the original dimension when some deforming force is applied to it. It is a type not an example. **1**

3. The upward force exerted by a fluid (liquid or gas) on an object immersed in the fluid. **1**

4. Time period decreases as effective value of acceleration due to gravity increases (*i.e.*, $g' = g + a$) and $(T \propto 1/\sqrt{g'})$. **1**

5.
$$\text{Intensity} = (\text{Amplitude})^2 \propto \frac{1}{(\text{distance})^2}$$

$\therefore$
$$\text{Amplitude} \propto \frac{1}{\text{distance}}$$

Hence,
$$\frac{A_1}{A_2} = \frac{r_2}{r_1} = \frac{25}{8}$$ **1**

6. Relative error in ρ is given by

$$\frac{\Delta\rho}{\rho} = 3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta\rho}{\rho} + \frac{\Delta d}{d}$$ **1**

so, percentage error is

$$\frac{\Delta\rho}{\rho}\times 100 = \{3\frac{\Delta\rho}{\rho} + 2\frac{\Delta\rho}{\rho} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}\}\times 100$$
$$= (3 \times 1\%) + (2 \times 3\%) + (1/2 \times 4\%) + (1 \times 2\%)$$
$$= 3\% + 6\% + +2\% + 2\%$$
$$= 13\%$$ **1**

7. Mass given by the equation $a = \frac{F}{M}$, *i.e.*, $M = \frac{F}{a}$ is called inertial mass. Clearly, higher the mass means lesser the acceleration. Thus mass of a body resists the acceleration, *i.e.*, rate of change in velocity due to external force or in other words it is the measure of inertia of a body. **2**

Or

$$|\vec{a}+\vec{b}|^2 - (|\vec{a}| + |\vec{b}|)^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta - |\vec{a}|^2 - |\vec{b}|^2 - 2|\vec{a}||\vec{b}|$$
$$= 2|\vec{a}||\vec{b}|(1 - \cos\theta)$$ **½**

$$= 2|\vec{a}||\vec{b}| \sin^2\frac{\theta}{2}$$ **1**

= a negative quanity

Hence, $$|\vec{a}+\vec{b}| < |\vec{a}|+|\vec{b}|$$ ½

8. It should be greater in night. In the day time, the body is pulled by the Earth and the Sun in two opposite directions. This will result into decrease in weight. **1**

During night time the earth and the sun pull the body in the same direction so the weight will increase in night. **1**

9. $$C_V = \frac{F}{2}R$$ (Where F in degree of freedom)

For monoatomic gas, F = 3

$$C_V = \frac{3}{2}R$$

$$C_P - C_V = R$$

$$\therefore \quad C_P = \left(1+\frac{3}{2}R\right)$$

$$C_P = \frac{5}{2}R$$

$$\gamma = \frac{C_P}{C_V} = \frac{5}{3} = 1.67$$ **1**

For diatomic gases, F = 5

$$C_V = \frac{5}{2}R, C_P = \frac{7}{2}R$$

$$\gamma = \frac{C_P}{C_V} = \frac{7}{5} = 1.4$$ 1

10. Motion of piston in a cylinder is another example of a vibrating system.

Resturing force in this case is given by :

$$F = -mg = -kx$$ **1**

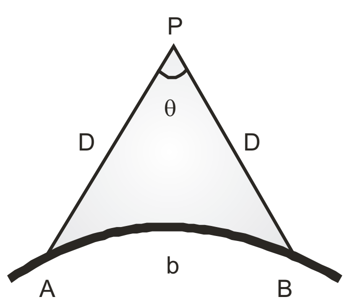
where m is mass of the piston and x is the displacement,

Time period of vibration $$T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$$

$$= 2\pi\sqrt{\frac{m}{k}}$$ **1**

11. Let there be a planet P far away from the Earth. The planet is observed from two different observation points A and B on Earth called observatories. If distance AB = b and angle APB = θ radian, then θ being

very small $\frac{b}{D} << 1$. Angle θ is called parallax or parallactic angle. **1**



1

Now radius :

$$AP = BP = D = \frac{b}{\theta}$$

So that $$b = D\theta.$$ **1**

12. **(a)** Since the motion of the train between two distant stations is smooth throughout so keeping in view the long distance covered between the two stations in reasonable duration of time, the size of the train is neglected and it is considered as a point object. **1**

(b) The distance covered by the monkey in reasonable duration of time is more so it is considered as a point object. **1**

(c) Since, the turning of the ball is not smooth but sharp so the distance covered by it in reasonable duration of time is not large so this ball cannot be treated as a point object. **½**

(d) Since the beaker is tumbling and then slips off. So, the distance covered by it in reasonable duration of time is not large. So it is not treated as a point object. **½**

13. When the stone is dropped, it falls freely under the acceleration due to gravity g. With respect to the Earth the acceleration of the stone is g. **1**

Inside the carriage, the stone possesses two accelerations :

(i) Horizontal acceleration 'a' due to the motion of carriage. **1**

(ii) Vertical downward acceleration 'g' due to gravity.

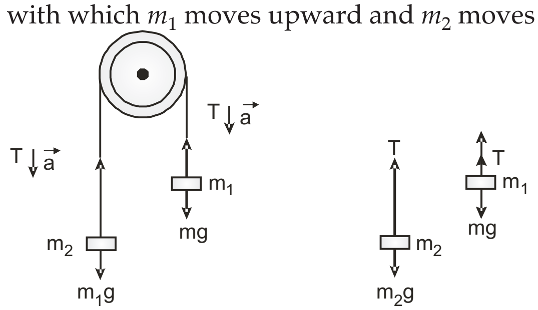
The acceleration of the stone w.r.t. the carriage $= \sqrt{a^2 + g^2}$ **1**

14. Suppose m_1 and m_2 be the masses suspended at the ends of a light inextensible string passing over the pulley.

$\therefore$ $m_1 = 8$ kg, $m_2 = 12$ kg

Suppose T = Tension in the string

a = common acceleration with which m_1 moves upward and m_2 moves downward = ?



The equations of motion of m_1 and m_2 are given by

$$T = m_1 a + m_1 g \quad \text{....(i)}$$

and

$$m_2 g - T = m_2 a \quad \text{....(ii)} \ \tfrac{1}{2}$$

Adding equations (i) and (ii), we get

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2} \quad \text{....(ii)} \ \tfrac{1}{2}$$

$\therefore$ From equations (i) and (iii), we get

$$T = m_1 g + m_1 \frac{(m_2 - m_1)g}{m_1 + m_2} \quad \tfrac{1}{2}$$

$$\Rightarrow \quad T = \frac{m_2 g}{m_1 + m_2}(m_1 + m_2 + m_2 - m_1)$$

$$\Rightarrow \quad T = \frac{2m_1 m_2}{m_1 + m_2} g \quad \text{...(iv)} \ \tfrac{1}{2}$$

Putting $m_1 = 8$ kg and $m_2 = 12$ kg and $g = 10$ ms^{-2} in equations (iii) and (iv), we get

$$a = \left(\frac{12 - 8}{8 + 12}\right) \times 10$$

$$\Rightarrow \quad = \frac{4}{20} \times 10 = 2 \text{ ms}^{-2} \quad \tfrac{1}{2}$$

From eqn. (i), $T = m_1 a + m_1 g$

$$= 8 \times 2 + 8 \times 10 = 96\text{N} \quad \tfrac{1}{2}$$

15. (a)
$$v = \frac{dx_A}{dt} = 2$$
and
$$a = \frac{d^2x_A}{dt^2} = 0$$
As per above equation the particle has no acceleration at all. **1**

(b)
$$v = \frac{dx_B}{dt}$$
$$= 3 \times 2t + 2 + 0$$
$$= 6t + 2$$
$$a = \frac{d^2x_B}{dt^2} = 6$$
Here acceleration is uniform. **1**

(c)
$$v = \frac{dx_c}{dt}$$
$$= 5 \times 3t^2 + 4$$
$$= 15t^2 + 4$$
$$a = \frac{dv}{dt} = \frac{d^2x_C}{dt^2}$$
$$= 15 \times 2t = 30t$$
Here, acceleration depends upon time so it is not uniform. **1**

16.

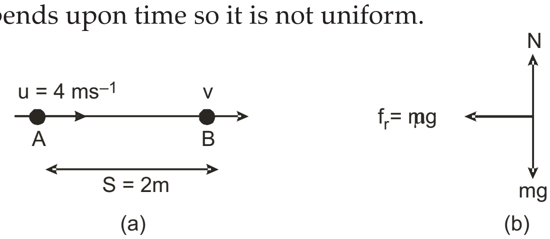


Figure (a) here shows the initial position A and the final position B of the object. Let the speed of particle at B be v. The particle does work against force of friction which is a non-conservative force.
$$K_i - K_f = W$$ **1**
where W is the work done against the force of friction.
Figure (b) shows all the forces acting on the object,
$$W = f_r \times s = \mu mg.s$$
$$= (0{\cdot}15 \times 0{\cdot}1 \times 10 \times 2)\ J$$
$$= 0{\cdot}3\ J$$ **1**

Hence
$$\frac{1}{2} \times 0.1 \times [4^2 - v^2] = 0{\cdot}3$$
Or
$$16 - v^2 = 6$$
$$v = \sqrt{10}\ ms^{-1} = 3{\cdot}16\ ms^{-1}$$ **1**

17. Velocity of centre of mass of a system,
$$\vec{v}_{cm} = \frac{d\vec{r}}{dt}$$
$$\text{Total external force } \vec{F} = M\frac{d^2\vec{r}}{dt^2}$$
$$= M\frac{d\vec{v}_{CM}}{dt} \quad \text{....(1)}$$ **1**

Internal forces cancel out in pairs, so velocity of centre of mass is not affected by internal force.

Puting $\vec{F} = 0$ in equation (i),

$$\frac{d\vec{v}_{CM}}{dt} = 0, (\because M \neq 0) \qquad \textbf{1}$$

i.e., $$v_{cm} = \text{constant}$$

If resultant external force is zero, the velocity of centre of mass remains same. **1**

Or

The concept of angular momentum : Expression for angular momentum in cartesian co-ordinates : In order to express torque as the rate of change of some quantity, we rewrite expression for torque rotating a particle in XY plane as

$$\tau = xF_y - yF_x \qquad ...(1)\ ½$$

If $P_x = mv_x$ and $P_y = mv_y$ are the x and y components of linear momentum of the body, then According to Newton's 2nd laws of motion

$$F_x = \frac{dP_x}{dt} = \frac{d}{dt}(mv_x)$$

$$= \frac{mdv_x}{dt} \qquad ½$$

$$F_y = \frac{dP_y}{dt} = \frac{d}{dt}(mv_y) = \frac{mdv_y}{dt}$$

Substituting in (1), we get $$\tau = x\frac{mdv_x}{dt} - y\frac{mdv_y}{dt}$$

Or $$\tau = m\left[x\frac{dv_x}{dt} - y\frac{dv_y}{dt}\right] \qquad ...(2)\ ½$$

Differentiating both the sides

$$\frac{d}{dt}(xv_x - yv_x) = x\frac{dv_y}{dt} + v_y\frac{dx}{dt} - y\frac{dv_x}{dt} - v_x\frac{dy_y}{dt}$$

$$= x\frac{dv_y}{dt} + v_yv_x - y\frac{dv_x}{dt} - v_xv_y \left[\therefore \frac{dv}{dt} = v_x\ \frac{dy}{dt} = v_y\right]$$

$$= x\frac{dv_y}{dt} - y\frac{dv_x}{dt} \qquad ...(3)\ ½$$

Substituting (3) in (2)

$$\tau = m\frac{d}{dt}(xv_y - yv_x) \qquad ½$$

$$\tau = \frac{d}{dt}(xmv_y - ymv_x)$$

As $$mv_y = P_y$$
and $$mv_x = P_x$$

$$\tau = \frac{d}{dt}(xP_y - yP_x) \qquad ...(4)$$

$$xP_y - yP_x = L$$

$\Rightarrow$ $$\tau = \frac{dL}{dt} \qquad ½$$

18. Given :

$$\alpha = 1.2 \times 10^{-5}\ °C^{-1}$$
$$\Delta t = 30 - 20 = 10\ °C$$
$$T = 25$$

From the relation

$$\Delta l = l\,\alpha\Delta t,$$

or $$\frac{T'}{T} = \alpha\Delta t$$
$$= 1.2 \times 10^{-5} \times 10$$
$$=1.2 \times 10^{-4} \quad ...(i)\ ½$$

$$T = 2\pi\sqrt{\frac{l}{g}} \quad ...(i)$$

If T' be the time period of the pendulum, when l increase by Δl.

$\therefore$ $$T' = 2\pi\sqrt{\frac{l+\Delta l}{g}}$$
$$= 2\pi\sqrt{\frac{l}{g}\left(1+\frac{\Delta l}{l}\right)} \quad ...(iii)\ \mathbf{1}$$

Dividing eqn. (iii) by eqn. (ii),

$$\frac{T'}{T} = \sqrt{1+\frac{\Delta l}{l}}$$
$$= \sqrt{1+1.2\times10^{-4}} \quad ½$$

$\therefore$ Loss in time in one oscillation = T' – T

Therefore loss in time in one day is given by

$$= \frac{T'-T}{T} \times 24 \times 3600 \text{ s.}$$
$$= \left(\frac{T'}{T}-1\right) \times 24 \times 3600 \text{ s.}$$
$$= \left[\sqrt{1+1.2\times10^{-4}}-1\right] \times 24 \times 3600 \text{ s.}$$
$$= \left[1+\frac{1}{2}\times1.2\times10^{-4}-1\right] \times 24 \times 3600 \text{ s.}$$
$$= \frac{1.2\times10-4\times24\times3600}{2} \text{ s.}$$
$$= 5.18 \text{ s.} \quad \mathbf{1}$$

19. Displacement : Consider a reference particle moving on a circle of reference of radius a with uniform angular velocity ω. Let the particle start from the point X and trace angular θ radian in time t as it reaches the point P.

Therefore, $$\omega = \frac{\theta}{t} \text{ or } \theta = \omega t. \quad ½$$

Let the projection of the particle P on diameter YOY' be at M. Then OM = y is the displacement in S.H.M. at time t.

In Δ OPM, $$\sin\theta = \frac{OM}{OP} = \frac{y}{a}$$

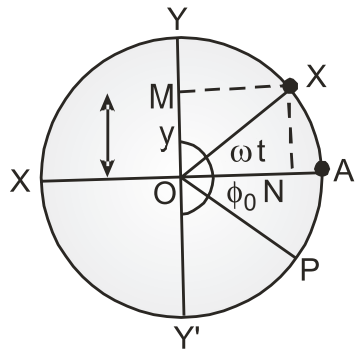
or $$y = a\sin\theta = a\sin\omega t \quad ....(i)\ ½$$

Important notes : (1) If projection of P is taken on diameter XOX′, then point N will be executing S.H.M. Here, ON = x = displacement in S.H.M. at time t

In Δ ONP, $$\cos\theta = \frac{ON}{OP}$$

or $$x = a\cos\theta = a\cos\omega t \quad ....(ii)\ ½$$

(2) If A is the starting position of the particle of reference such that $\angle AOX = \phi_0$ and $\angle AOP = \omega t$.



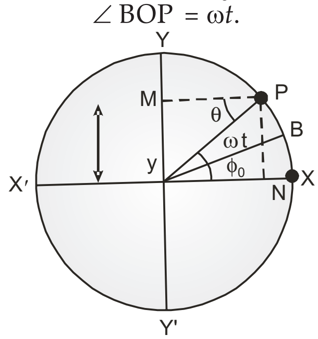
½

$$\theta = \angle XOP = \omega t - \phi_0$$

Here (–) ϕ_0 is called the initial phase or epoch of S.H.M.

If B is the starting position of the particle of reference such that

$$\angle BOX = \phi_0$$
$$\angle BOP = \omega t.$$



½

$$\theta = \angle XOP = \omega t + \phi_0$$

From equation (i), $y = a\sin(\omega t + \phi_0)$

From equation (ii), $x = a\cos(\omega t + \phi_0)$ ½

Here (+) ϕ_0 is called initial phase or epoch of S.H.M.

20. **(a)** The formula for velocity of sound in air is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad ....(1)$$

where γ = constant for a given gas, *i.e.*, air

ρ = density of the gas

From gas equation,

$$PV = RT \quad ....(1)$$

Or $$P = \frac{RT}{V} \quad ....(2)$$

∴ From equations (1) and (2), we get

$$v = \sqrt{\frac{\gamma RT}{\rho V}} = \sqrt{\frac{\gamma RT}{M}} \quad ....(3)$$

where $\rho V = M$ = molecular mass of air or gas.

For a given gas, m = constant

R is also constant.

When T = constant, then from equation (3), we conclude that v is independent of the pressure of air (gas) if temperature remains constant. **1**

(b) **Effect of temperature :** We know that

$$PV = RT$$

Or $$P = \frac{RT}{V}$$

Also $$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{\rho V}} = \sqrt{\frac{\gamma RT}{M}}$$

where M = ρV = molecular mass of the gas. Since γ, R and M are constants, so

$$v \propto \sqrt{T}\ .$$

It means velocity of sound in a gas is directly proportional to the square root of its temperature, hence we conclude that the velocity of sound in air increases with increase in temperature. **1**

(c) Effect of humidity : The presence of water vapours in air changes the density of air, thus the velocity of sound changes with humidity of air.

Let ρ_m = density of moist air

ρ_d = density of dry air

v_m = velocity of sound in moist air

v_d = velocity of sound in dry air

From relation $v = \sqrt{\dfrac{\gamma P}{\rho}}$,

we get

$$v_m = \sqrt{\frac{\gamma P}{\rho_m}} \quad \text{....(i)}$$

and

$$v_d = \sqrt{\frac{\gamma P}{\rho_d}} \quad \text{....(ii)}$$

Dividing equation (i) by equation (ii), we get

$$\frac{v_m}{v_d} = \sqrt{\frac{\rho_d}{\rho_m}}$$

Also we know that density of water vapours is less than the density of dry air. It means dry air is heavier than water vapours as the molecular mass of water is less than that of N_2 (28) and O_2 (32), so

$$\rho_m < \rho_d$$

Or

$$\frac{\rho_d}{\rho_m} > 1 \quad \text{....(iv)}$$

∴ from equations (iii) and (iv), we get

$$\frac{v_m}{v_d} > 1$$

∴ $$v_m > v_d$$

It means velocity of sound in air increases with humidity, *i.e.*, velocity of moist air is greater than the velocity of sound in dry air. That is why sound travels faster on rainy day than a dry day. **1**

21. Work done against gravity $W_1 = mgh = V\rho gh$ ½

Work done against pressure difference is

$$W_2 = \Delta P \times V$$
$$= h\rho g \times V$$
$$= V\rho gh \quad \mathbf{1}$$

Total work done $W = W_1 + W_2$

$$= 2V\rho gh \quad \text{½}$$

∴ Power required $= \dfrac{W}{t} = \dfrac{2V\rho gh}{t}$

$$= \frac{2 \times (300 \times 10^{-3}) \times 10^3 \times 10 \times 6}{60}$$

= 600 watt. **1**

22. Molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one gram mole of the substance through a unit degree. By definition, one mole of any substance is a quantity of the substance whose mass in gram is numerically equally to the molecular mass M.

Thus $C = Mc$ ....(i) **1**

$n \rightarrow$ no. of moles

$m \rightarrow$ mass of substance in grams.

i.e., $$n = \frac{m}{M}$$

or $$m = nM$$ **1**

From equation (i), $$c = \frac{\Delta\theta}{m(\Delta t)} = \frac{\Delta\theta}{nM(\Delta t)}$$

$$Mc = \frac{1}{n}\left(\frac{\Delta\theta}{\Delta t}\right)$$

Using equation (i) $$C = \frac{1}{n}\left(\frac{\Delta\theta}{\Delta t}\right)$$ **1**

23. **(a)** Rahim loves animals and feeds them. He takes care to be at a safe distance from the animal. **1**

(b) **(i)** The tension developed in the string when the monkey climbs up with an acceleration of $6\ m/s^2$ is given by $T = m(g + a) = 40(10 + 6) = 640$ N. **1**

(ii) The tension developed when the monkey climbs down with an acceleration of $4\ m/s^2$ is given by $T = m(g - a) = 40(10 - 4) = 40 \times 6 = 240$ N. ½

(iii) When the monkey climbs with uniform speed of 5 m/s, acceleration is zero and the tension in the string is $T = mg = 40 \times 10 = 400$ N. ½

(iv) As the monkey falls down the rope nearly under gravity, the tension in the string is given by $T = m(g - a) = m(g - g) = 0$ ½

Since the string can withstand a maximum tension of 600 N, hence the rope will break only in the first case (i) ½

24. **(a)** Let a constant force $\vec{F}$ be applied to a body moving with initial velocity $\vec{u}$, so that its velocity becomes $\vec{v}$ along the direction of force when s is its displacement. Using Newton's second law of motion we get magnitude of force, $F = ma$ and from equation of motion, we get $v^2 - u^2 = 2as$, where a is the acceleration of the body. **1**

Multiplying both sides by m, we get

$$mv^2 - mu^2 = 2mas$$

i.e., $$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$

i.e., $$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs = W \qquad (\because ma = F)$$

i.e., $$K.E._{(f)} - K.E._{(i)} = W$$ **1**

where $K.E._{(f)}$ is final kinetic energy and $K.E._{(i)}$ is initial kinetic energy.

Thus work done on a body by a net force is equal to the change in kinetic energy of the body. **1**

(b) Suppose, ΔW be the amount of work done in a small time interval Δt, when P_{av} be the average power, then

$$P_{av} = \frac{\Delta W}{\Delta t} \qquad \text{....(i)}$$ ½

When P be the instantaneous power, then by definition 'S'.

$$P = \underset{\Delta t \to 0}{Lt}\ P_{av}$$

$$= \underset{\Delta t \to 0}{Lt}\ \frac{\Delta W}{\Delta t}$$ ½

$$\Rightarrow \qquad P = \frac{dW}{dt} = \frac{d}{dt}(W) \qquad \text{....(ii)}$$

Now $$W = \vec{F}.\vec{s} \qquad \text{....(iii)}$$ ½

Where, F = constant force producing a displacements

$\therefore$ From equations (ii) and (iii), we get

$$P = \frac{d}{dt}(\vec{F}.\vec{S})$$

$$= \vec{F}\frac{d\vec{S}}{dt} = \vec{F}.\vec{v}$$ ½

Or

(a) (i) In physics work is not said to be done, if (a) the applied force (F) is zero. A body moving with uniform velocity on a smooth surface has some displacement but no external force, so in this case work done is zero. **1**

(ii) The displacement (*s*) is zero. A labourer standing with a load on his head does no work. ½

(iii) The angle between force and displacement (θ) is π/2 rad or 90°. Then cos θ = cos 90° = 0. Thus work done is also zero.

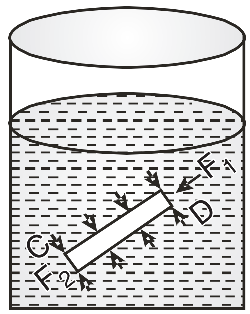
In circular motion, instantaneous work done is always zero because of this reason. **1**

(iv) The change in kinetic energy (ΔKE) is zero. ½

(b) If roads were to go straight up, the slope (θ) would have been large, the frictional force ($\mu mg \cos \theta$) would be small. The wheels of the vehicle would slip. Also for going up a large slope, a greater power shall be required. **2**

25. It states that if gravity effect is neglected, the pressure at every point of liquid in equilibrium or rest is same. ½

Proof : Consider two points C and D inside the liquid in a container which is in equilibrium or rest. Imagine a right circular cylinder with axis CD of uniform cross-section area A such that points C and D lie on float faces of the cylinder in figure. **1**



½

The liquid inside the cylinder is in equilibrium under the action of forces exerted by the liquid outside the cylinder. These forces are acting every where ⊥ to the surface of the cylinder. Thus force on the flat faces of the cylinder at C and D will be ⊥ to the forces on the curved surface of the cylinder. Since the liquid is in equilibrium therefore, the sum of forces acting on the curved surface of the cylinder must be zero. If P_1 and P_2 are the pressure at points C and D and F_1 and F_2 are the forces acting on the flat faces of the cylinder due to liquid, then ½

$$F_1 = P_1A$$

and

$$F_2 = P_2A$$

Since the liquid is in equilibrium,

$$F_1 = F_2$$

or

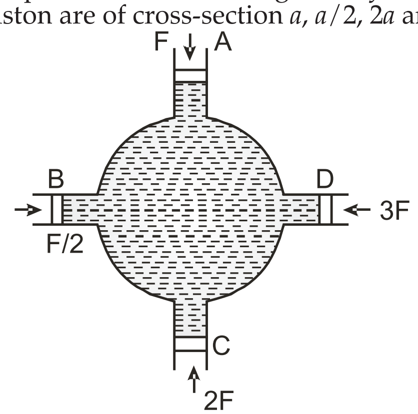
$$P_1A = P_2A$$

or

$$P_1 = P_2$$

It means the pressure at C and D are the same. ½

Experimental Proof : Consider a spherical vessel having four cylindrical tubes A, B, C and D each fitted with air tight frictionless piston are of cross-section *a*, *a*/2, 2*a* and 3*a* respectively.



½

Fill the vessel with an incompressible liquid so that no air gap is left inside the vessel and piston fitted in the various cylindrical tubes. Push the piston A with force F. The pressure developed on the liquid = F/*a* = P (say) ½

It is seen that all other pistons will be pushed outwards. To keep the pistons at their respective original positions, the force F/2, 2F and 3F respectively required to be applied on pistons of tubes B, C and

D respectively to hold them. Now pressure developed on liquid in tubes B, C and D are F/a, 2F/2a, 3F/3a *i.e.*, each equal to F/a. This indicates that the pressure applied is transmitted equally to all parts of liquid. This proves Pascal's law. **1**

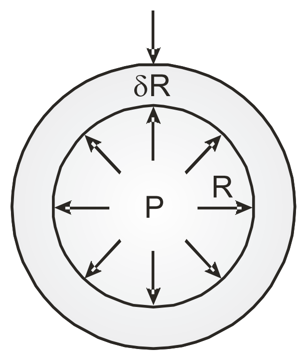
Or

(i) Excess Pressure nside a liquid drop : Consider a liquid drop of radius R. The molecules lying on the surface of liquid drop, due to surface tension will experience a resultant force acting inward ⊥ to the surface.

Let S = Surface tension of liquid drop

P = excess pressure inside the drop

Due to excess of pressure, let there be an increase in the radius of the drop by a small quanity δR, as shown in figure.



½

Then work done by the excess pressure,

$$W = \text{force} \times \text{displacement}$$
$$= (\text{excess pressure} \times \text{area} \times \text{increase in radius})$$
$$= P \times 4\pi R^2 \times \delta R \quad \text{....(i)} \quad ½$$

Increase in surface area of the drop

$$= \text{final surface area} - \text{initial surface area}$$
$$= 4\pi(R + \delta R)^2 - 4\pi R^2$$
$$= 4\pi[R^2 + 2R(\delta R) + (\delta R)^2 - R^2]$$
$$= 8\pi R.\delta R \text{ [Neglecting, } (\delta R)^2 \text{ being very very small]} \quad ½$$

∴ Increase in surface energy= increase in surface area × surface tension

$$= 8\pi R\,(\delta R) \times S \quad \text{....(ii)} \quad ½$$

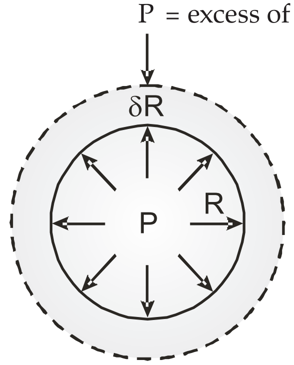
As the increase in surface energy is at the cost of work done by the excess pressure, therefore from (i) and (ii),

$$P \times 4\pi R^2 \times \delta R = 8\pi R\delta R \times S \quad ½$$
$$P = \frac{2S}{R}$$

(ii) Inside a liquid bubble : Consider a soap bubble of radius R, the molecules lying on the surface of liquid bubble will experience a resultant force acting on water ⊥ to the surface due to the surface tension.

Let S = surface tension of the soap solution

P = excess of pressure inside the bubble.



½

Due to it, let there be an increase in the radius of the bubble by a small amount δR, as shown in fig., then work done, W = force ×placement

= (excess pressure × area) × increase in radius

$= P \times 4\pi R^2 \times \delta R$ ....(iii) ½

The soap bubble has two free surfaces one outside the bubble and one inside the bubble, when soap solution and air are in contact.

∴ The effective increase in surface area of the bubble

= 2 [final S.A – initial S.A.]

$= 2\,[4\pi(R + \delta R)^2 - 4\pi R^2]$

$= 2 \times 4\pi[R^2 + 2R(\delta R) + (\delta R)^2 - R^2]$

$= 8\pi \times 2R(\delta R)$, [Neglecting $(dR)^2$, being very small]

$= 16\pi R.\delta R$ ½

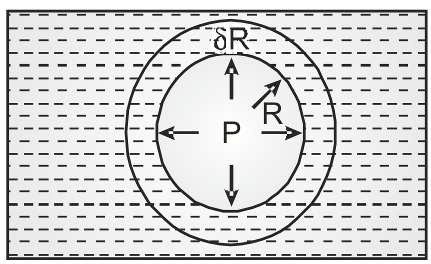
∴ Increase in surface energy is as the cost of work done by the excess pressure therefore from (iii) and (iv), we get

$$P \times 4\pi R^2 \times (\delta R) = 16\pi R\,(\delta R) \times S$$

Or $$P = \frac{4S}{R}$$ ½

(iii) Gross Pressure inside a bubble in liquid : Consider an air bubble of radius R. Just inside a liquid of surface tension S. The air bubble will have only one free surface as shown in figure. It can be shown that the pressure inside the air bubble is given by,

$$P = \frac{2S}{R}$$ ½



½

26. (a) Assumptions of Kinetic theory :

(i) Size of a molecule of a gas is very small as compared to intermolecular distance.

(ii) Behaviour of the molecules of the gas is perfectly elastic.

(iii) Molecules do not exert any force on each other except in collision.

(iv) Velocities of the molecules are in all directions.

(v) Molecules are in random motion.

(vi) The duration of collision is instantaneous.

(vii) The molecules move in a straight line between two successive collisions.

(viii) Collisions of the molecules with one another and the walls of the container are perfectly elastic. **2**

(b) Boyle's law :

Using the relation $$P = \frac{1}{3}\rho v^2 = \frac{1}{3}\frac{M}{V}v^2$$

$$PV = \frac{1}{3}Mv^2$$ **1**

Here M is fixed and $v^2 \propto T$

If temperature T is fixed then PV = constant

Or $P \propto 1/V$

i.e., It temperature of a given mass of a gas is kept constant, its pressure is inversely proportional to its volume.

Charle's law :

Using the relation $$P = \frac{1}{3}\rho v^2 = \frac{1}{3}\frac{M}{V}v^2$$

$$V = \frac{1}{3}\frac{M}{P}v^2$$ **1**

$$V^2 \propto T \text{ and M is fixed}$$

$$V \propto T$$

If pressure of a given mass of gas is kept constant its volume is directly proportional to the temperature of the gas. **1**

Or

(a) Pressure exerted by one mole of gas, $$P = \frac{1}{3}\rho v^2 = \frac{1}{3}\frac{M}{V}v^2$$

Or $$PV = \frac{1}{3}Mv^2$$

But $$PV = RT$$ **1**

$\therefore$ $$\frac{1}{3}Mv^2 = RT$$

Or $$\frac{1}{2}Mv^2 = \frac{3}{2}RT$$

Or $$\frac{1}{2}(Nm)v^2 = \frac{3}{2}RT \qquad (\because M = Nm)$$

Or $$\frac{1}{2}mv^2 = \frac{\frac{3}{2}RT}{N} = \frac{3}{2}KT \qquad \left(\text{Where } \frac{R}{K} = K\right)$$ **1**

$\therefore$ Total random K.E. for one mole $= \frac{3}{2}RT$

and average K.E. per molecule $= \frac{3}{2}KT.$ **1**

(b) Root mean square velocity is defined as the square root of the average of the squares of the individual velocities of the gas molecules *i.e.*, ½

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + ... + v_n^2}{n}}$$

$$= \sqrt{\overline{v}^2}$$ ½

where $v_1, v_2, v_3, ..., v_n$ are individual velocities

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{Nm}}$$ *i.e.*,

$$v_{rms} \propto \sqrt{T}.$$ **1**

❑❑❑

SOLUTIONS

SAMPLE QUESTION PAPER - 8

Self Assessment______________________________

Time : 3 Hours **Maximum Marks : 70**

1. 1 kWh = 1000 × 3600 Ws = $3{\cdot}6 \times 10^6$ J. **1**

2. The normal force exerted by a liquid on any surface is called thrust. **1**

3. **(a)** PV^{γ} = constant or TV^{r-1} = constant. **½**

(b) $W = -nC_v\,(T_f - T_i)$. **½**

4. The strong smelling molecules of a gas move rapidly to intermingle with air molecules through the surrounding. This process takes little longer time because of collision of gas and air molecules. This keeps the odour persisting for some time. **1**

5. Condition **(i)** is not sufficient, because it gives no reference of the direction of acceleration, whereas in S.H.M. the acceleration is always in a direction opposite to that of the displacement. **1**

6. It is not necessary that a precise measurement has to be more accurate.

Let true value of certain length be 3 cm. This length is measured with a measuring instrument of limit of resolution of 0·1 cm and the measured value is obtained as 3·1 cm. This length is again measured with another measuring instrument of resolution 0·01 cm and the length is measured as 2·8 cm. **1**

In this case first measurement has more accuracy because it is closer to the true value but less precision because the resolution is only 0·1 cm whereas in the second it was 0·01 cm. **1**

7. This graph can be for a ball dropped vertically from a height d. It hits the ground with some downward velocity and bounces upto height $d/2$ where its upward velocity becomes zero. **2**

8. As the vectors $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ are perpendicular to each other, therefore **½**

$$(\vec{A}+\vec{B}).(\vec{A}-\vec{B}) = 0$$

$$\vec{A}.\vec{A}-\vec{A}.\vec{B}+\vec{B}.\vec{A}-\vec{B}.\vec{B} = 0$$

Or $$A^2 - B^2 = 0$$ **½**

$\Rightarrow$ $$A = B, \qquad (\because \vec{A}.\vec{B} = \vec{B}.\vec{A})$$ **1**

Or

Static friction (f_s) comes into play when the horse applies force to start the motion in the cart, on the other hand, kinetic friction (f_k) comes into play when the cart is moving $f_s > f_k$. **2**

9. **(a)** The inability of a body to change its state of uniform rotation about an axis is called rotational inertia or M.I. of the body. **½**

It plays the same role in rotatory motion as is played by the mass in translatory motion, *i.e.*, it is rotational analogue of mass. ½

(b) M.I. of a body about a fixed axis of rotation is defined as the sum of the products of the masses and square of the perpendicular distance of various constituent particles from the axis of rotation. **1**

10. When a simple harmonic system oscillates with a constant amplitude which does not change with time, its oscillations are called undamped oscillations. When a simple harmonic system oscillates with a decreasing amplitude, with time its oscillations are called damped oscillations. **2**

11. Let the measured values be :

$$\text{Mass of block, } (m) = 39{\cdot}3 \text{ g}$$
$$\text{Length of block, } (l) = 5{\cdot}12 \text{ cm}$$
$$\text{Breadth of block, } (b) = 2{\cdot}56 \text{ cm}$$
$$\text{Thickness of block, } (t) = 0{\cdot}37 \text{ cm}$$

The density of the block is given by

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{m}{l \times b \times t}$$
$$= \frac{39{\cdot}3 \text{ g}}{5{\cdot}12 \text{ cm} \times 2{\cdot}56 \text{ cm} \times 0{\cdot}37 \text{ cm}}$$
$$= 8{\cdot}1037 \text{ g cm}^{-3}$$

Now, uncertainty in $m = \pm 0{\cdot}01$ g
uncertainty in $l = \pm 0{\cdot}01$ cm
uncertainty in $b = \pm 0{\cdot}01$ cm
uncertainty in $t = \pm 0{\cdot}01$ cm **1**

Maximum relative error in the density value is, therefore given by

$$\frac{\Delta\rho}{\rho} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta t}{t} + \frac{\Delta m}{m}$$
$$= \frac{0{\cdot}01}{5{\cdot}12} + \frac{0{\cdot}01}{2{\cdot}56} + \frac{0{\cdot}01}{0{\cdot}37} + \frac{0{\cdot}1}{39{\cdot}3}$$
$$= 0{\cdot}0019 + 0{\cdot}0039 + 0{\cdot}027 + 0{\cdot}0024$$
$$= 0{\cdot}0358$$ **1**

Hence, $\rho = 0{\cdot}0358 \times 8{\cdot}1037 \approx 0{\cdot}3 \text{ g cm}^{-3}$.

We cannot, therefore, report the calculated value of ρ (= $8{\cdot}1037$ gm^{-3}) upto the fourth decimal place. Since $\rho = 0{\cdot}3$ g cm^{-3} the value of ρ can be regarded as accurate upto the first decimal place only. Hence the value of ρ must be rounded off as $8{\cdot}1$ g cm^{-3} and the result of measurements should be reported as $\rho = (8{\cdot}1 + 0{\cdot}3)$ g cm^{-3}. **1**

12. **(a)** Bold As the ball is moving under the effect of gravity, the direction of acceleration due to gravity remain vertically downwards. ½

(b) If the ball is at the highest point of its motion, its velocity becomes zero and the acceleration is equal to the acceleration due to gravity = $9{\cdot}8$ ms^{-2} in vertically downward direction. ½

(c) If the highest point is chosen as the location for $x = 0$ and $t = 0$ and vertically downward direction to be the positive direction of x-axis.

For upward motion, sign of position is negative, sign of velocity is negative and the sign of acceleration is positive, *i.e.*, $v < 0$, $a > 0$.

For downward motion, sign of position is positive, sign of velocity is positive and the sign of acceleration is also positive, *i.e.*, $v > 0$, $a > 0$. ½

(d) Suppose, t = time taken by the ball to reach the highest point.

H = height of the highest point from the ground.

During vertically upward motion of the ball,

$\therefore$ Initial velocity, $u = -29{\cdot}4$ ms^{-1}

$a = g = 9{\cdot}8$ ms^{-2}.

Final velocity $v = 0$, $s = \text{H} = ?$, $t = ?$

Applying the relation, $v^2 - u^2 = 2as$, we get

$$0^2 - (29{\cdot}4)^2 = 2 \times 9{\cdot}8 \text{ H}$$

Or $$H = -\frac{29{\cdot}4\times29{\cdot}4}{2\times9{\cdot}8} = -44{\cdot}1 \text{ m}$$ ½

where negative sign indicates that the distance is covered in upward direction.

Using relation $v = u + at$, we get

$$0 = -29{\cdot}4 + 9{\cdot}8\times t$$

$\therefore$ $$t = \frac{29{\cdot}4}{9{\cdot}8} = 3 \text{ s.}$$

i.e., Time of ascent = 3 s. ½

It is also well known that when the object moves under the effect of gravity alone, the time of ascent is always equal to the time of descent.

$\therefore$ Total time after which the ball returns to the player's hand. ½

$$= 2t = 2\times3 = 6 \text{ s.}$$

13. **Step I :** Using $$H = \frac{u^2\sin^2\theta}{2g}$$ ½

when $H = 25$ m, $u = 40$ m/s

and $g = 9{\cdot}8$ m/s^2

$$25 = \frac{40^2\sin^2\theta}{2\times9{\cdot}8}$$ ½

i.e., $$\sin^2\theta = \frac{490}{40^2}$$

$$\sin\theta = \frac{\sqrt{490}}{40} = 0{\cdot}5534$$

i.e., $\theta = 33{\cdot}6°$ **1**

Step 2 : $$R = \frac{u^2\sin2\theta}{g}$$

$$R = \frac{40^2\sin2\,(33{\cdot}6)}{9{\cdot}8}$$

$$= \frac{40^2\sin67{\cdot}2}{9{\cdot}8} = \frac{40^2\times0{\cdot}9219}{9{\cdot}8}$$

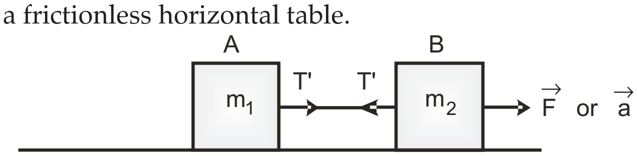
$$= 150{\cdot}514$$ **1**

14. Given, $F = 600$ N

Suppose $m_1 = 10$ kg

and $m_2 = 20$ kg

be the masses lying on a frictionless horizontal table.



Suppose T be tension in the string and 'a' be the acceleration of the system, in the direction of force applied.

(a) If force is applied on the heavier mass.

Then equation of motion of A and B are

$$m_1a = T \quad ....(i)$$

$$m_2a = F - T \quad ....(ii)$$

Dividing equation (ii) by equation (i), we get

$$\frac{m_2}{m_1} = \frac{F - T}{T} = \frac{F}{T} - 1$$ ½

$$\Rightarrow \quad \frac{20}{10} = \frac{F}{T} - 1$$

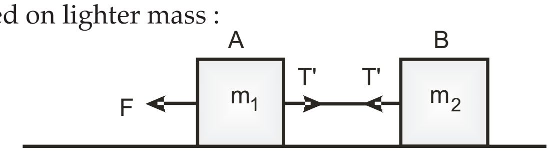
$$\Rightarrow \quad \frac{F}{T} = 2 + 1 = 3$$

$$\Rightarrow \quad T = \frac{F}{3}$$

$$= \frac{600}{3}$$

$$\Rightarrow \quad = 200 \text{ N}$$ ½

(b) If the force is applied on lighter mass :



½

Suppose T be the tension in the string in this case, then equations of motion of A and B are,

$$F - T' = m_1 a \qquad \text{....(iii)}$$

and $$T' = m_2 a \qquad \text{....(iv)}$$ ½

Equations (iii) and (iv) give

$$\frac{F - T'}{T'} = \frac{m_1 a}{m_2 a}$$

$$\Rightarrow \quad \frac{F}{T'} - 1 = \frac{m_1}{m_2} = \frac{10}{20} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{F}{T'} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$\Rightarrow \quad T' = \frac{3}{2}F = \frac{2}{3} \times 600$$

$$= 400 \text{ N}$$ ½

$$\Rightarrow \quad T' = \frac{2}{3} \times 600 = 400 \text{ N}$$ ½

15. If m is mass of the gas molecule and M is the mass of the wall. The K.E. of the molecule before collision

$$E_1 = \frac{1}{2} m (200)^2$$

$$= 2 \times 10^4 \text{ m J}$$ **1**

The total K.E. after collision

$$E_2 = (100^2 + 100^2)\text{m}$$

$$= (2 \times 100^2)\text{m}$$

$$= (2 \times 10^4)\text{m}$$ **1**

Hence, the total kinetic energy of the molecule remains conserved during the collision. Therefore the collision is elastic. **1**

16. Using equation for position vector, $$\vec{r} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

Let $$\vec{r} = 0,$$ **1**

then $$m_1 \vec{r}_1 + m_2 \vec{r}_2 = 0$$

i.e., $$m_2 \vec{r}_2 = - m_1 \vec{r}_1$$

i.e., $$r_2 = -\frac{m_1}{m_2}\vec{r_1}$$ **1**

Clearly, $\vec{r_2}$ and $\vec{r_1}$ are opposite to each other. It indicates when centre of mass lies on origin, m_1 lies on the left side of origin and m_2 on the right side, *i.e.,* they lie on a straight line. **1**

17. As per Kepler's third law, $$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$ **½**

Let 1 denotes the planet and 2 denotes the Earth

$$R_1{}^3 = \frac{T_1^2}{T_2^2}\times R_2^3$$ **1**

As the planet is revolving twice as fast as the Earth,

$$T_1 = \frac{T_2}{2}$$

$\Rightarrow$ $$R_1{}^3 = \frac{\left(\frac{T_2}{2}\right)^2}{T_2^2}\times(R_2)^3$$

$\Rightarrow$ $$R_1{}^3 = \frac{1}{4}(R_2)^3$$

$$R_1 = \left(\frac{1}{4}\right)^{1/3} R_2$$

$$R_1 = \frac{1}{\sqrt[3]{4}}R_2$$

$$R_1 = 0{\cdot}62996\ R_2 \text{ A.U.} = 0{\cdot}63\ R_2 \text{ A.U.}$$ **1**

This result can be stated as the orbital radius of the planet is 0·63 times the orbital radius of the Earth. **½**

18. If the temperature changes from 30°C to 10°C, then change in length of the wire is

$$\Delta l = l\alpha\,\Delta t = l \times 1.7 \times 10^{-5} \times 20$$
$$= 3.4 \times l \times 10^{-4} \text{ m.}$$

Given :

$\rho = 9 \times 10^3$ kgm^{-3}, V = 1.4×10^{11} Nm^{-2}, $\Delta t = 30 - 10 = 20$ °C,

$$\alpha = 1.7 \times 10^{-5}\ °C^{-1}$$

Now $$Y = \frac{F/a}{\Delta l/l}$$

or $$F = Y\,a\frac{\Delta l}{l}$$ **1**

where a = area of cross-section of the wire

or $$F = \frac{1.4\times10^{11}\times a\times l\times3.4\times10^{-4}}{l}$$

$\Rightarrow$ $$F = 4.76 \times 10^7 \times a\text{N.}$$

Now $$v = \sqrt{\frac{T}{m}}$$ **1**

where $$m = a,\ \rho = 9 \times 10^3 \text{ kg m}^{-3}$$

$\Rightarrow$ $$v = \sqrt{\frac{4.76\times10^7\times a}{a\times9\times10^3}}$$

$$v = 72.72$$

So, $$v = 72 \text{ ms}^{-1}$$ **1**

Or

Node (N) is a point where the amplitude of oscillation is zero (and pressure is maximum).
Antinode (A) is a point where the amplitude of oscillation is maximum (and pressure is minimum).
These nodes and antinodes do not coincide with pressure nodes and antinodes. **1**
In fact, N coincides with pressure antinode and A coincides with pressure node, as is clear from the definitions. **1**
The separation between a node and the nearest antinode is $\lambda/4$.
As the phase difference between two points separated by A is 2π radian, therefore phase difference between two points separated by $\lambda/10$ is $2\pi/10 = \pi/5$ radian. **1**

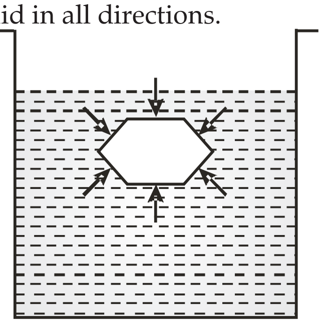
19. It states, that change in pressure is produced in any part of an enclosed fluid, the same is transmitted equally to all points of liquid in all directions. **1**

Or

Ignoring the effect of gravity, pressure in a fluid at rest is the same at all points.

Or

Equal pressure is exerted by a liquid in all directions.



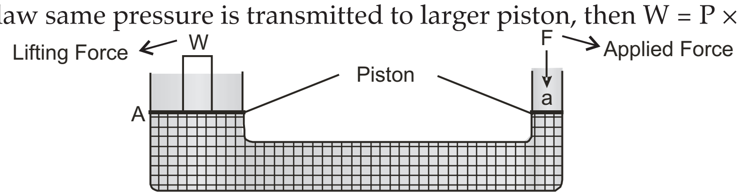
½

Applications : This principle is used to manufacture hydraulic lift. It consist of two cylinders, one of larger area of cross-section A and the other that of smaller area '*a*'. Force is applied to smaller piston to produce a pressure.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

½

As per Pascal's law same pressure is transmitted to larger piston, then W = P × A.



Clearly large area A is producing more lifting force W.
Hydraulic brakes are also based upon Pascal's law where force is pressure is transmitted to brake drum.
The large force then operates the brake shoe. **1**

20. **(a)** Streamline flow of a liquid is that flow in which every particle of the liquid follows exactly the path of its preceding particle and has the same velocity in magnitude and direction as that of its preceding particle while crossing through that point.

The streamline flow is accompanied by streamlines.

A streamline is the actual path followed by the procession of particles in a steady flow, which may be straight or curved such that tangent to it at any point indicates the direction of flow of liquid at the point. **1½**

(b) **Important properties of streamlines :**

(i) In a streamline flow, no two streamlines can cross each other. If they do so, the particles of the liquid at the point of intersection will have two different directions for their flow, which will destroy the steady nature of the liquid flow.

(ii) The greater is the crowding of streamlines at a place, the greater is the velocity of liquid particles at that place and vice-versa. **1½**

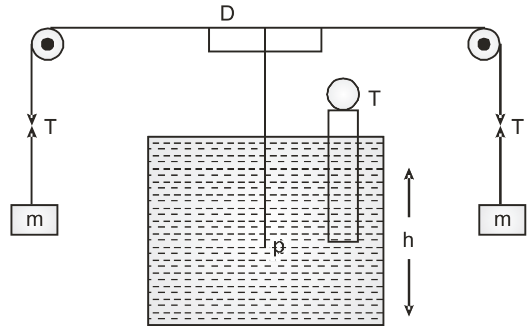
21. Joule observed that amount of mechanical work W when disappeared will appears in the form of heat, *i.e.*,

$$W \propto H$$

$$W = JH$$

Or $$J = W/H = 4{\cdot}18 \text{ Joule/calorie}$$ **1**

He made two equal weights to fall through some heights. This makes the drum D rotate and thus paddle P will push the water inside the calorimeter. **½**



½

This makes the water heated up. The rise in temperature is noted with thermometer T. The height of fall of weights helps in working out the mechanical work W. The temperature helps in calculating the heat energy H. It was observed that,

$$W \propto H.$$ **1**

22. (a) Kelvin's statement : It is not possible to get continuous work done by cooling a body to a temperature lower than that of coldest of its surroundings. **1**

Celsius statement : It is not possible to transfer heat from a body at lower temperature to another at higher temperature without the help of some external energy. **1**

(b) It is defined as the ratio of the amount of heat removed in a cycle from the refrigerator to the work done by some external agency to help the removal of this heat, *i.e.*,

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$ **½**

Also, $$\beta = \frac{T_2}{T_1 - T_2}$$

For higher efficiency of refrigerator β should be higher. **½**

23. (a) Sabita is sincere and hard working and having a scientific temper. **1**

(b) The restoring force F_1 acting on the mass m due to the spring having force constant k_1 will be,

$$F_1 = k_1 x_1$$

Here x_1 is the elongation.

The restoring force F_2 acting on the mass m due to the spring having force constant k_2 will be,

$$F_2 = k_2 x_2$$

Here x_2 is the elongation.

The tension in the two springs will be same.

So, $$k_1 x_1 = k_2 x_2$$

But the total extension is $x_1 + x_2$.

Thus the effective spring constant k of the combination will be,

$$k = \frac{F}{x}$$

$$= \frac{F}{(x_1 + x_2)}$$

$$= \frac{1}{[x_1/F + x_2/F]}$$

$$= \frac{1}{[1/k_1 + 1/k_2]}$$

$$= \frac{k_1 k_2}{k_1 + k_2}$$ **1**

Thus the time period of an object of mass m on a spring executes a simple harmonic motion is given by, $$T = 2\pi\sqrt{\frac{m}{k}}$$ ½

So frequency f on the oscillation would be,

$$f = \frac{1}{T}$$

$$= \frac{1}{\left(2\pi\sqrt{\frac{m}{k}}\right)}$$

$$= \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$ (Since, $T = 2\pi\frac{\sqrt{m}}{k}$) **1**

To find out the frequency of oscillation f, substitute $\frac{k_1 k_2}{k_1 + k_2}$ for the spring constant k in the equation $f = \frac{1}{2\pi}\sqrt{\frac{m}{k}}$,

we get,

$$f = \frac{1}{2\pi}\sqrt{\frac{m}{k}},$$

$$= \frac{1}{2\pi}\frac{\sqrt{\left(\frac{k_1 k_2}{k_1 + k_2}\right)}}{m}$$

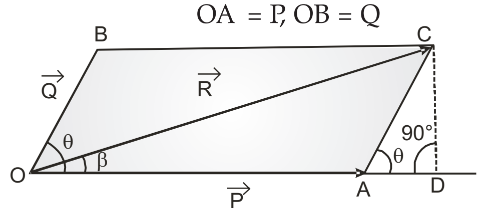
$$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$ **1**

24. It states that, if two vectors can be represented completely (*i.e.*, both in magnitude and direction) by the two adjacent sides of a parallelogram drawn from a point then their resultant is represented completely by its diagonal drawn from the same point. ½

Proof : Let $\vec{P}$ and $\vec{Q}$ be the two vectors represented completely by the adjacent sides OA and OB of the parallelogram OACB *s.t.*, such that,

$$\vec{OA} = \vec{P}, \vec{OB} = \vec{Q}$$

Or $$OA = P, OB = Q$$



½

$$\theta = \angle AOB$$

If R be their resultant, then it will be represented completely by the diagonal OC through point O such that, $$\vec{OC} = \vec{R}.$$

Magnitude of $\vec{R}$: Draw CD ⊥ to OA produced

$$\therefore \quad \angle DAC = \angle AOB = \theta$$

Now in right angled triangle ODC,

$$OC^2 = OD^2 + DC^2$$
$$= (OA + AD)^2 + DC^2$$
$$= OA^2 + AD^2 + 2OA.AD + DC^2$$
$$= OA^2 + (AD^2 + DC^2) + 2OA.OD \quad ....(i)$$

Also in right angled triangle ADC,

$$AC^2 = AD^2 + AC^2 \quad \text{....(ii)}$$

Also, $$\frac{AD}{AC} = \cos\theta$$

Or $$AD = AC\cos\theta \quad \text{....(iii)}$$

and $$\frac{DC}{AC} = \sin\theta$$

Or $$DC = AC\sin\theta \quad \text{....(iv)}$$

∴ From eqns. (i), (ii), (iii), we get

$$OC^2 = OA^2 + AC^2 + 2OA.AC\cos\theta$$

Or $$OC = \sqrt{OA^2 + AC^2 + 2OA\,.AC\cos\theta} \quad \text{....(v)}$$

As $$OC = R,\ OA = P,\ AC = OB = Q \quad \text{....(vi)}$$

∴ From equations (v) and (vi), we get

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \quad \text{....(vii)}$$

Equation (iii) gives the magnitude of $\vec{R}$. **1**

Direction of R : Let β the angle made by $\vec{R}$ with $\vec{P}$.

∴ In right angled triangle ODC,

$$\tan\beta = \frac{DC}{OD} = \frac{DC}{OA + AD}$$

$$= \frac{AC\sin\theta}{OA + AC\cos\theta}, \quad \text{[by using (iii) and (iv)]}$$

$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta} \quad \text{....(viii)}$$

Special cases : (a) When two vectors are acting in same direction, then

$$\theta = 0°$$

∴ $$R = \sqrt{(P+Q)^2}$$

$$= P + Q$$

and $$\tan\theta = \frac{Q.0}{P+Q} = 0$$

Or $$\beta = 0°$$

Thus, the magnitude of the resultant vector is equal to the sum of the magnitudes of the two vectors acting in the same direction and their resultant acts in the direction of P and Q. **1**

(b) When two vectors act in the opposite direction, then

$$\theta = 180°$$

∴ $$\cos\theta = -1 \text{ and } \sin\theta = 0$$

∴ $$R = \sqrt{P^2 + Q^2 + 2PQ(-1)}$$

$$= \sqrt{P^2 + Q^2 - 2PQ}$$

$$= \sqrt{(P-Q)^2} \text{ or } \sqrt{(Q-P)^2}$$

$$= (P - Q) \text{ or } (Q - P)$$

and $$\tan\beta = \frac{Q \times 0}{P + Q(-1)} = 0$$

$$= \tan 0 \text{ or } \tan 180°$$

∴ $$\beta = 0° \text{ or } 180°.$$

Thus, the magnitude of the resultant of two vectors acting in the opposite direction is equal to the difference of the magnitude of two vectors and it acts in the direction of bigger vector. **1**

(c) If θ = 90°, *i.e.*, if $\vec{P} \perp \vec{Q}$, then cos 90° = 0

and $$\sin 90° = 1$$

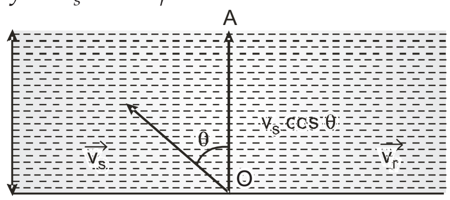
$$\therefore \qquad R = \frac{1}{\sqrt{P^2+Q^2}}$$

and $$\tan\beta = \frac{0}{P}$$ **1**

Or

(a) Let $\vec{v_s}$ and $\vec{v_r}$ be the velocities of swimmer and river respectively.

Let $\vec{v}$ = resultant velocity of v_s an d v_r.



½

(i) Let the swimmer begins to swim at an angle θ with the line OA where OA is ⊥ to the flow of river. If t = time taken to cross the river, then,

$$t = \frac{l}{v_s \cos\theta}$$ ½

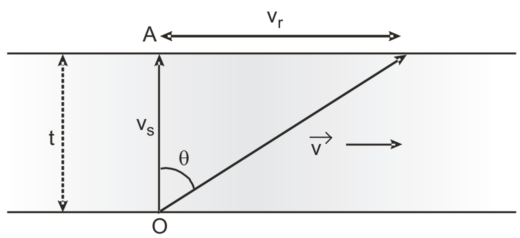
where l = breadth of river

For t to be minimum, cos θ should be maximum,

i.e., $\cos\theta = 1.$

This is possible, if $\theta = 0°.$

Thus, we conclude that the swimmer should swim in a direction ⊥ to the direction of flow of river.



(ii) $$v = \sqrt{v_s^2 + v_r^2}$$ ½

(iii) $$\tan\theta = \frac{v_r}{v_s} = \frac{X}{l}$$

or $$X = l\frac{v_r}{v_s}$$ ½

(iv) $$t = \frac{l}{v_s}$$ ½

(b) It is defined as a vector having zero magnitude and acting in the arbitrary direction. It is denoted by $\vec{0}$. ½

Properties of null vector :

(i) The addition or subtraction of zero vector from a given vector is again the same vector.

i.e., $\vec{A} + 0 = \vec{A} - 0 = \vec{A}$

(ii) The multiplication of zero vector by a non-zero real number is again the zero vector.

i.e., $\pi 0 = 0$ ½

(iii) If $n_1\vec{A} = n_2\vec{B}$, where n_1 and n_2 are non-zero real numbers, then the relation will hold good.

If $\vec{A} = \vec{B} = 0$

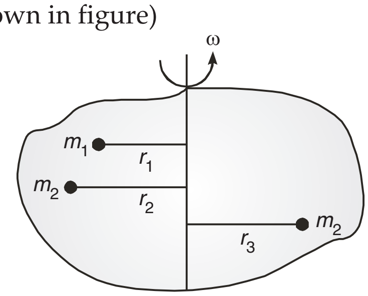
i.e., both $\vec{A}$ and $\vec{B}$ are null vectors. ½

Physical significance of null vector : It is useful in describing the physical situation involving vector quantities.

e.g.
$$\vec{A} - \vec{A} = 0$$
$$\vec{0} \times \vec{A} = 0$$ ½

25. Torque and moment of inertia : Consider a rigid body rotating about a given axis with a uniform angular acceleration a, under the action of a torque. ½

Let the body consist of particles of masses $m_1, m_2, m_3, ..., mn$ at ⊥ distances $r_1, r_2, r_3,...., r_n$ respectively from the axis of rotation (as shown in figure) **1**



As the body is rigid, angular acceleration 'a' of all the particles of the body is the same. However, the linear accelerations of the particles from the axis. differ. If $a_1, a_2, a_3,..., a_n$ are the respective linear accelerations of the particle, then

$$a_1 = r_1\alpha,\ a_2 = r_2\alpha,\ a_3 = r_3\alpha,\ ....$$ **1**

Force on particle of mass m, is

$$f_1 = m_1 a_1 = m_1 r_1 \alpha$$

Moment of this force about the axis of rotation

$$f_1 \times r_1 = (m_1 r_1 \alpha) \times r_1 = m_1 r_1^2 \alpha$$ **1**

Similarly, moment of forces on other particles about the axis of rotation are $m_2 r_2^2\alpha, m_3 r_3^2\alpha, ..., m_n r_n^2\alpha$.

∴ Torque acting on the body,

$$\tau = m_1 r_1^2\alpha + m_2 r_2^2\,\alpha + m_3 r_3^2\,\alpha,...., m_n r_n^2\alpha$$
$$\tau = (m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_n r_n^2)\alpha$$

So,
$$\tau = \left(\sum_{i=1}^{i=n} m_i r_i^2\right)\alpha$$
$$\tau = I\alpha$$ **1**

where $m_1 r_1^2$ = I = moment of inertia of this body about the given axis of rotation.

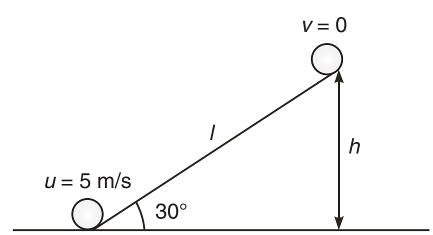
If $\alpha = 1, \tau = I$

$$\vec{\tau} = I\,\vec{\alpha}\,.$$ **1**

Or

If 'a' is the acceleration (*i.e.*, deceleration) in the present case

$$a = \frac{g \sin 30°}{\left(1 + \frac{K^2}{r^2}\right)}$$ **1**



In case of a solid cylinder

$$k = \frac{1}{\sqrt{2}}r$$ ½

$$a = \frac{g\sin 30°}{\left(1+\frac{\frac{1}{2}r^2}{r^2}\right)}$$

$$= \frac{g\frac{1}{2}}{1+\frac{1}{2}} = \frac{g}{3}$$ ½

From $v^2 = u^2 + 2as$

$$0 = (5)^2 + 2\left(\frac{-g}{3}\right)l$$

$$0 = 25 - \frac{2g}{3}l$$

$$l = \frac{25\times 3}{2g} = \frac{75}{2\times 9.8}$$

$$= 3{\cdot}8 \text{ m}$$ **1**

Now time for going up and returning must be equal.

From $s = ut + \frac{1}{2}at^2$,

we get, $l = 0\times t + \frac{1}{2}\left(\frac{-g}{3}\right)t^2$

$$t^2 = \frac{6l}{g} = \frac{6\times 3.8}{9.8}$$

$$t = \sqrt{\frac{6\times 3.8}{9.8}}$$

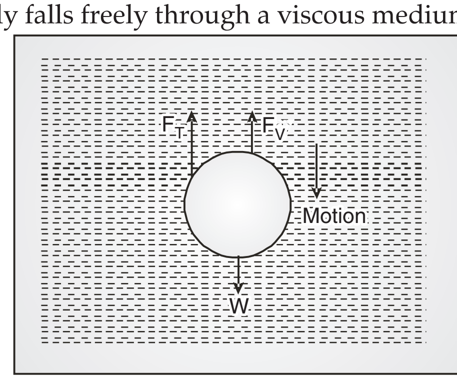
$$= 1{\cdot}5 \text{ sec.}$$ **1**

Total time for going up and coming down $= 2t$

$$= 2\times 1{\cdot}5 = 3 \text{ s}$$ **1**

26. **Terminal velocity :** It is the maximum constant velocity acquired by a body while falling freely in a viscous medium.

When a small spherical body falls freely through a viscous medium, three forces act on it.



(i) Weight of the body acting vertically downwards. ½

(ii) Upward thrust due to buoyancy equal to weight of liquid displaced. ½

(iii) Viscous drag acting in the direction opposite to the motion of body.

According to Stoke's law,

$$F \propto v,$$

i.e., the opposing viscous drag goes on increasing with the increasing velocity of the body. ½

As the body falls through a medium, its velocity goes on increasing due to gravity. Therefore, the opposing viscous drag which acts upwards also goes on increasing. A stage reaches when the true weight of the body is just equal to the sum of the upward thrust due to buoyancy and the upward viscous drag. At this stage, there is no net force to accelerate the body. Hence it starts falling with a constant velocity, which is called terminal velocity. **1**

Let ρ be the density of the material of the spherical body of radius r and ρ_0 be the density of the medium.

$\therefore$ True weight of the body,

$$W = \text{volume} \times \text{density} \times g = \frac{4}{3}\pi r^3 \rho g$$

Upward thrust due to buoyancy,

F_T = weight of the medium displaced

$\therefore$ F_T = volume of the medium displaced × density × g = $\frac{4}{3}\pi r^3 \rho_0 g$ **1**

If v is the terminal velocity of the body, then according to Stoke's law.

$$\text{upward viscous drag } F_V = 6\pi\eta r v$$

When body attains terminal velocity, then

$$F_T + F_V = W$$

$\therefore$

$$\frac{4}{3}\pi r^3 \rho_0 g + 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g$$

Or

$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \rho_0) g$$

Or

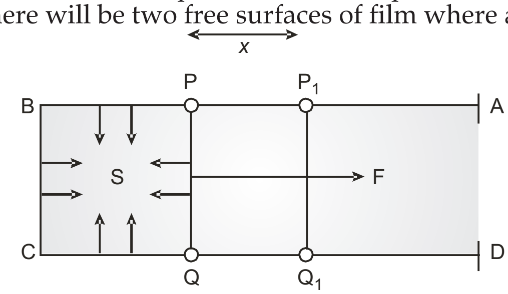
$$v = \frac{2r^2(\rho - \rho_0)g}{9\eta}$$ **1**

Factors it depend upon : The terminal velocity varies directly as the square of the radius of the body and inversely as the coefficient of viscosity of the medium. It also depends upon densities of the body and the medium. ½

Or

Surface energy is defined as the amount of the work done against the force of surface tension, in forming the liquid surface of a given area at a constant temperature. **1**

To obtain a expression for surface energy, take a rectangular frame ABCD having a wire PQ which can slide along the sides AB and CD. Dip the frame in soap solution and form a soap film BCQP on the rectangular frame. There will be two free surfaces of film where air and soap are in contact. ½



Let

S = surface tension of the soap solution.

l = length of the wire PQ. ½

Since there are two free surfaces of the film and surface tension acts on both of them, hence total inward force on the wire PQ is

$$F = S \times 2l$$ ½

To increase the area of the soap film, we have to pull the sliding wire PQ outwards with a force F. Let the film be stretched by displacing wire PQ through a small distance x to the position P_1Q_1.

The increase in area of film PQQ_1 $\quad P_1 = a$

$$= 2(l \times x)$$ ½

$\therefore$ Work done in stretching film is,

$$E = \text{force applied} \times \text{distance moved}$$
$$= (S \times 2l) \times x$$
$$= S \times (2lx)$$
$$= S \times a,$$ ½

where $2lx$ = a = increase in area of the film in both sides. ½

If temperature of the film remains constant in this process, this work done is stored in the film as its surface energy. Thus surface energy = Surface tension × increase in area. **1**

❑❑❑

SOLUTIONS

SAMPLE QUESTION PAPER - 9

Self Assessment

Time : 3 Hours **Maximum Marks : 70**

1. The value of g on the moon is small, therefore, the weight of the moon travellers will also be small. **1**

2. Since there is no water under the block to exert an upward force on it, therefore, there is no buoyant force. **1**

3. At the boiling point, vapour pressure of a liquid is equal to the atmospheric pressure. So, when the atmospheric pressure on the surface of the liquid increases, the liquids boil at higher temperature to generate greater vapour pressure. **1**

4. Since there is no acceleration in the body at the mean position, hence the resultant force on it will be zero, *i.e.*, the force applied by the spring will be exactly equal to the weight of the body. **1**

5. Yes, it is applicable to electromagnetic waves. **1**

6. The dimensions of L.H.S.,

$$F.\ S. = [MLT^{-2}].\ [L] = [ML^2T^{-2}] \qquad \frac{1}{2}$$

The dimensions of R.H.S.,

$$\frac{1}{2}mv^2 \text{ or } \frac{1}{2}mu^2 = [M]\ [LT^{-1}]^2$$

$$= [ML^2T^{-2}] \qquad \frac{1}{2}$$

So, dimensions of L.H.S. = R.H.S

So, the given equation is dimensionally correct. **1**

7.

$$1\ N = 1\ kg \times 1 m/s^2$$
$$= 1000\ gm \times 100\ cm/s^2$$
$$= 10^5\ \ gm\text{-}cm/s^2$$
$$= 10^5 \text{ dyne.} \qquad \mathbf{1}$$

The dyne is a unit of force specified in the centimeter-gram-second (cgs) system of units. One dyne is equal to exactly 10^{-5} newtons. Further, the dyne can be defined as " the force required to accelerate a mass of one gram at a rate of one centimeter per second squared. **1**

Or

$$|\vec{a} - \vec{b}|^2 - (|\vec{a}| + |\vec{b}|)^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}| \qquad \frac{1}{2}$$

$$\cos\theta - |\vec{a}|^2 - |\vec{b}|^2 - 2|\vec{a}||\vec{b}|$$

$$= 4|\vec{a}||\vec{b}|\cos^2\frac{\theta}{2}$$ **1**

= a negative quantity

Hence $$|\vec{a}-\vec{b}| < |\vec{a}|+|\vec{b}|$$ **½**

8. It's the power measured at some instant of time. Since power is energy per unit of time, it follows that

$$\text{Instantaneous power} = \frac{dE}{dt}.$$ **2**

9. Put the given graph for a stress of 150×10^6 Nm^{-2}, the strain is 0·002.

(a) From the formula, Young's modulus of the material (Y) is given by

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{150\times10^6}{0{\cdot}002} = \frac{150\times10^6}{2\times10^{-3}}$$

$$= 75 \times 10^9 \text{ Nm}^{-2}$$

$$= 7{\cdot}5 \times 10^{10} \text{ Nm}^{-2}.$$ **1**

(b) Yield strength of a material is defined as the maximum stress it can sustain without crossing the elastic limit.

$\therefore$ From the graph, the approximate yield strength of the given material

$$= 300 \times 10^6 \text{ Nm}^{-2} = 3 \times 10^8 \text{ Nm}^{-2}.$$ **1**

10. The P. E. of a particle executing S.H.M. is given by :

$$E_P = \frac{1}{2}m\omega^2 y^2.$$ **½**

E_P is maximum when $y = r$ = amplitude of vibration *i.e.*, the particle is passing from the extreme position and is minimum when $y = 0$, *i.e.*, the particle is passing from the mean position. **½**

The K.E. of a particle executing S.H.M. is given by,

$$E_K = m\omega^2(r^2 - y^2)$$

E_K is maximum when y = 0, *i.e.*, the particle is passing from the mean position and E_K is minimum when $y = r$, *i.e.*, the particle is passing from the extreme position. **1**

11. Size or diameter of a planet can be measured with the help of parallax method. Distance D of the planet from the Earth is measured with the help of relation $D = h/\theta$, where h is the distance between two observatories on the Earth and θ is the angle between two directions along which the planet was viewed from two observations. **1**

Let d be the diameter of planet and α the angle subtended by the diameter d at a point E on the Earth, then α being very small, $d/D < 1$. Let AB be an arc (of length d) of a circle with centre E, then distance

$$AB = d = D\alpha$$ **1**

Or $$\alpha = \frac{d}{D}$$

$\therefore$ Diameter, $$d = \alpha D$$ **1**

12. (i) Velocity at $t = 0$, $u = 0$

Velocity at $t = 2$ sec, $v = 20$ m/s

So, from $$v = u + at$$

We get $$a = 10 \text{ m/s}^2$$

So distance covered between 0 to 2 sec.

$$s = ut + \frac{1}{2}at^2$$

$$= 0\times2 + \frac{1}{2}\times10(2)^2$$

$$= 20 \text{ m}$$

From 2 sec to 5 sec, velocity is same 20 m/s.

So, $$\text{distance travelled} = 20 \times 3 = 60 \text{ m}$$ **1**

$\therefore$ Total distance covered between 0 to 5 sec

$$= 20 + 60 = 80 \text{ m}$$ **1**

(ii) At $t = 5$ sec, $u = 20$ m/s,

At $t = 10$ sec, $v = 0$ m/s

So, from $\quad v = u + at$

$\Rightarrow \quad 0 = 20 + a \times 5$

$\Rightarrow \quad a = -4 \text{ m/s}^2$

So, distance covered from $v^2 = u^2 + 2as$,

$$(0)^2 = (20)^2 - 2 \times 4 \times s$$

$$\Rightarrow \quad s = \frac{400}{8} = 50 \text{ m}$$

So, the total distance covered $= 80 + 50$ **1**

$= 130$ m.

13. The nature of a vector may or may not be changed when it is multiplied, by a physical quantity **1**
For example, when a vector is multiplied by a pure number like 1, 2, 3,...., etc., then the nature of the vector does not change. **1**
On the other hand, when a vector is multiplied by a scalar physical quantity, then the nature of the vector changes. For example, when acceleration (vector) is multiplied by mass (scalar) of a body, then it gives force (a vector quantity) whose nature is different than acceleration. **1**

14. Given, F = 50 N, $m_1 = 5$ kg, $m_2 = 10$ kg, $m_3 = 15$ kg.

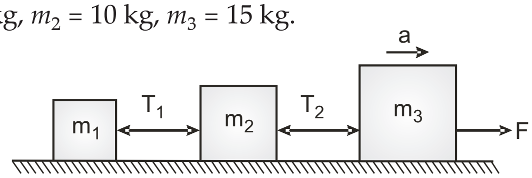


Fig. (a)

Since, the three blocks move with an acceleration 'a'.

So, $$a = \frac{F}{m_1 + m_2 + m_3}$$

$$\Rightarrow \quad a = \frac{50}{5+10+15} = \frac{50}{30}$$

$$= \frac{5}{3} \text{ ms}^{-2}.$$ **1**

To determine T_2 : Imagine the free body diagram (a).

Here $\vec{F}$ and $\vec{T_2}$ act towards right and left respectively.

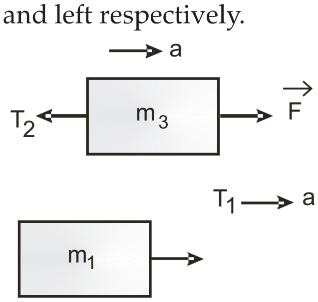


Fig. (b)

Since, the motion is towards the right side, so according to Newton's Second law of motion :

$$F - T_2 = m_3 a$$

$$\Rightarrow \quad 50 - T_2 = 15 \times \frac{5}{3} = 25$$

$$\Rightarrow \quad T_2 = 50 - 25 = 25 \text{ N.}$$ **1**

To determine T_1 : Consider the free body diagram (b).

Here $m_1 a = T_1$

Or $T_1 = m_1 a = 50 \times \frac{5}{3} = \frac{25}{3}$

$= 8.33$ N. **1**

15. Following are some important points about uniform motion :'

(i) The velocity in uniform motion does not depend upon the time interval $(t_2 - t_1)$. ½

(ii) The velocity in uniform motion is independent of the choice of origin. ½

(iii) No net force acts on the object having uniform motion. **1**

(iv) Velocity is taken to be positive when the object moves towards right of the origin and it is taken – ve if object moves toward left of the origin. **1**

16. The vehicle stops when its kinetic energy is spent in working against the force or friction between the tyres and the road. This force of friction varies directly with the weight of the vehicle. **1**

As the K.E. = Work done = Force of friction × distance

$E = f \times d$

Or $d = E/f$

Where, E = Kinetic energy

f = Force **1**

For given kinetic energy, distance d will be smaller, where F is larger, such as in the case of truck. Thus truck stops earlier. **1**

17. Using theorem of perpendicular axes,

$I_z = I_x + I_y$

As I_x and I_y are along the two diameters of disc so using symmetry, **1**

$I_x = I_y$.

So,zz $I_z = 2I_x$, But $I_2 = \frac{MR^2}{2}$ **1**

So, $I_x = \frac{I_z}{2} = \frac{MR^2}{4}$. **1**

Or

Momentum conservation and motion of the centre of the mass.

When a system of n particles is under the action of a total force f, then according to Newton's second law, ½

$$\vec{f} = \sum_{i=1}^{i=n} \frac{d}{dt}(m_i \vec{v_i}) \quad \text{....(1)}$$

$$\vec{f} = \frac{d}{dt}(m_1\vec{v_1} + m_2\vec{v_2} + ... + m_n\vec{v_n}) \quad ½$$

If no external force acts on the system, then total force, $\vec{f} = 0$. From (1) we obtain,

$$\frac{d}{dt}(m_1\vec{v_1} + m_2\vec{v_2} + ... + m_n\vec{v_n}) = 0$$

$$(m_1\vec{v_1} + m_2\vec{v_2} + ... + m_n\vec{v_n}) = k \quad \text{....(2)} \ ½$$

If M is the total mass of the system concentrated at the centre of mass, whose position vector is r, then

$$M\frac{d^2\vec{r}}{dt^2} = \vec{f}$$

Or

$$M\frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \vec{f}$$

But

$$\frac{d\vec{r}}{dt} = v_{c.m.}$$

= velocity of centre of mass of the system

$$\therefore \quad M\frac{d}{dt}(\vec{v}_{c.m.}) = \vec{f} \qquad ....(3)\ ½$$

If no external force acts on the system then $\vec{f} = 0$

From eqn. (3), we get

$$M\frac{d}{dt}(\vec{v}_{c.m.}) = 0 \qquad ½$$

$$M \neq 0$$

$$\therefore \quad \frac{d}{dt}(\vec{v}_{c.m.}) = 0$$

$$\Rightarrow \quad \vec{v}_{c.m.} = \text{constant.} \qquad ½$$

18. The minimum energy required for a satellite to leave its orbit around the Earth and escape to infinity is called the binding energy.

Expression :

A satellite revolving around the Earth has potential energy (P.E.) as well as kinetic energy (k.E.) P.E. due to gravitational field of the Earth and K.E. because of its motion.

$$\Rightarrow \quad \text{P.E.} = -\frac{GM_e m}{Re} \qquad ...(i)\ ½$$

$$\text{K.E.} = \frac{1}{2}\left(\frac{-GM_e m}{Re}\right) \qquad ...(ii)\ ½$$

$$\text{Total energy (T.E.)} = \text{P.E.} + \text{K.E.,}$$

$$= \frac{-GM_e m}{R_e} + \frac{1}{2}\frac{GM_e m}{R_e} \qquad \mathbf{1}$$

$$= \frac{-1}{2}\frac{GM_e m}{R_e} \qquad \mathbf{1}$$

19. (i) $v_{max} = A\omega.$

If amplitude A is doubled, then value of the maximum velocity becomes double. **1**

(ii) Total energy,

$$E = \frac{1}{2}m\omega^2 A^2.$$

If A is doubled, then E becomes four times. **1**

(iii) Time period, $T = 2\pi\sqrt{\frac{m}{k}}$,

since m and k do not change with the change in amplitude of oscillation, hence period of oscillator remains unchanged with change in amplitude of oscillations. **1**

20. Here,

$$y = 2\cos 2\pi(10t - 0{\cdot}0080x + 0{\cdot}35)$$

$$= 2\cos [2\pi(10\,t - 0{\cdot}0080x) + 2\pi\,(0{\cdot}35)]$$

$$= 2\cos\left[2\pi\times0.0080\left(\frac{10}{0.0080}t - x\right) + 2\pi0.35\right] \qquad ½$$

Standard equation for a travelling wave is

$$y = r\cos\left[\frac{2\pi}{\lambda}(vt - x) + \phi\right]$$

Here $\quad \phi = \frac{2\pi}{\lambda}x = 2\pi \times 0{\cdot}008x \qquad \left[\because \frac{2\pi}{\lambda} = 0.008\right]\ ½$

(a) When x = 4 m = 400 cm,

$$\phi = 2\pi \times 0{\cdot}008 \times 400$$
$$= 6{\cdot}4\ \pi \text{ rad.}$$ ½

(b) When x = 0·5 m = 50 cm,

$$\phi = 2\pi \times 0{\cdot}008 \times 50$$
$$= 0{\cdot}8\ \pi \text{ rad.}$$ ½

(c) When $x = \frac{\lambda}{2}$,

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \text{ rad.}$$ ½

(d) When $x = \frac{3}{4}\lambda$,

$$\phi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3}{2}\ \pi \text{ rad.}$$ ½

21. (a) The height of the blood column in the human body is more at the feet than at the brain. That is why, the blood exerts more pressure at the feet than at the brain ($\because$ pressure = $h\rho g$). **1**

(b) We know that the density of air is maximum near the surface of earth and decreases rapidly with height and at a height of about 6 km, it decreases to nearby half its value at the sea level. Beyond 6 km height, the density of air decreases very slowly with height. Due to this reason, the atmospheric pressure at the height of about 6 km decreases to nearly half of its value at sea level. **1**

(c) Since, due to applied force on liquid, the pressure is transmitted equally in all directions inside the liquid. That is why there is no fixed direction for pressure due to liquid. Hence hydrostatic pressure is a scalar quantity. **1**

22. (a) The gas would rush from A to B. The change in pressure or volume will take place under adiabatic conditions. The final pressure in the two cylinders would be 0·5 atm. ½

(b) The change in internal energy of the gas will be zero. ½

(c) The change in temperature will be zero. **1**

(d) Since the process is rapid, the intermediate states are not equilibrium states and hence do not satisfy the gas equation. So, the intermediate states of the system do not lie on the P – V – T surface. **1**

23. (a) Radha takes care of things and has concern for others. Practical in finding the solutions to problems. **2**

(b) When the wheel is rolling, the angular momentum is conserved. However, due to frictional force, it continues to decrease. Thus, the wheel can stay upright on its rim only for a certain interval of time. In the stationary position, the wheel falls due to unstable equilibrium. **2**

24. (a) Using the relation for kinetic energy,

$$\text{K.E.} = \frac{1}{2}\ mv^2$$

we get rate of change of K.E. with respect to time as

$$\frac{d}{dt}(\text{K.E.}) = \frac{d}{dt}\left[\frac{1}{2}mv^2\right]$$
$$= \frac{1}{2}\ m.2\ \frac{dv}{dt}\ v$$
$$= \frac{mdv}{dt}v$$

But $$\frac{mdv}{dt} = ma = F.$$ **1**

where a is acceleration and F is force.

$$\therefore \quad \frac{d}{dt}\text{ K.E.} = F.v.$$

$$= F\frac{dx}{dt}$$

Or $$d(\text{K.E.}) = Fdx$$ **1**

Integrating between the intial and final energies, *i.e.*, K.E. and K.E.$_f$ and also positions, *i.e.*, x_i and x_f respectively, we get

$$\int_{\text{K.E}f}^{\text{K.E.}_i} d\text{K.E.} = \int_{x_i}^{x_f} F.dx$$

$$\therefore \quad \text{K.E.}_f - \text{K.E.}_i = W.$$ **1**

The work energy theorem is verified.

(b) Potential energy will increase. This is because in bringing two protons closer, work has to be done against the force of repulsion. This is stored up in the form of potential energy. **1**

However, the potential energy will decrease when a proton and an electron are brought nearer. Work will be done by the force of attraction between them. **1**

Or

(a) Let mass of the block = m

After breaking, $$m_1 = \frac{2}{5}m \text{ and } m_2 = \frac{3}{5}m$$

Initial momentum, $$P_f = m_1\vec{v}_1 + m_2\vec{v}_2$$ **½**

According to law of conservation of momentum

$$P_f = P_i$$

$$\Rightarrow \quad m_1\vec{v}_1 + m_2\vec{v}_2 = 0$$ **1**

$$\vec{v}_1 = \text{velocity of smaller part}$$

$$\vec{v}_2 = \text{velocity of bigger part}$$

$$\Rightarrow \quad \frac{2}{5}m(8\hat{i}+6\hat{j})+\frac{3}{5}m(\vec{v}_2) = 0$$ **½**

$$\Rightarrow \quad \frac{3}{5}m\vec{v}_2 = -\frac{1}{5}m(16\hat{i}+12\hat{j})$$

$$\vec{v}_2 = -\left(\frac{16}{3}\hat{i}+4\hat{j}\right)$$ **1**

(b) As energy associated with discharge of a single neuron is 10^{-10} J, therefore total energy in a nerve impulse, where 10^5 neurons are fired is $10^{-10} \times 10^5$ J = 10^{-5} J. **2**

25. **(a)** When we blow over a piece of paper, velocity of air above the paper becomes more than that below it. Since, K.E. of air above the paper increases, so in accordance with Bernoulli's theorem $P+\frac{1}{2}\rho v^2$ = constant), its pressure energy and hence its pressure decreases. Because of greater value of pressure below the piece of paper – atmospheric pressure, it remains horizontal and does not fall.

While we blow under the paper, the pressure on the lower side decreases. The atmospheric pressure above the paper will therefore bend the paper downward. So the paper will not remain horizontal. **1**

(b) This can be cleared from the equation of continuity, *i.e.*, $a_1v_1 = a_2v_2$. We try to close a water tap with our fingers, the area of cross-section of the outlet of water jet is reduced considerably to the openings between our fingers provide constriction (*i.e.*, regions of smaller area). Hence, velocity of water increases greatly and fast jets of water come through the openings between our fingers. **1**

(c) From Bernoulli's theorem, we know that

$$P + \frac{1}{2}\rho v^2 + gh = \text{constant} \qquad \text{....(i)}$$

Here the size of the needle controls the velocity of flow and the thumb pressure controls pressure.

Now P occurs with power 1 and velocity (v) occuring with power 2 in equation (i) therefore, the velocity has more influence. That is why the needle of syringe has a better control over the flow rate. **1**

(d) If a fluid is flowing out of a small hole in a vessel, it acquires a large velocity and hence possesses large momentum. Since no external force is acting on the system, a backward velocity must be attained by the vessel (according to the law of conservation of momentum). As a result of it, an impulse (backward thrust) is experienced by the vessel. **1**

(e) This is because of Magnus effect, let a ball moving to the right be given a spin at the top of the ball. The velocity of air at the top is higher than the velocity of air below the ball. So according to Bernoulli's theorem, the pressure above the ball is less than the pressure below the ball. Thus there is a net upward force on the spinning ball, so the ball follows a curved path. This effect is known as magnus effect. **1**

Or

Given, Atmospheric pressure, $P_0 = 76$ cm of Hg.

(a) In figure (a) pressure head,

$$h_1 = 20 \text{ cm of Hg.}$$

$\therefore$ Absolute pressure (P) of the gas is greater than the P_0,

i.e,
$$\begin{aligned} P &= P_0 + h_1\rho g \\ &= 76 \text{ cm of Hg} + 20 \text{ of Hg} \\ &= 96 \text{ cm of Hg.} \end{aligned}$$ **1**

Gauge pressure is the difference between the absolute pressure and the atmospheric pressure. ½

It means,
$$\begin{aligned} \text{Gauge pressure} &= P - P_0 \\ &= 96 \text{ cm of Hg} - 76 \text{ cm of Hg} \\ &= 20 \text{ cm of Hg} \end{aligned}$$

In figure (b), pressure head,

$$h_2 = -18 \text{ cm of Hg.}$$ ½

$\therefore$ The absolute pressure of the gas (is lesser than the atmospheric pressure) is given by

$$\begin{aligned} P &= P_0 + h_2\rho g \\ &= 76 \text{ cm of Hg} + (-18 \text{ cm}) \text{ of Hg} \\ &= 58 \text{ cm of Hg} \end{aligned}$$

$$\begin{aligned} \text{Gauge pressure} &= \text{absolute pressure} - \text{atmoshperic pressure} \\ &= 58 \text{ cm of Hg} - 76 \text{ cm of Hg} \\ &= -18 \text{ cm of Hg} \end{aligned}$$ ½

It means, gauge pressure is simply equal to h cm of Hg. ½

(b) Given 13·6 cm of water added in the right limb is equivalent to $\frac{13.6}{13.6} = 1$ cm of Hg column.

i.e., h= 1 cm of Hg column, which can be calculated as follows,

$$h_w = 13{\cdot}6 \text{ cm of water}$$

Suppose h_m = height of Hg column equivalent to 13·6 cm of water, thus equilibrium,

$$h_m\rho_m g = h_w\rho_w g$$ ½

Or $$h_m = h_w \frac{\rho_w}{\rho_m} = \frac{h_w}{\left(\frac{\rho_m}{\rho_w}\right)}$$

$$= \frac{13.6}{13.6} = 1 \text{ cm of Hg.}$$ ½

The mercury will rise in the left limb such that the difference in the height of Hg column in the two limbs

$$= (20 - 1) \text{ cm}$$
$$= 19 \text{ cm of Hg column.}$$ **1**

26. (a) It states that if the pressure remains constant, then the volume of a given mass of a gas increases or decreases by its volume at 0°C for each 1°C rise or fall of temperature.

Let V_0 be the volume of the given mass of gas at °C. According to Charle's law its volume at 1°C, **1**

$$V_1 = V_0 + \frac{V_0}{273.15}$$

$$V_0 = \left[1 + \frac{1}{273.15}\right]$$ **1**

Volume of the gas at 2°C,

$$V_2 = V_0\left[1+\frac{2}{273.15}\right]$$

$\therefore$ Volume of the gas at t°C,

$$V_1 = V_0\left[1+\frac{t}{273.15}\right]$$

$$= V_0\left[\frac{273.15 + t}{273.15}\right]$$ **1**

If T_0 and T are temperatures on kelvin scale corresponding to 0°C and t°C, then

$$T_0 = 273{\cdot}15 + 0 = 273{\cdot}15$$
$$T = 273{\cdot}15 + t$$

$$V_t = V_0 \frac{T}{T_0}$$

$$\frac{V_t}{T} = \frac{V_0}{T_0}$$

$$\frac{V}{T} = \text{constant}$$

i.e., $$V \propto T.$$ **1**

(b) This equation give the relation between pressure P, volume V and absolute temperature T of a gas,

$$PV = nRT.$$ ½

Derivation. According to Boyle's law,

$$V \propto \frac{1}{P} \quad \text{....(1)}$$

According to Charle's law,

$$V \propto T \quad \text{....(2)}$$

Comparing (1) and (2), we have

$$\frac{PV}{T} = \text{constant}$$

As $$PV = RT.$$

For n moles of gas $\quad PV = nRT$ ½

This is perfect or ideal gas equation.

Or

(a) Numerical value of R : Consider one mole of a gas at S.T.P., then

$$R = \frac{P_0 V_0}{T_0}$$ ½

Standard pressure, $P_0 = 0{\cdot}76$ m of Hg column

$$= 0{\cdot}76 \times 13{\cdot}6 \times 10^3 \times 9{\cdot}8 \text{ N/m}^2$$

Standard temperature $= T_0 = 273{\cdot}15$ K

Volume of one mole of gas at $= 22{\cdot}4 \times 10^{-3}$ m^3

$$R = \frac{0.76 \times 13.6 \times 10^3 \times 9.8 \times 22.4 \times 10^{-3}}{273.15}$$

$$= 8{\cdot}01 \text{ J mole}^{-1} \text{ K}^{-1}.$$ **1**

In the C.G.S. system,

$$R = \frac{8.31}{4.2} \text{ cal mole}^{-1} \text{ °C}^{-1}$$

$$= 1{\cdot}98 \text{ cal mole}^{-1} \text{ °C}^{-1}.$$ ½

Numerical value of K_B : We know that

$$K_B = \frac{R}{NA}$$

$$K_B = \frac{8.31 \text{ J mole}^{-1} \text{ K}^{-1}}{6.02 \times 10^{23} \text{ mole}^{-1}}$$ ½

$$= 1{\cdot}38 \times 10^{-23} \text{ J/K}.$$ ½

(b) Suppose a polyatomic gas molecule has n degrees of freedom.

Total energy associated with the gram molecule of gas,

$$E = n \times \frac{1}{2}RT = \frac{n}{2}RT$$

i.e.,

As $$C_v = \frac{dE}{dT}$$

$\therefore$ $$C_v = \frac{d}{dT}(E)$$ ½

$$= \frac{d}{dT}\left(\frac{n}{2}RT\right)$$

$$= \frac{n}{2}R$$

As $$C_P = C_V + R$$

$$C_P = \frac{n}{2}R + R$$

$$= \left(\frac{n}{2}+1\right)R$$ ½

As $$\gamma = \frac{C_P}{C_V}$$

$$\gamma = \frac{\left(\frac{n}{2}+1\right)R}{\frac{n}{2}R} = \frac{2}{n}\left(\frac{n}{2}+1\right)$$

$$\gamma = 1 + \frac{2}{n}.$$ **1**

❑❑❑

SOLUTIONS

SAMPLE
QUESTION PAPER - 10

Self Assessment

Time : 3 Hours **Maximum Marks : 70**

1. Its value remains same as $6{\cdot}67 \times 10^{11}$ Nm^2 kg^{-2} as it is a universal constant and it does not depend on value of *g*. **1**

2. Compressibility is the reciprocal of the bulk modules, *i.e.*, compressibility = 1/K. **1**

3. **(a)** PV = constant, since $\Delta T = 0$ ½

(b) $W = 2{\cdot}303\, nRT \log \left(\frac{V_f}{V_i}\right)$ ½

4. It gives an evidence for the existence of atoms. **1**

5. Yes, when a ball is dropped from a height on a perfectly elastic plane surface, the motion of a ball is oscillatory but not simple harmonic. **1**

6. The conventional rules are :

(i) If the insignificant digit to be dropped is more than 5 the preceeding digit is increased by 1 but if it is less than 5 then proceeding digits is not changed, *e.g.*, 1·748 is rounded off to 3 significant figures as 1·75 and 1·742 as 1·74. **1**

(ii) If the insignificant digit to be dropped is 5 then this digit is simply dropped, if the prceeding digit is even but if odd then the preceeding digit is increased by 1.

e.g., the number 1·845 rounded off to three significant digits is 1·84 but for number 1·875 it is 1·88. **1**

7. In one dimensional motion, the initial instant of time or zero time (t = 0) is considered at the beginning of the observation of motion. It may not be the instant of beginning of motion. Suppose a ball is thrown from a height of 500 m. It becomes visible say at a height of 100 m. Then origin or initial instant of time or zero time (t = 0) is this instant only and not the instant when motion actually began. **1**

Thus, if zero instant is considered as present then times, before the instant is past and time after this instant is future. **1**

8. Let θ is the angle between vectors $\vec{A}$ and $\vec{B}$

$\therefore$ $$\vec{A} \cdot \vec{B} = AB\cos\theta \quad \text{....(i)}$$ ½

and $$\vec{B} \cdot \vec{A} = BA\cos\theta \quad \text{....(ii)}$$ ½

From eqns. (i) and (ii), we get

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$ **1**

Or

In this case force $\vec{F}$ refers to the total external force, on the system (internal forces in the system are not included in $\vec{F}$) and $\vec{a}$ refers to the acceleration of the whole system, *i.e.*, acceleration of the centre of mass of the system. **2**

9. **(a)** The rotating wheels of a bicycle possess angular momentum. In the absense of an external torque, neither the magnitude nor the direction of angular momentum can change. The direction of angular momentum is along the axis of the wheel. So, the bicycle does not get tilted. **1**

(b) The cycle wheel is constructed in such a way so as to increase the M.I. of the wheel with minimum possible mass, which can be achieved by using spokes and the M.I. is increased to ensure the uniform speed. **1**

10. **(i)**
$$\sin \omega t - \cos \omega t = \sqrt{2}\left[\sin \omega t \cos \frac{\pi}{4} - \cos \omega t \sin \frac{\pi}{4}\right]$$
$$= \sqrt{2} \sin\left(\omega t - \frac{\pi}{4}\right) \qquad \tfrac{1}{2}$$

This function represents S.H.M. and periodic having period
$$T = \frac{2\pi}{\omega} \text{ and initial phase} = -\frac{\pi}{4} \text{ rad.} \qquad \tfrac{1}{2}$$

(ii)
$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$$
$$= \frac{1}{2} - \frac{1}{2}\cos 2\omega t. \qquad \tfrac{1}{2}$$

The function is periodic, having a period,
$$T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

but does not represent S.H.M. ½

11. It is the indirect method of measuring the distances of the order of 10^{-10} m which is the size of an atom, *i.e.*, small distances. An atom is a tiny sphere. When such atoms lie packed in any substance, empty spaces are left in between. According to Avogadro's hypothesis, volume of all atoms in one gram of substance is 2/3 of the volume occupied by one gram of the substance.
$$V' = \frac{2}{3} V \qquad \text{....(i)}$$

V = Actual volume of one gram mass.
V' = Volume occupied by atoms in 1 gram mass.
ρ = Density of the substance
$$V = \frac{1}{\rho} \qquad \text{....(ii)} \quad \mathbf{1}$$

Let M = atomic weight of the substance
N = Avogadro's no.

$\therefore$ No. of atoms in 1 gm of the substance $= \dfrac{N}{M}$

If r be the radius of each atom, then

V' = no. of atoms in gm × volume of each atom
$$V' = \frac{N}{M} \times \frac{4}{3}\pi r^3 \qquad \text{....(iii)} \quad \mathbf{1}$$

$\therefore$ From eqns. (i), (ii) and (iii), we get
$$\frac{N}{M} \times \frac{4}{3}\pi r^3 = \frac{2}{3} \times \frac{1}{\rho}$$

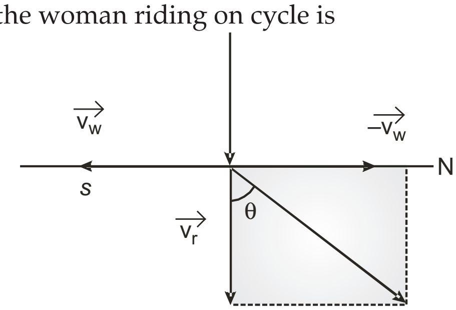
$$r = \left(\frac{M}{2\pi N\rho}\right)^{1/3}$$ **1**

12. Figure (a) does not represent one dimensional motion of particle because the particle has two different positions at the same instant which is not the case of one dimensional motion. Figure (d) also does not represent one dimensional motion of the particle because here the total path length is shown to decrease with time which is not possible in one dimensional motion. **1**

Graph (b) does not represent one dimensional motion because at the same instant a particle cannot have positive and negative velocity if the motion is one dimensional. **1**

Graph (c) does not represent one dimensional motion because speed cannot be negative as shown in graph. **1**

13. Velocity of rain relative to the woman riding on cycle is



1

$$\vec{v}_{rw} = \vec{v}_r - \vec{v}_w$$

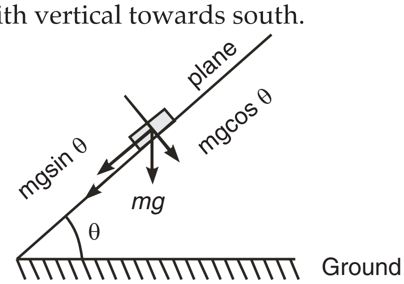
$$\tan\theta = \frac{v_w}{v_r} = \frac{10}{30} = \frac{1}{3}$$ **1**

$$= 0{\cdot}3333$$

$$\theta = 18°26'$$ **½**

∴ She should hold her umbrella with vertical towards south. **½**

14.



½

Given :

$$\text{Angle of sliding} = 20°$$

$$\mu = \text{coefficient of friction}$$
$$= \tan 20°$$
$$= 0.3647$$
$$s = 1.2 \text{ m}$$

Suppose, a is the acceleration when the inclination is increased to 30° and F be the value of limiting friction, then

$$ma = mg\sin\theta - F$$ **1**

Now $$F = \mu R = \mu mg\cos\theta$$

∴ From equation (1),

$$ma = mg\sin\theta - \mu mg\cos\theta$$ **½**

or $$a = g\sin\theta - \mu g\cos\theta$$

⇒ $$a = 9.8\,(\sin 30° - 0.3647 \times \cos 30°)$$

⇒ $$a = 9.8 \times \left(\frac{1}{2} - 0.3647 \times \frac{\sqrt{3}}{2}\right)$$

$\Rightarrow$ $a = 1.81 \text{ ms}^{-2}$

$\therefore$ Downward force,

$$F' = ma$$
$$= 10 \times 1.81 = 0.81 \text{ N}$$

Work done, $W = Fs = 1.81 \times 1.2$

$= 21.72$ J. **1**

15. We are given that

T_{max} = maximum tension in the string so that it does not break

$= 2 \text{ kg } \omega t = 2 \times 10 \text{ N} = 20 \text{ N}$

Let T_1 be the tension in the string when the stone is in its lowest position of its circular path. We know that

$$T_1 = \frac{mv_1^2}{l} + mg. \quad \textbf{1}$$

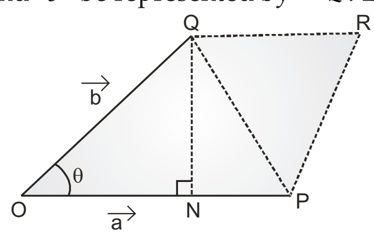
T_1 would have its minimum value when v_1 equal its minimum value = $\sqrt{5gl}$, needed by the stone, to complete its vertical circular path. **1**

Hence, $(T_1) \text{ min} = mv_{\text{min}}^2 + mg$

$$= 6mg$$
$$= 6 \times 0{\cdot}4 \times 10$$
$$= 24 \text{ N}$$

We thus see that $(T_1)_{min}$ is more than the breaking strength of the string. Hence the particle cannot describe the vertical circle. **1**

16. Let $\vec{a}$ be represented by $\overrightarrow{OP}$ and $\vec{b}$ be represented by $\overrightarrow{OQ}$. Let $\angle POQ = \theta$



Complete the parallelogram OPRQ. Join PQ and draw QN $\perp$ OP.

In ΔQNO, $\sin\theta = \frac{QN}{OQ} = \frac{QN}{b}$

$$QN = b \sin\theta$$

Now, by definition,

$$|\vec{a} \times \vec{b}| = ab \sin\theta$$
$$= (OP)(QN)$$
$$= \frac{2(OP)(QN)}{2}$$
$$= 2 \times \text{area of } \Delta OPQ$$

$\therefore$ Area of $\Delta OPQ = \frac{1}{2}|\vec{a} \times \vec{b}|$ **1**

17. **(a)** The linear speed ($v = \omega R$) changes because the distance, *i.e.*, (R) of the comet from the Sun changes due to it elliptical orbit around the Sun. ½

(b) The angular speed of the comet also changes because it covers different angle in equal intervals of time. ½

(c) The angular momentum of the comet is same throughout due to the conservation of angular momentum in the absence of any torque. ½

(d) Kinetic energy changes because linear speed is different at different points. ½

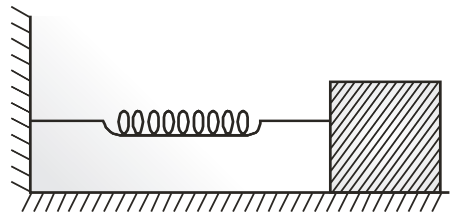
(e) The potential energy at different points is different because the comet is not at the same distance from the Sun (the orbit is not circular). ½

(f) The total energy of comet remains the same throughout the motion. ½

18. **(i)** Using , $f = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$, we get

$$f = \frac{1}{2\pi}\sqrt{\frac{1200}{3}}$$

$$= 3{\cdot}18 \text{ s}^{-1}$$



(ii) Maximum acceleration $= r\omega^2$ **1**

$$= r(2\pi f)^2$$
$$= 4\pi^2 f^2 r$$
$$= 4\pi^2 (3{\cdot}18)^2 \times 0{\cdot}02$$
$$= 7{\cdot}98 \text{ ms}^{-2}.$$ **1**

(iii) Maximum speed $= r\omega$

$$= r \times 2\pi f$$
$$= 2\pi \times 3{\cdot}18 \times 0{\cdot}02$$
$$= 0{\cdot}04 \text{ ms}^{-1}$$ **1**

Or

According to Newton's law, speed of sound,

$$v = \sqrt{\frac{K_{isothermal}}{\rho}} \quad ...(i)$$

where $K_{isotermal}$ = pressure P for isothermal change

$$v = \sqrt{\frac{P}{\rho}} \quad ...(ii)$$ **1**

where $P = 76$ cm of Hg

$= 76 \times 13{\cdot}6 \times 980$ dyne cm^{-2}

and $\rho = 1{\cdot}293 \times 10^{-3}$ g cm^{-3}

$\therefore$ v at N.T.P. $= 280$ ms^{-1} (from ii)

Laplace suggested that $K_{isothermal}$ should be replaced with $K_{adiabitic}$.

$$v = \sqrt{\frac{K_{adiabitic}}{\rho}}$$

Here $PV^\gamma = \text{constant}$ **1**

Differentiating, we have

$$P\gamma V^{\gamma-1}\, dV + V^\gamma\, dP = 0$$

i.e., $\gamma P dV = -V dP$

Or $\frac{dV}{V} = -\frac{dP}{\gamma P}$

But $K_{adiabitic} = \frac{dp}{dv/v} = \gamma P$

$\therefore$ $v = \sqrt{\frac{\gamma P}{\rho}}$

$$v_{\text{N.T.P.}} = \sqrt{\frac{(1\cdot41)(76\times13\cdot6\times980)}{1\cdot293\times10^{-3}}}$$

$= 333\ \text{ms}^{-1}$. **1**

19. Given, $Y = 12\cdot5 \times 10^{11}$ dyne cm^{-2}

$= 12\cdot5 \times 10^{10}\ \text{Nm}^{-2}$

Diameter, $D = 2\cdot5\ \text{mm} = 2\cdot5 \times 10^{-3}\ \text{m}$

$\therefore$ Radius, $r = \frac{D}{2} = 1\cdot25 \times 10^{-3}$ m.

$F = 100$ kgf

$= 100 \times 9\cdot8$ N

$= 980$ N

$$\frac{\Delta L}{L}\times100 = ?$$

$A = \pi r^2 = \pi\,(1\cdot25 \times 10^{-3})^2\ \text{m}^2$.

$$Y = \frac{F/A}{\Delta L/L}$$ **1**

$$\%\ \text{increase in length} = \frac{\Delta L}{L}\times100$$

$$= \frac{F}{AY}\times100$$

$$= \frac{F}{\pi r^2\,Y}\times100$$

$$= \frac{980}{3\cdot142\times(1\cdot25\times10^{-3})^2\times12\cdot5\times10^{10}}\times100$$

$= 15\cdot96 \times 10^{-2}$

$= 0\cdot16\%$ **1**

20. Let a be the area of cross-section of the rod, V be the volume of mass attached, when rod is floating on water, then

weight of float = weight of water displaced

$= (a \times 3 + V) \times 1 \times g$ ....(1) ½

When rod is floating in a liquid.

Weight of liquid displaced $= (a \times 3\cdot15 + V)\ 0\cdot9 \times g$ ....(2)

Or $3a + V = 3\cdot15\,a + 0\cdot9\,V$

Or $V = 1\cdot5\,a$ ½

Let x be the depth of the rod immersed in a liquid of sp. gravity 1·2, then weight of liquid displaced

$= (xa + V)1\cdot2 \times g$ **1**

$\therefore$ $(xa + V)\ 1\cdot2 \times g = (a \times 3 + V)\ g$

$(xa + 1\cdot5\,a) \times 1\cdot2 = 3\,a + 1\cdot5\,a = 4\cdot5\,a$

Or $1\cdot2\,x + 1\cdot8 = 4\cdot5$

$x = 2\cdot25$ cm. **1**

21. Using Newton's Law of Cooling :

Rate of loss of heat = (K)* (difference in temperature of surrounding and body)

Taking average of 80 °C and 50 °C *i.e.,* 65 ° C as the temperature of body.

$$\frac{(80-50)}{5\,\text{min}} = K \times (65 \times 20)$$

$$6 = K \times 45$$

This gives the value of $K = \frac{6}{45}$ **1**

Now assume that the second change takes about "t min."

And we see that the avg. temp of body now is $\frac{60+30}{2} = \frac{90}{2} = 45$

from Newton's Law,

$$\Rightarrow \quad \frac{(60-30)}{t} = \frac{6}{45} \times (45-20)$$

Solving the above for t gives $t = 9$ min. **1**

22. For using the internal energy of sea water to operate the engine of a ship, the internal energy of the sea water has to be converted into mechanical energy. The whole of the internal energy cannot be converted into mechanical energy, a part has to be rejected to a colder body (sink). Since, no such body is available, the internal energy of the sea water cannot be used to operate the engine of the ship. **3**

23. **(a)** Adarsh is hardworking, thinks logically, having scientific temper, able to find solutions with patience. **2**

(b) Since, length of the pendulum l is proportional to g, the length of the pendulum on the surface of the moon will be 1/6 m. **2**

24. **Proof.** When we can show that Newton's first and third laws are contained in the second law, then we can say that it is the real law of motion.

(i) **First law is contained in second law :** According to Newton's second law of motion,

$$\vec{F} = m\vec{a} \qquad \text{....(1)}$$

where m = mass of the body on which an external force $\vec{F}$ is applied and a is acceleration produced in it.

When no external force is applied on the body, *i.e.*,

When $\vec{F} = 0$, then from equation (1), we get

$$m\vec{a} = 0, \text{ but as } m \neq 0$$

$$\therefore \quad \vec{a} = 0$$ **½**

It means that there will be no acceleration in the body if no external force is applied. This represents that a body at rest will remain at rest and a body in uniform motion will continue to move along the same straight line in the absence of an external force. This corresponds to Newton's first law of motion. So, first law of motion is contained in second law of motion. **½**

(ii) **Third law is contained in second law :** Consider an isolated system of two bodies A and B. Let they interact enternally.

Suppose,

F_{AB} = force applied on body A by body B.

and F_{BA} = force applied on body B by body A.

When $\frac{d\vec{P}_A}{dt}$ = rate of change of momentum of body A.

and $\frac{d\vec{P}_B}{dt}$ = rate of chage of momentum of body B.

Then, from Newton's second law of motion,

$$F_{AB} = \frac{d\vec{P}_A}{dt} \qquad \text{....(2)}$$

$$F_{BA} = \frac{d\vec{P}_B}{dt} \qquad ....(3) \quad \mathbf{1}$$

Equations (2) and (3) give,

$$\vec{F}_{AB}+\vec{F}_{BA} = \frac{d}{dt}(\vec{P}_A)+\frac{d}{dt}(\vec{P}_B) = \frac{d}{dt}(\vec{P}_A+\vec{P}_B) \qquad \mathbf{1}$$

Since no external force acts on the system ($\because$ it is isolated), therefore according to Newton's second law of motion,

$$\frac{d}{dt}(\vec{P}_A+\vec{P}_B) = 0$$

Or $$\vec{F}_{AB}+\vec{F}_{BA} = 0$$

Or $$\vec{F}_{AB} = -\vec{F}_{BA}$$

Action force = – Reaction force. **1**

It means that action and reaction are equal and opposite. It is the statement of Newton's third law of motion. Thus, 3rd law is contained in the second law of motion.

As both first and third laws are contained in second law, so second law is the real law of motion. **1**

Or

(a) Impulse-momentum theorem states that the impulse of force on a body is equal to the change in momentum of the body.

i.e., $$\vec{J} = \vec{F}\,t = \vec{P}_2-\vec{P}_1 \qquad \mathbf{1}$$

Proof. According to Newton's second law of motion, we know that

$$\vec{F} = \frac{d\vec{P}}{dt}$$

Or $$\vec{F}\,dt = d\vec{P} \qquad ....(i) \quad \tfrac{1}{2}$$

When $\vec{F}$ = constant force acting on the body.

Suppose $\vec{P}_1$ and $\vec{P}_2$ be the linear momentum of the body at time t = 0 and t respectively.

$\therefore$ Integrating equation (i) within these limits, we get

$$\int_0^t \vec{F}\,dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{P} \qquad \tfrac{1}{2}$$

$\Rightarrow$ $$F\int_0^t dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{P}$$

$$\vec{F}\,[t]_0^t = [P]_{p_1}^{p_2}$$

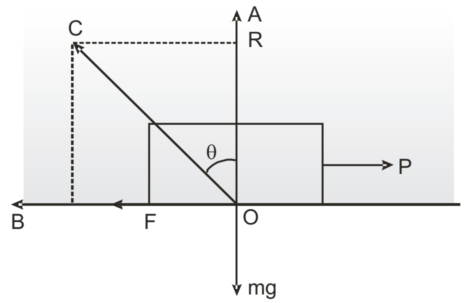
$$\vec{F}\,t = \vec{P}_2-\vec{P}_1$$

$$\vec{J} = \vec{P}_2-\vec{P}_1\,. \qquad \mathbf{1}$$

(b) The coefficient of friction between any two surfaces in contact is defined as the ratio of the force of limiting friction and normal reaction between them.

$$\mu = \frac{F}{R} \qquad \tfrac{1}{2}$$

Angle which the resultant of force of limting friction F and normal reaction R makes with the direction of normal reaction R is the angle of reaction. ½



Relation in Δ AOC

$$\tan\theta = \frac{AC}{OA}$$

$$= \frac{OB}{OA}$$

$$= \frac{F}{R} = \mu$$

Hence $\mu = \tan\theta.$ **1**

25. Suppose two balls A and B of masses m_1 and m_2 are moving initially along the same striaght line with velocities u_1 and u_2 respectively.

When $u_1 > u_2$, relative velocity of approach before collision,

$$= u_1 - u_2$$ ½

When the two balls collide, let the collision be perfectly elastic. After collision, suppose v_1 is velocity of A and v_2 is velocity of B along the same straight line, When $v_2 - v_1$, relative velocity seperation after collision,

$$= v_2 - v_1$$ ½

Linear momentum of the two balls before collision,

$$= m_1u_1 + m_2u_2$$ ½

Linear momentum of the two balls after collision,

$$= m_1v_1 + m_2v_2$$ ½

As linear momentum is conserved in an elastic collision, therefore ½

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 \quad ....(i)$$

Or $$m_2(v_2 - u_2) = m_1(u_1 - v_1) \quad ....(ii)$$

Total K.E. of the two balls before collision

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \quad ....(iii)$$

Total K.E. of the two balls after collision

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad ...(iv)$$ ½

As K.E. is also conserved in an elastic collision, therefore from (iii) and (iv),

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_2^2 + \frac{1}{2}m_2u_2^2$$ ½

Or $$\frac{1}{2}m_2(v_2^2 - u_2^2) = \frac{1}{2}m_1(u_1^2 - v_1^2)$$

Or $$m_2(v_2{}^2 - u_2{}^2) = m_1(u_1{}^2 - v_1{}^2) \quad ....(v)$$ ½

Dividing (v) by (ii), we get

$$\frac{m_2(v_2^2 - u_2^2)}{m_2(v_2 - u_2)} = \frac{m_1(u_1^2 - v_1^2)}{m_1(u_1 - v_1)}$$

$$\frac{(v_2 + u_2)(v_2 - u_2)}{(v_2 - u_2)} = \frac{(u_1 + v_1)(u_1 - v_1)}{(u_1 - v_1)}$$

Or $$v_2 + u_2 = u_1 + v_1$$

Or $$v_2 - v_1 = u_1 - u_2 \quad \text{....(vi)} \quad ½$$

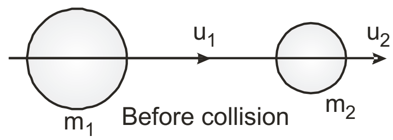
Hence in one dimensional elastic collision, relative velocity of separation after collision is equal to relative velocity of approach before collision.

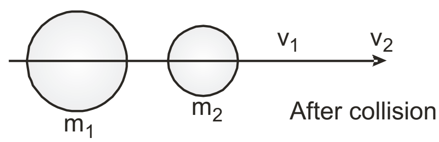
From, $$\frac{v_2 - v_1}{u_1 - u_2} = 1$$

By definition, $$\frac{v_2 - v_1}{u_1 - u_2} = e = 1. \quad ½$$

Or

Figure shows two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 respectively, along a single axis. They collide involving some loss of kinetic energy. Therefore, the collision is inelastic. Let v_1 and v_2 be the velocities of the two bodies after collision. ½





As the two bodies form one system, which is closed and isolated, we can write the law of conservation of linear momentum for the two body system as :

Total momentum before the collision (P_i) = Total momentum after the collision (P_f)

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad ½$$

For perfectly inelastic collision between two bodies of masses m_1 and m_2 : The body of mass m_2 happen to be initially at rest ($u_2 = 0$) we refer to this body as the target. The incoming body of mass m_1, moving with intial velocity u_1 is referred to as the projectile. After the collision, the two bodies move together with a common velocity v. The collision is perfectly inelastic. As the total linear momentum of the system cannot change, therefore $P_i = P_f$. ½

i.e., $$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

Or $$m_1 u_1 = (m_1 + m_2)v, \quad (\because \boldsymbol{u}_2 = 0)$$

Or $$v = \frac{m_1u_1}{m_1 + m_2} \quad \text{...(i)} \quad \mathbf{1}$$

Knowing m_1, m_2 and u_1, we can calculate the final velocity v. As the mass ratio

$$\frac{m_1}{m_1 + m_2} < 1,$$

$\therefore$ therefore $$v < u_1 . \quad \mathbf{1}$$

We can calculate loss of K.E. in this collision.

Total K.E. before collision, $$E_1 = \frac{1}{2}m_1u_1^2$$

Total K.E. after collision, $$E_2 = \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2}(m_1 + u_2)\left(\frac{m_1u_1}{m_1 + m_2}\right)^2 \quad \text{Using (i)}$$

$$= \frac{m_1^2u_1^2}{2(m_1 + m_2)} \quad ½$$

Loss of K.E. $= E_1 - E_2$

$$= \frac{1}{2}m_1u_1^2 - \frac{m_1^2u_1^2}{2(m_1+m_2)}$$

$$= \frac{m_1^2u_1^2 + m_1m_2u_1^2 - m_1^2u_1^2}{2(m_1+m_2)}$$

$$= \frac{m_1m_2u_1^2}{2(m_1+m_2)}.$$

1

26. Consider an incompressible non-viscous liquid entering the cross-section A_1 at A with a velocity v_1 and coming out at a height h_2 at B with velocity v_2.

The P.E. and K.E. increase since h_2 and v_2 are more than h_1 and v_1 respectively. These are done by the pressure doing work on the liquid. If P_1 and P_2 are the pressures at A and B and a small displacement at A and B.

The work done on the liquid $\quad A = (P_1A_1)$

$$\Delta x_1 = P_1A_1v_1\Delta t$$

The work done by the liquid at B $= -(P_2A_2)$

$$\Delta x_2 = -P_1A_2v_2\,\Delta t \qquad \mathbf{1}$$

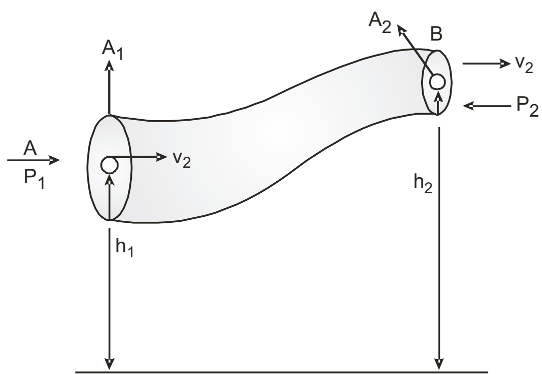
(Considering a small time Δt so that area may be same.)

Net work done by pressure $= (P_1 - P_2)Av\Delta t, \qquad [\because A_1v_1 = A_2v_2]$

From the conservation of energy,

$$(P_1 - P_2)Av\Delta t = \text{change in (K.E. + P.E.)} \qquad \mathbf{1}$$

$$(P_1 - P_2)Av\Delta t = Av\rho\Delta tg(h_2 - h_1) + \frac{1}{2}Av\rho\Delta tg(v_2^2 - v_1^2) \qquad \text{½}$$



$$(P_1 - P_2) = \rho g(h_2 - h_1) + \frac{\rho}{2}(v_2^2 - v_1^2) \qquad \text{½}$$

i.e.,
$$P_1 + \rho gh_1 + \frac{\rho}{2}v_1^2 = P_2 + \rho hg_2 + \frac{\rho}{2}v_1^2$$

i.e.,
$$\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant.} \qquad \mathbf{1}$$

Limitations of bernoulli's theorem :

(a) While deriving Bernoulli's theorem, it is assumed that velocity of every particle of liquid across any cross section of tube is uniform. Practically it is incorrect.

(b) The viscous drag of the liquid which comes into play when liquid is in motion has not been taken into account.

(c) While deriving the equation, it is assumed that there is no loss of energy when liquid is in motion. **1**

Or

Refer Ans 25, Sample Question Paper 7.

❑❑❑

(c) $R = \sqrt{P^2 + Q^2}$ and $\alpha = \tan^{-1}\dfrac{Q}{P}$ ½

Or

Component of force along horizontal,

$$F_x = F\cos 60°$$
$$= 72 \times \frac{1}{2} = 36 \text{ dyne} \quad 1$$

Using $\quad F_x = ma_x$

We get $\quad a_x = \dfrac{F_x}{m} = \dfrac{36}{9}$

$$= 4 \text{ cms}^{-2}. \quad 1$$

9. If there is only one propeller, the helicopter will start rotaing in a direction opposite to that of the rotation of the propeller so as to conserve angular momentum. **2**

10. Sound waves are mechanial waves whose velocity is given by :

$$v = \sqrt{\gamma RT/M} \quad 1$$

Light waves are non-mechanical waves or electromagnetic waves for which $c = 1/\sqrt{\mu_0\varepsilon_0}$, where μ_0 is the absolute magnetic permeability of free space and ε_0 is the absolute electrical permittivity of the free space. Therefore, v depends upon T, but c does not. **1**

11. Here $Y = \dfrac{mgl^3}{4bd^3\delta}$, g is constant.

$\therefore$ Maximum relative error in Y is given by :

$$\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{3\Delta l}{l} + \frac{\Delta b}{b} + \frac{3\Delta d}{d} + \frac{\Delta\delta}{\delta} \quad 2$$

Thus clearly m, l, b, d and δ introduce the maximum error in the measurement of Y. **1**

12. **(a)** A lives closer to the school than B because B has to cover higher distance [OP < OQ].

(b) A starts from the school earlier than B because $t = 0$ for A but B has some finite time.

(c) B walks faster than A because it covers more distance in less duration of time [slope of B is greater than that of A].

(d) A and B reach home at the same time.

(e) B overtakes A on the road once (at X, *i.e.*, the point of intersection). **3**

13. The position vector $(\vec{r})$ of the particle is

$$\vec{r} = 3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k} \text{ m} \quad ....\text{(i)}$$

(a) velocity $\vec{v}(t)$ of the particle is given by :

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(\vec{r})$$
$$= \frac{d}{dt}(3.0t\,\hat{i} - 2.0t^2\,\hat{j} + 4.0\,\hat{k})$$
$$= 3\hat{i} - 4t\,\hat{j} + 0 \quad ....\text{(ii)} \quad 1$$

Also, acceleration $\vec{a}(t)$ of the particle is given by :

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \frac{d}{dt}\vec{v}(t)$$
$$= \frac{d}{dt}(3\hat{i} - 4t\,\hat{j}) \quad \text{[by using (ii)]}$$

$$= 0 - 4\hat{j}$$

$$\vec{a}\ (t) = -4\hat{j} \qquad \text{....(iii)} \quad \tfrac{1}{2}$$

(b) At time t, the veocity of the particle is given by using equation (ii).

$$\vec{v}\ (t) = 3\hat{i} - 4t\hat{j}$$

$\therefore$ At $t = 2\ s$,
$$v = 3.0\hat{i} - 4 \times 2\hat{j}$$
$$= 3.0\hat{i} - 8.0\hat{j} \qquad \tfrac{1}{2}$$

$\therefore$ Its magnitude is :

$$v = \sqrt{3^2 + (-8)^2} = \sqrt{9 + 64}$$
$$= \sqrt{73} = 8{\cdot}544\ \text{ms}^{-1} \qquad \tfrac{1}{2}$$

and, direction of v is given by,

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-8}{3}\right)$$
$$= 70° \text{ with } x\text{-axis.} \qquad \tfrac{1}{2}$$

14. Speed of train,
$$v = 54\ \text{km/hr}$$
$$= \frac{54 \times 1000}{60 \times 60} = 15\ \text{m/s} \qquad \tfrac{1}{2}$$

$$\text{Mass of train} = 10^6\ \text{kg}$$
$$\text{Angle of banking } \theta = ? \qquad \tfrac{1}{2}$$

The centripetal force is provided by the lateral thrust by the outer rail. According to Newton's third law of motion, the train exerts (*i.e.* causes) an equal and opposite thrust on the outer rail causing its wear and tear.

$$\text{Centripetal force} = mv^2/r$$
$$= \frac{10^6 \times 15^2}{30} = 75 \times 10^5\text{N} \qquad \mathbf{1}$$

Angle of banking,
$$\theta = \tan^{-1}(v^2/rg)$$
$$= \tan^{-1}\frac{15^2}{30 \times 9.8}$$
$$\theta = 37^\circ \qquad \mathbf{1}$$

15. The ratio of relative velocity of separation after collision to the relative velocity of approach before collision is called coefficient of restitution. ½

Coefficient of restitution,
$$e = \frac{v_2 - v_1}{u_1 - u_2} \qquad \tfrac{1}{2}$$

where u_1 and u_2 are initial velocities of the two colliding bodies and v_1, v_2 are their final velocities after collision. ½

(i) For elastic collision, velocity of separation is equal to the velocity of approach.

$\therefore$
$$e = 1 \qquad \tfrac{1}{2}$$

(ii) For inelastic collision, velocity of separation is not zero but always less than the velocity of approach.

$\therefore$
$$0 < e < 1 \qquad \tfrac{1}{2}$$

(iii) For perfectly inelastic collision, the colliding bodies do not separate out but move with same velocity.

$$e = 0 \qquad \tfrac{1}{2}$$

16. Using Newton's second law of motion for a system of N particles, total force,

$$\vec{\text{F}} = \sum_{i=1}^{i=N} \frac{d}{dt}(m_i\,\vec{v}_i)$$

$$= \frac{d}{dt}(m_1 \vec{v_1} + m_2 \vec{v_2} + ... + m_n \vec{v_n})$$ **1**

Internal forces acting on the particles cancel out in pairs. Taking external force also to be zero.

i.e., $\vec{F} = 0$ **1**

We get $\frac{d}{dt}(m_1\vec{v_1} + m_2\vec{v_2} + ... + m_N\vec{v_N}) = 0$

or $(m_1\vec{v_1} + m_2\vec{v_2} + ... + m_n\vec{v_n}) = \text{constant.}$

The above expression is called principle of conservation of linear momentum. **1**

17. **(a)** No, from the formula $v_e \sqrt{\frac{2GM}{R}} =$, it is clear that escape velocity does not depend on the mass of the body. **1**

(b) The escape velocity depends upon the value of gravitational potential at the point from where the body is projected. The gravitational potential energy of body $E = -\sqrt{\frac{2Gm}{R}}$ is slightly different at different points.

($\therefore$ the earth is not a perfect sphere and hence R is different at different points). Because of this escape velocity depend slightly on the latitude of the place from where the body is projected. **1**

(c) The escape velocity of a body does not depend upon its direction of projection. ½

(d) Since the gravitational potential energy at a point at the height h from the earth surface is $\frac{GMm}{(R+h)}$, the escape velocity will be different for different value of h. ½

18. **(a)** Here at $t = 0$, OP makes an angle with x-axis. As motion is clockwise, so $\phi = \frac{+\pi}{2}$ radian. So the x-projection of OP at time t will give us the equation of S.H.M. given by,

$$x = A\cos\left(\frac{2\pi t}{T} + \phi\right)$$ ½

$$= 3\cos\left(\frac{2\pi t}{T} + \frac{\pi}{2}\right),$$ **($\because$ A = 3 cm, T = 2 s)**

$$\Rightarrow \quad x = 3\cos\left(\pi t + \frac{\pi}{2}\right)$$

$$= -3\sin \pi t$$ **(x is in cm)**

$$\therefore \quad x = -3 \sin \pi t \text{ cm.}$$ **1**

(b) T = 4 s, A = 2 m

At $t = 0$, OP makes an angle π with the positive direction of x-axis, *i.e.*, $\phi = +\pi$.

Hence, the x-projection of OP at time t will give us the equation of S.H.M.

As, $$x = A\cos\left(\frac{2\pi t}{T} + \phi\right)$$ ½

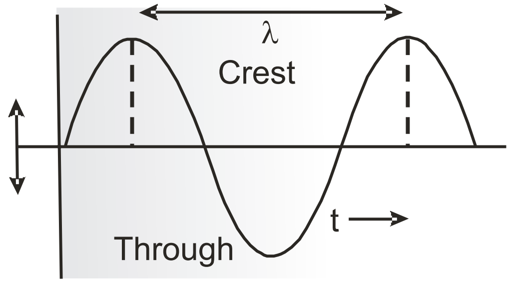
$$= 2\cos\left(\frac{2\pi}{T} + \pi\right)$$

$$= +2\cos\left(\frac{\pi}{2}t + \pi\right)$$

Or $$x = -2\cos\left(\frac{\pi}{2}t\right) \text{ m.}$$ **1**

Or

If the particles of a medium vibrate in a direction normal to the direction of wave, the motion is called transverse wave motion, *e.g.* streched string of violin, guitar, sitar sonometer etc. Electromagnetic waves are also transverse in nature. **1 + 1**



Crests and trough are formed when transverse wave propagates. Distance between consecutive troughs or crests is called wavelength. **1**

19. **(a)** While deriving Bernoulli's equation, we say that

Descrease in pressure energy per second = increase in K.E. /sec + increase in P.E. /sec ½

Consider that viscous forces are absent. Thus as the fluid flows from lower to upper edge there is a fall of pressure energy due to the fall of pressure. If dissipating force are present, then a part of this pressure energy will be used in overcoming these forces during the flow of fluid. Hence there shall be greater drop of pressure as the fluid moves along the tube. **1**

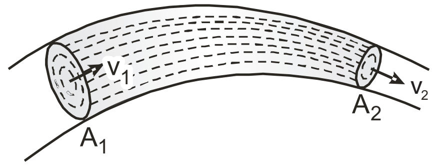
(b) Yes, the dissipative forces become more important as the fluid velocity increases. ½

From the Newton's law of viscous drag, we know that :

$$F = \eta A \frac{dv}{dx}$$

Clearly as v increases, velocity gradient increases and hence, viscous drag *i.e.* dissipative force also increases. **1**

20. **Equation of continuity :** Consider a non-viscous liquid in srteam line flow through a tube AB of varying cross-section. Let a_1, a_2 = areas of cross-sections of the tube at A and B respectively.



v_1, v_2 = velocities of flow of liquid at A and B respectively.
ρ_1, ρ_2 = density of liquid at A and B respectively.
$\therefore$ Volume of liquid entering per second at A $= a_1 v_1$ ½
Mass of liquid entering per second at A $= a_1 v_1 \rho_1$ ½
Similarly, mass of liquid leaving per second at B

$$= a_2 v_2 \rho_2$$

If there is no loss of liquid in the tube and the flow is steady, then
mass of the liquid entering per second at A = mass of the liquid leaving per second at B
Or $a_1 v_1 \rho_1 = a_2 v_2 \rho_2$ ....(1) ½
If the liquid is incompressible, then

$$\rho_1 = \rho_2$$

From (1), $a_1 v_1 = a_2 v_2$
Or av = constant ....(2) ½
This is known as equation of continuity.

From (2), $v \propto \frac{1}{a}$. It means the larger is the area of cross-section, the smaller will be the velocity of liquid flow and vice-versa. It is due to this reason that (a) deep water runs slow or slow water runs deep, (b) the jet of falling water becomes narrow as it goes down. **1**

21. Refer Ans 15 Sample Question Paper 4.

22. **(i)** At the triple point, *i.e.*, temperature = – 56·6°C and pressure = 5·11 atm, the vapour, liquid and the solid phase of CO_2 exist in equilibrium. ½

(ii) If the pressure decreases, both fusion point and boiling point of CO_2 decreases. ½

(iii) The critical temperature and pressure of CO_2 are 31·1°C and 73·0 atm respectively. If the temperature of CO_2 is more than 31·1°C, it cannot be liquified, how soever large pressure we may apply to it. ½

(iv) **(a)** CO_2 will be a vapour at – 70°C at a pressure of 1 atm. ½

(b) CO_2 will be a solid at – 60°C at a pressure of 10 atm. ½

(c) It will be a liquid at 15°C at a pressure of 56 atm. ½

23. **(a)** Navigator, he is a responsible citizen, he is duty minded, having presence of mind. **2**

(b) Apparent frequency received by an enemy submarine,

$$v' = \{(v + v_0)/v\}$$
$$v = \{(1450 + 100)/1450\} \times 40 \times 10^3 \text{ Hz}$$
$$= 4{,}276 \times 10^4 \text{ Hz.}$$

This frequency is reflected by the enemy submarine (source) and is observed by SONAR (now observer)

In this case apparent frequency, $v'' = \{v/(v - v_s)\} \times v$

$$= [1450/1450 - 100)] \times 4.276 \times 10^4 \text{ Hz} = 45.9 \text{ kHz.}$$ **2**

24. **(i)** $v = u + at$

Derivation : By definition of acceleration, we know that

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$

Or $v_2 - v_1 = a(t_2 - t_1)$

Or $v_2 = v_1 + a(t_2 - t_1)$ ....(i)

where v_1 and v_2 are the velocities of an object at time t_1 and t_2 respectively.

If $v_1 = u$ (initial velocity of the object) at $t_1 = 0$

$v_2 = v$ (final velocity of the object) at $t_2 = t$.

Then (i) reduces to $v = u + at$ **1**

(ii) $v^2 = u^2 + 2as$

Derivation : We know that acceleration is given by,

$$a = \frac{v_2 - v_1}{t_2 - t_1}$$, where v_1 and v_2, t_1 and t_2 are as in (i),

Or $t_2 - t_1 = \dfrac{v_2 - v_1}{a}$ ....(i)

Also we know that

$$x_2 - x_1 = v_1(t_1 - t_2) + \frac{1}{2} a(t_2 - t_1)^2$$ ....(ii)

From (i) and (ii), we get

$$x_2 - x_1 = v_1 \frac{v_2 - v_1}{a} + \frac{1}{2}a\left[\frac{v_2 - v_1}{a}\right]^2$$
$$= \frac{v_1v_2 - v_1^2}{a} + \frac{v_2^2 + v_1^2 - 2v_1v_2}{2a}$$
$$= \frac{2v_1v_2 - 2v_1^2 + v_1^2 + v_2^2 - 2v_1v}{2a}$$
$$= \frac{v_2^2 - v_1^2}{2a}$$

Or $$v_2^2 - v_1^2 = 2a\,(x_2 - x_1) \quad \text{....(iii)}$$

Now if
$$v_1 = u \text{ at } t_1 = 0$$
$$v_2 = v \text{ at } t_2 = t \quad \text{....(iv)}$$
$$x_2 - x_1 = s$$

Then from (iii) and (iv), we get
$$v^2 - u^2 = 2as \quad \text{....(v)} \quad \textbf{2}$$

Or $$v^2 = u^2 + 2as$$

(iii) $$s = ut + \frac{1}{2}at^2$$

Derivation : Let

x_1, v_1 = position and velocity of the object at time t_1
x_2, v_2 = position and velocity of the object at time t_2
a = uniform acceleration of the object

Also let v_{av} = average velocity in $t_2 - t_1$ interval.

∴ By definition,
$$v_{av} = \frac{x_2 - x_1}{t_2 - t_1}$$

Or $$x_2 - x_1 = v_{av}\,(t_2 - t_1) \quad \text{....(i)}$$

Also we know that $$v_{av} = \frac{v_2 - v_1}{2} \quad \text{....(ii)}$$

∴ From eqns. (i) and (ii), we get
$$x_2 - x_1 = \frac{v_2 - v_1}{2}\,(t_2 - t_1) \quad \text{....(iii)}$$

Also we know that $v_2 = v_1 + a\,(t_2 - t_1)$ ....(iv)

∴ From eqns. (iii) and (iv), we get
$$v_2 - x_1 = \frac{1}{2}\,[v_1 + v_1 + a\,(t_2 - t_1)]\,(t_2 - t_1)]$$
$$= v_1\,(t_2 - t_1) + \frac{1}{2}\,a\,(t_2 - t_1)^2 \quad \text{....(v)}$$

Now, if
$$x_1 = x_0 \text{ at } t_1 = 0$$
$$x_2 = x \text{ at } t_2 = \text{t}$$
$$v_1 = u \text{ at } t_1 = 0 \quad \text{....(iv)}$$
$$v_2 = v \text{ at } t_2 = t$$

∴ From eqns. (v) and (vi), we get
$$x - x_0 = ut + \frac{1}{2}at^2$$

If $x - x_0 = s$, then
$$s = ut + \frac{1}{2}at^2. \quad \textbf{2}$$

Or

(a) Let v_1 and v_2 = finishing velocities of car A and car B
and t_1 and t_2 = finishing time intervals for car A and car B
$$v = v_1 - v_2 \quad \text{....(i)}$$
$$t = t_2 - t_1 \quad \text{....(ii)}$$

Let d = distance covered during race by each car.

Using eqn. $$s = \left(\frac{u+v}{2}\right)t$$

For car A, $$d = \left(\frac{0+v_1}{2}\right)t_1$$
$$= \frac{v_1 t_1}{2}, \quad (\because \text{u} = 0) \quad \text{....(iii)}$$

For car B, $$d = \left(\frac{0+v_2}{2}\right)t_2$$

$$= \frac{v_2 t_2}{2}$$ (∵ u = 0) ....(iv) ½

From (iii) and (iv), we get

$$d = \frac{v_1 t_1}{2} = \frac{v_2 t_2}{2}$$

$$v_1 = \frac{2d}{t_1}$$

and $$v_2 = \frac{2d}{t_2}$$ ....(v) ½

Also using eqn.

$$s = ut + \frac{1}{2}at^2$$

$$d = \frac{1}{2}a_1 t_1^2 = \frac{1}{2}a_2 t_2^2$$ (∵ $u = 0$)

$$a_1 = \frac{2d}{t_1^2}$$

and $$a_2 = \frac{2d}{t_2^2}$$ ....(vi) ½

Since, $$a = \frac{v}{t} = \frac{v_1 - v_2}{t_2 - t_1} = \frac{\frac{2d}{t_1} - \frac{2d}{t_2}}{t_2 - t_1}$$

$$= \frac{2d(t_2 - t_1)}{t_1 t_2 (t_2 - t_1)} = \frac{2d}{t_1 t_2}$$

$$= \sqrt{\left(\frac{2d}{t_2 t_1}\right)^2}$$

Or $$\frac{v}{t} = \sqrt{\frac{2d}{t_1^2} \times \frac{2d}{t_2^2}}$$

$$= \sqrt{a_1 \times a_2}$$

$$v = t\sqrt{a_1 . a_2}\ .$$ **1**

(b) Given, $$t = \sqrt{x} + 3$$

Or $$\sqrt{x} = t - 3$$ ....(i)

Squaring on both sides of equation (i), we get

$$x = (t-3)^2$$ ....(ii)

$$= t^2 + 9 - 6t$$

If v be the velocity of the particle, then

$$v = \frac{dx}{dt} = \frac{d}{dt}(x)$$ ½

$$= \frac{d}{dt}(t^2 - 6t + a)$$

$$= 2t - 6$$

When $v = 0$, $2t - 6 = 0$

Or $\quad t = 3$ sec. ....(iii) **1**

$\therefore$ From equations (ii) and (iii), we get

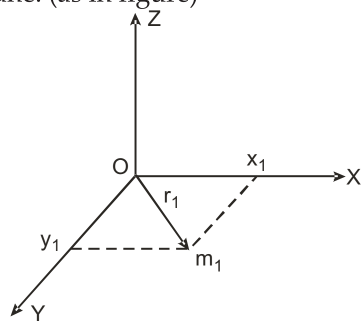
$$x = 3^2 + 9 - 6 \times 3$$

$$= 18 - 18$$

$$x = 0.$$ **1**

25. **Theorem of perpendicular axes :** According to this theorem, the moment of inertia of a plane lamina about any axis OZ ⊥ to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about any two mutually ⊥ axes OX and OY in the plane of the lamina, meeting at a point where the given axis OX passes through the lamina. ½

Suppose, the lamina is in XY plane. (as in figure)



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I_x = moment of inertia of the lamina about OX

I_y = moment of inertia of the lamina about OY

I_z = moment of inertia of the lamina about OZ

According to this theorem,

$$I_z = I_x + I_y$$ **1**

Proof : Suppose the lamina consists of n particles of masses $m_1, m_2, ..., m_n$ at ⊥ distances $r_1, r_2, r_3, ..., r_n$ respectively from the axis OZ.

Suppose the corresponding ⊥ distances of these particles from the axis OY are $x_1, x_2, x_3, ..., x_n$ and from the axis OX are $y_1, y_2, y_3, y_4, ..., y_n$ respectively.

$\therefore$
$$I_x = m_1y_1^2 + m_2y_2^2 + ... + m_ny_n^2$$

$$= \sum_{i=1}^{i=n} m_iy_i^2 \quad ....(1)$$

$$I_y = m_1x_1^2 + m_2x_2^2 + ... + m_nx_n^2$$

$$= \sum_{i=1}^{i=n} m_ix_i^2 \quad ....(2)$$

$$I_z = m_1r_1^2 + m_2r_2^2 + ... + m_nr_n^2$$

$$= \sum_{i=1}^{i=n} m_ir_i^2 \quad ....(3)$$

Adding (1) and (2), we get

$$I_x + I_y = \sum_{i=1}^{i=n} m_iy_i^2 + \sum_{i=1}^{i=n} m_ix_i^2$$

$$= \sum_{i=1}^{i=n} m_i(y_i^2 + x_i^2)$$ ½

As clear from fig.,

$$r_i^2 = x_i^2 + y_i^2$$

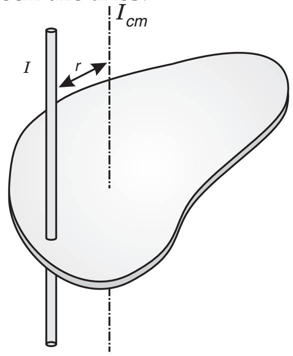
$\therefore$
$$I_x + I_y = \sum_{i=1}^{i=n} m_ir_i^2 = I_z$$

$\therefore$
$$I_x + I_y = I_z$$

Now
$$I_z = I_x + I_y.$$ ½

Or

Theorem of parallel axes : The theorem determines the moment of inertia of a rigid body about any given axis, given that moment of intertia about the parallel axis through the center of mass of an object and the perpendicular distance between the axes.



1

The moment of inertia about Z-axis can be represented as :

$$I_z = I_{c.m.} + mr^2$$

Where $I_{c.m.}$ is the moment of inertia of the object about its centre of mass, m is the mass of the object and r is the perpendicular distance between the two axes.

Proof :

Assume that the perpendicular distance between the axes lies along the x-axis and the centre of mass lies at the origin. The moment of inertia relative to z-axis that passes through the centre of mass, is represented as

$$I_{c.m.} = \int (x^2 + y^2)dm$$ **1**

Moment of inertia relative to the new axis with its perpendicular distance r along the x-axis, is represented as :

$$I_z = \int ((x-r)^2 + y^2)dm$$ **1**

We get,

$$I_z = \int (x^2 + y^2)dm + r^2 \int dm - 2r \int xdm$$ **1**

The first term is $I_{c.m.}$ the second term is mr^2 and the final term is zero as the origin lies at the centre of mass. Finally,

$$I_z = I_{c.m.} + mr^2$$ **1**

26. (a) Let,

W = water equivalent of calorimeter and stirrer
t_1 = initial temperature of water and calorimeter
m_1 = mass of water
m_2 = mass of substance
C = specific heat of the substance
t_2 = temperature of the substance

Rise in temperature of water and calorimeter ½

$$= (t - t_1)$$

Fall in temperature of the substance = $(t_2 - t)$ ½

Heat gained by water and calorimeter = $(m_1 + w)(t - t_1)$ ....(i) ½

Heat lost by the substance = $C.m_1(t_2 - t)$ ....(ii) ½

If we assume that there is no stray loss of heat, then

$$\text{Heat lost} = \text{Heat gained}$$

$$Cm_2.(t_2 - t) = (m_1 + W)(t - t_1)$$

$$C = \frac{(m_1 + W)(t - t_1)}{m_2(t_2 - t)}$$ ½

(b) Consider a cube of side x and area of each face 'A'. The opposite faces of the cube are maintained at temperatures, θ_1 and θ_2, where $\theta_1 > \theta_2$. Heat gets conducted in the direction of the fall of temperature.

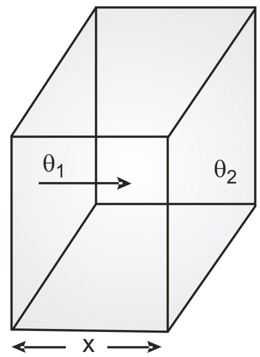
$$Q \propto A$$ ½

$$Q \propto (\theta_1 - \theta_2)$$ ½

$$Q \propto \tau$$

$$Q \propto \frac{1}{x} \propto \frac{A(\theta_1 - \theta_2)t}{x}$$

$$Q = \frac{KA(\theta_1 - \theta_2)t}{x}$$ ½



Here K is a constant called the coefficient of thermal conductivity of the material of the cube and t stands for time interval.

We can also write as

$$H = KA\left[\frac{\Delta\theta}{\Delta x}\right]$$ ½

Where, H = Heat flow per second

$\frac{\Delta\theta}{\Delta x}$ = temperature gradient

$T = \theta$ = temperature

If, $(\theta_1 - \theta_2) = 1°C$

$t = 1$ s

$x = 1$ cm

then $\theta = K$. ½

Or

(a) The quantity $\frac{\theta_1 - \theta_2}{x}$ or $\frac{d\theta}{dx}$ respresents the rate of fall w.r. to distance.

The quantity $\frac{d\theta}{dx}$ represents the rate of change of temperature w. r. to distance and is called temperature gradient.

$$Q = -KA\left[\frac{d\theta}{dx}\right]t$$ ½

Dimensions of K.

Q represents energy and its dimensions are

$$[Q] = [ML^2T^{-2}]$$

$$[dx] = [L]\ [A] = [L^2]$$

$$[d\theta] = [\theta]\ [t] = [T]$$

$$[K] = \frac{[ML^2T^{-2}][L]}{[L^2][\theta][T]}$$

$$= [MLT^{-3}\ \theta^{-1}]$$ ½

(b) Consider a compound wall (or a slab) made of two materials A and B of thickness d_1 and d_2. Let K_1 and K_2 be the coefficients of thermal conductivity and θ_1 and θ_2 are the temperature of the end faces $(\theta_1 > \theta_2)$ and θ is the temperature of the surface in contact. ½

For material A :

$$Q_1 = \frac{K_1A_1(\theta_1 - \theta)}{d_1} \quad \text{....(i) ½}$$

For the material B :

$$Q_2 = \frac{K_2A_2(\theta - \theta_2)}{d_2} \quad \text{....(ii) } \mathbf{1}$$

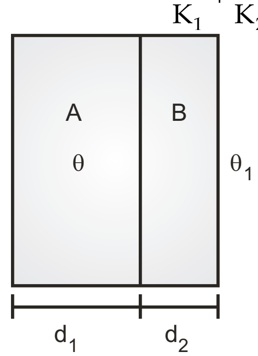
From eqns. (i) and (ii), we get

$$\frac{K_1A_1(\theta_1 - \theta)}{d_1} = \frac{K_2A_2(\theta - \theta_2)}{d_2} \quad ½$$

$$= \frac{\frac{K_1\theta_1}{d_1} + \frac{K_2\theta_2}{d_2}}{\frac{K_1}{d_1} + \frac{K_2}{d_2}} \quad ½$$

Putting the value of θ in eqn. (i),

$$\theta = \frac{A(\theta_1 - \theta_2)}{\frac{d_1}{K_1} + \frac{d_2}{K_2}}$$



½

In general for any number of walls,

$$\theta = \frac{A_1(\theta_1 - \theta_2)}{\Sigma\left(\frac{d}{K}\right)} . \quad ½$$

❑❑❑

SOLUTIONS

SAMPLE
QUESTION PAPER - 7

Self Assessment______________________________

Time : 3 Hours **Maximum Marks : 70**

1. It is called so because this law holds good irrespective of the nature of interacting bodies at all places and at all times. **1**

2. It is the ratio of change in dimension of a body to the original dimension when some deforming force is applied to it. It is a type not an example. **1**

3. The upward force exerted by a fluid (liquid or gas) on an object immersed in the fluid. **1**

4. Time period decreases as effective value of acceleration due to gravity increases (*i.e.*, $g' = g + a$) and $(T \propto 1/\sqrt{g'})$. **1**

5.
$$\text{Intensity} = (\text{Amplitude})^2 \propto \frac{1}{(\text{distance})^2}$$

$\therefore$
$$\text{Amplitude} \propto \frac{1}{\text{distance}}$$

Hence,
$$\frac{A_1}{A_2} = \frac{r_2}{r_1} = \frac{25}{8}$$ **1**

6. Relative error in ρ is given by

$$\frac{\Delta\rho}{\rho} = 3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta\rho}{\rho} + \frac{\Delta d}{d}$$ **1**

so, percentage error is

$$\frac{\Delta\rho}{\rho}\times 100 = \{3\frac{\Delta\rho}{\rho} + 2\frac{\Delta\rho}{\rho} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}\}\times 100$$
$$= (3 \times 1\%) + (2 \times 3\%) + (1/2 \times 4\%) + (1 \times 2\%)$$
$$= 3\% + 6\% + +2\% + 2\%$$
$$= 13\%$$ **1**

7. Mass given by the equation $a = \frac{F}{M}$, *i.e.*, $M = \frac{F}{a}$ is called inertial mass. Clearly, higher the mass means lesser the acceleration. Thus mass of a body resists the acceleration, *i.e.*, rate of change in velocity due to external force or in other words it is the measure of inertia of a body. **2**

Or

$$|\vec{a}+\vec{b}|^2 - (|\vec{a}|+|\vec{b}|)^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta - |\vec{a}|^2 - |\vec{b}|^2 - 2|\vec{a}||\vec{b}|$$
$$= 2|\vec{a}||\vec{b}|(1 - \cos\theta)$$ **½**

$$= 2|\vec{a}||\vec{b}|\ \sin^2\frac{\theta}{2}$$ **1**

= a negative quanity

Hence, $$|\vec{a}+\vec{b}| < |\vec{a}|+|\vec{b}|$$ ½

8. It should be greater in night. In the day time, the body is pulled by the Earth and the Sun in two opposite directions. This will result into decrease in weight. **1**

During night time the earth and the sun pull the body in the same direction so the weight will increase in night. **1**

9. $$C_V = \frac{F}{2}R$$ (Where F in degree of freedom)

For monoatomic gas, F = 3

$$C_V = \frac{3}{2}R$$

$$C_P - C_V = R$$

$\therefore$ $$C_P = \left(1+\frac{3}{2}R\right)$$

$$C_P = \frac{5}{2}R$$

$$\gamma = \frac{C_P}{C_V} = \frac{5}{3} = 1.67$$ **1**

For diatomic gases, F = 5

$$C_V = \frac{5}{2}R, C_P = \frac{7}{2}R$$

$$\gamma = \frac{C_P}{C_V} = \frac{7}{5} = 1.4$$ 1

10. Motion of piston in a cylinder is another example of a vibrating system.

Resturing force in this case is given by :

$$F = -mg = -kx$$ **1**

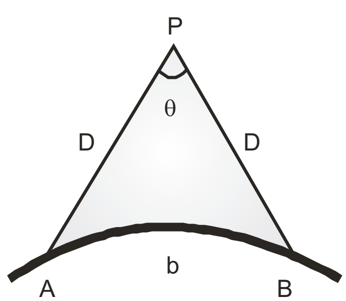
where m is mass of the piston and x is the displacement,

Time period of vibration $$T = 2\pi\sqrt{\frac{\text{Inertia factor}}{\text{Spring factor}}}$$

$$= 2\pi\sqrt{\frac{m}{k}}$$ **1**

11. Let there be a planet P far away from the Earth. The planet is observed from two different observation points A and B on Earth called observatories. If distance AB = b and angle APB = θ radian, then θ being

very small $\frac{b}{D}$ << 1. Angle θ is called parallax or parallactic angle. **1**



1

Now radius :

$$AP = BP = D = \frac{b}{\theta}$$

So that $$b = D\theta.$$ **1**

12. **(a)** Since the motion of the train between two distant stations is smooth throughout so keeping in view the long distance covered between the two stations in reasonable duration of time, the size of the train is neglected and it is considered as a point object. **1**

(b) The distance covered by the monkey in reasonable duration of time is more so it is considered as a point object. **1**

(c) Since, the turning of the ball is not smooth but sharp so the distance covered by it in reasonable duration of time is not large so this ball cannot be treated as a point object. ½

(d) Since the beaker is tumbling and then slips off. So, the distance covered by it in reasonable duration of time is not large. So it is not treated as a point object. ½

13. When the stone is dropped, it falls freely under the acceleration due to gravity g. With respect to the Earth the acceleration of the stone is g. **1**

Inside the carriage, the stone possesses two accelerations :

(i) Horizontal acceleration 'a' due to the motion of carriage. **1**

(ii) Vertical downward acceleration 'g' due to gravity.

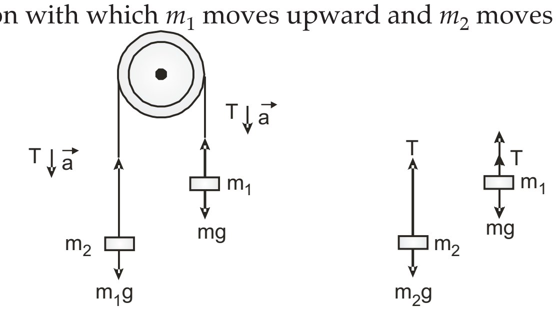
The acceleration of the stone w.r.t. the carriage $= \sqrt{a^2 + g^2}$ **1**

14. Suppose m_1 and m_2 be the masses suspended at the ends of a light inextensible string passing over the pulley.

$\therefore$ $m_1 = 8$ kg, $m_2 = 12$ kg

Suppose T = Tension in the string

a = common acceleration with which m_1 moves upward and m_2 moves downward = ?



The equations of motion of m_1 and m_2 are given by

$$T = m_1a + m_1g \quad \text{....(i)}$$

and

$$m_2g - T = m_2a \quad \text{....(ii)} \; ½$$

Adding equations (i) and (ii), we get

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2} \quad \text{....(ii)} \; ½$$

$\therefore$ From equations (i) and (iii), we get

$$T = m_1g + m_1\frac{(m_2 - m_1)g}{m_1 + m_2} \quad ½$$

$$\Rightarrow \quad T = \frac{m_2g}{m_1 + m_2}(m_1 + m_2 + m_2 - m_1)$$

$$\Rightarrow \quad T = \frac{2m_1m_2}{m_1 + m_2}g \quad \text{...(iv)} \; ½$$

Putting $m_1 = 8$ kg and $m_2 = 12$ kg and $g = 10$ ms^{-2} in equations (iii) and (iv), we get

$$a = \left(\frac{12 - 8}{8 + 12}\right) \times 10$$

$$\Rightarrow \quad = \frac{4}{20} \times 10 = 2 \text{ ms}^{-2} \quad ½$$

From eqn. (i), $T = m_1a + m_1g$

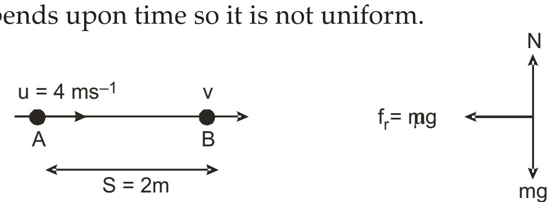
$$= 8 \times 2 + 8 \times 10 = 96\text{N} \quad ½$$

15. **(a)**
$$v = \frac{dx_A}{dt} = 2$$
and
$$a = \frac{d^2x_A}{dt^2} = 0$$
As per above equation the particle has no acceleration at all. **1**

(b)
$$v = \frac{dx_B}{dt}$$
$$= 3 \times 2t + 2 + 0$$
$$= 6t + 2$$
$$a = \frac{d^2x_B}{dt^2} = 6$$
Here acceleration is uniform. **1**

(c)
$$v = \frac{dx_c}{dt}$$
$$= 5 \times 3t^2 + 4$$
$$= 15t^2 + 4$$
$$a = \frac{dv}{dt} = \frac{d^2x_C}{dt^2}$$
$$= 15 \times 2t = 30t$$
Here, acceleration depends upon time so it is not uniform. **1**

16.



(a) (b)

Figure (a) here shows the initial position A and the final position B of the object. Let the speed of particle at B be v. The particle does work against force of friction which is a non-conservative force.
$$K_i - K_f = W$$ **1**
where W is the work done against the force of friction.
Figure (b) shows all the forces acting on the object,
$$W = f_r \times s = mg.s$$
$$= (0{\cdot}15 \times 0{\cdot}1 \times 10 \times 2) \text{ J}$$
$$= 0{\cdot}3 \text{ J}$$ **1**

Hence $$\frac{1}{2} \times 0.1 \times [4^2 - v^2] = 0{\cdot}3$$

Or $$16 - v^2 = 6$$
$$v = \sqrt{10} \text{ ms}^{-1} = 3{\cdot}16 \text{ ms}^{-1}$$ **1**

17. Velocity of centre of mass of a system,
$$\vec{v}_{cm} = \frac{d\vec{r}}{dt}$$
$$\text{Total external force } \vec{F} = M\frac{d^2\vec{r}}{dt^2}$$
$$= M\frac{d\vec{v}_{CM}}{dt} \quad ....(1)$$ **1**

Internal forces cancel out in pairs, so velocity of centre of mass is not affected by internal force.

Puting $\vec{F} = 0$ in equation (i),

$$\frac{d\vec{v}_{CM}}{dt} = 0, (\because M \neq 0) \quad \textbf{1}$$

i.e., $$v_{cm} = \text{constant}$$

If resultant external force is zero, the velocity of centre of mass remains same. **1**

Or

The concept of angular momentum : Expression for angular momentum in cartesian co-ordinates : In order to express torque as the rate of change of some quantity, we rewrite expression for torque rotating a particle in XY plane as

$$\tau = xF_y - yF_x \quad ...(1)\ ½$$

If $P_x = mv_x$ and $P_y = mv_y$ are the x and y components of linear momentum of the body, then According to Newton's 2nd laws of motion

$$F_x = \frac{dP_x}{dt} = \frac{d}{dt}(mv_x)$$

$$= \frac{mdv_x}{dt} \quad ½$$

$$F_y = \frac{dP_y}{dt} = \frac{d}{dt}(mv_y) = \frac{mdv_y}{dt}$$

Substituting in (1), we get $$\tau = x\frac{mdv_x}{dt} - y\frac{mdv_y}{dt}$$

Or $$\tau = m\left[x\frac{dv_x}{dt} - y\frac{dv_y}{dt}\right] \quad ...(2)\ ½$$

Differentiating both the sides

$$\frac{d}{dt}(xv_x - yv_x) = x\frac{dv_y}{dt} + v_y\frac{dx}{dt} - y\frac{dv_x}{dt} - v_x\frac{dy_y}{dt}$$

$$= x\frac{dv_y}{dt} + v_yv_x - y\frac{dv_x}{dt} - v_xv_y \left[\therefore \frac{dv}{dt} = v_x \frac{dy}{dt} = v_y\right]$$

$$= x\frac{dv_y}{dt} - y\frac{dv_x}{dt} \quad ...(3)\ ½$$

Substituting (3) in (2)

$$\tau = m\frac{d}{dt}(xv_y - yv_x) \quad ½$$

$$\tau = \frac{d}{dt}(xmv_y - ymv_x)$$

As $$mv_y = P_y$$
and $$mv_x = P_x$$

$$\tau = \frac{d}{dt}(xP_y - yP_x) \quad ...(4)$$

$$xP_y - yP_x = L$$

$\Rightarrow$ $$\tau = \frac{dL}{dt} \quad ½$$

18. Given : $$\alpha = 1.2 \times 10^{-5}\ °C^{-1}$$
$$\Delta t = 30 - 20 = 10\ °C$$
$$T = 25$$

From the relation

$$\Delta l = l\,\alpha\Delta t,$$

or $$\frac{T'}{T} = \alpha\Delta t$$

$$= 1.2 \times 10^{-5} \times 10$$

$$=1.2 \times 10^{-4}$$ ...(i) ½

$$T = 2\pi\sqrt{\frac{l}{g}}$$ ...(i)

If T' be the time period of the pendulum, when l increase by Δl.

$\therefore$ $$T' = 2\pi\sqrt{\frac{l+\Delta l}{g}}$$

$$= 2\pi\sqrt{\frac{l}{g}\left(1+\frac{\Delta l}{l}\right)}$$ ...(iii) **1**

Dividing eqn. (iii) by eqn. (ii),

$$\frac{T'}{T} = \sqrt{1+\frac{\Delta l}{l}}$$

$$= \sqrt{1+1.2\times 10^{-4}}$$ ½

$\therefore$ Loss in time in one oscillation = T' – T

Therefore loss in time in one day is given by

$$= \frac{T'-T}{T} \times 24 \times 3600 \text{ s.}$$

$$= \left(\frac{T'}{T}-1\right) \times 24 \times 3600 \text{ s.}$$

$$= \left[\sqrt{1+1.2\times 10^{-4}}-1\right] \times 24 \times 3600 \text{ s.}$$

$$= \left[1+\frac{1}{2}\times 1.2\times 10^{-4}-1\right] \times 24 \times 3600 \text{ s.}$$

$$= \frac{1.2\times 10-4\times 24\times 3600}{2} \text{ s.}$$

$$= 5.18 \text{ s.}$$ **1**

19. Displacement : Consider a reference particle moving on a circle of reference of radius a with uniform angular velocity ω. Let the particle start from the point X and trace angular θ radian in time t as it reaches the point P.

Therefore, $$\omega = \frac{\theta}{t} \text{ or } \theta = \omega t.$$ ½

Let the projection of the particle P on diameter YOY' be at M. Then OM = y is the displacement in S.H.M. at time t.

In Δ OPM, $$\sin\theta = \frac{OM}{OP} = \frac{y}{a}$$

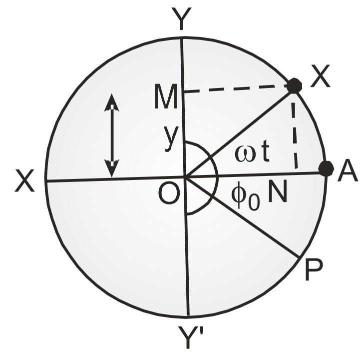
or $$y = a\sin\theta = a\sin\omega t$$ ....(i) ½

Important notes : (1) If projection of P is taken on diameter XOX′, then point N will be executing S.H.M. Here, ON = x = displacement in S.H.M. at time t

In Δ ONP, $$\cos\theta = \frac{ON}{OP}$$

or $$x = a\cos\theta = a\cos\omega t$$ ....(ii) ½

(2) If A is the starting position of the particle of reference such that $\angle AOX = \phi_0$ and $\angle AOP = \omega t$.



½

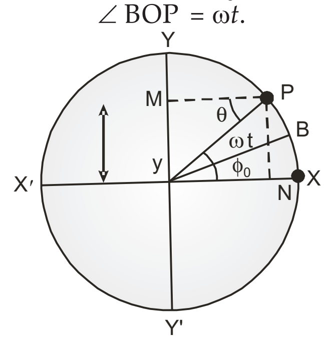
$$\theta = \angle XOP = \omega t - \phi_0$$

Here (–) ϕ_0 is called the initial phase or epoch of S.H.M.

If B is the starting position of the particle of reference such that

$$\angle BOX = \phi_0$$

$$\angle BOP = \omega t.$$



½

$$\theta = \angle XOP = \omega t + \phi_0$$

From equation (i), $y = a\sin(\omega t + \phi_0)$

From equation (ii), $x = a\cos(\omega t + \phi_0)$ ½

Here (+) ϕ_0 is called initial phase or epoch of S.H.M.

20. **(a)** The formula for velocity of sound in air is given by

$$v = \sqrt{\frac{\gamma P}{\rho}} \quad ....(1)$$

where γ = constant for a given gas, *i.e.*, air

ρ = density of the gas

From gas equation,

$$PV = RT \quad ....(1)$$

Or $$P = \frac{RT}{V} \quad ....(2)$$

∴ From equations (1) and (2), we get

$$v = \sqrt{\frac{\gamma RT}{\rho V}} = \sqrt{\frac{\gamma RT}{M}} \quad ....(3)$$

where $\rho V = M$ = molecular mass of air or gas.

For a given gas, m = constant

R is also constant.

When T = constant, then from equation (3), we conclude that v is independent of the pressure of air (gas) if temperature remains constant. **1**

(b) **Effect of temperature :** We know that

$$PV = RT$$

Or $$P = \frac{RT}{V}$$

Also $$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{\rho V}} = \sqrt{\frac{\gamma RT}{M}}$$

where M = ρV = molecular mass of the gas. Since γ, R and M are constants, so

$$v \propto \sqrt{T}\ .$$

It means velocity of sound in a gas is directly proportional to the square root of its temperature, hence we conclude that the velocity of sound in air increases with increase in temperature. **1**

(c) Effect of humidity : The presence of water vapours in air changes the density of air, thus the velocity of sound changes with humidity of air.

Let
ρ_m = density of moist air
ρ_d = density of dry air
v_m = velocity of sound in moist air
v_d = velocity of sound in dry air

From relation $v = \sqrt{\dfrac{\gamma P}{\rho}}$,

we get

$$v_m = \sqrt{\frac{\gamma P}{\rho_m}} \qquad \text{....(i)}$$

and

$$v_d = \sqrt{\frac{\gamma P}{\rho_d}} \qquad \text{....(ii)}$$

Dividing equation (i) by equation (ii), we get

$$\frac{v_m}{v_d} = \sqrt{\frac{\rho_d}{\rho_m}}$$

Also we know that density of water vapours is less than the density of dry air. It means dry air is heavier than water vapours as the molecular mass of water is less than that of N_2 (28) and O_2 (32), so

$$\rho_m < \rho_d$$

Or

$$\frac{\rho_d}{\rho_m} > 1 \qquad \text{....(iv)}$$

∴ from equations (iii) and (iv), we get

$$\frac{v_m}{v_d} > 1$$

∴

$$v_m > v_d$$

It means velocity of sound in air increases with humidity, *i.e.*, velocity of moist air is greater than the velocity of sound in dry air. That is why sound travels faster on rainy day than a dry day. **1**

21. Work done against gravity $W_1 = mgh = V\rho gh$ ½

Work done against pressure difference is

$$W_2 = \Delta P \times V = h\rho g \times V = V\rho gh$$ **1**

Total work done

$$W = W_1 + W_2 = 2V\rho gh$$ ½

∴ Power required

$$= \frac{W}{t} = \frac{2V\rho gh}{t} = \frac{2\times(300\times10^{-3})\times10^3\times10\times6}{60}$$

= 600 watt. **1**

22. Molar specific heat of a substance is defined as the amount of heat required to raise the temperature of one gram mole of the substance through a unit degree. By definition, one mole of any substance is a quantity of the substance whose mass in gram is numerically equally to the molecular mass M.

Thus

$$C = Mc \qquad \text{....(i)}$$ **1**

$n \rightarrow$ no. of moles

$m \rightarrow$ mass of substance in grams.

i.e.,

$$n = \frac{m}{M}$$

or $\quad m = nM$ **1**

From equation (i), $\quad c = \dfrac{\Delta\theta}{m(\Delta t)} = \dfrac{\Delta\theta}{nM(\Delta t)}$

$$Mc = \frac{1}{n}\left(\frac{\Delta\theta}{\Delta t}\right)$$

Using equation (i) $\quad C = \dfrac{1}{n}\left(\dfrac{\Delta\theta}{\Delta t}\right)$ **1**

23. **(a)** Rahim loves animals and feeds them. He takes care to be at a safe distance from the animal. **1**

(b) **(i)** The tension developed in the string when the monkey climbs up with an acceleration of 6 m/s^2 is given by $T = m(g + a) = 40(10 + 6) = 640$ N. **1**

(ii) The tension developed when the monkey climbs down with an acceleration of 4 m/s^2 is given by $T = m(g - a) = 40(10 - 4) = 40 \times 6 = 240$ N. ½

(iii) When the monkey climbs with uniform speed of 5 m/s, acceleration is zero and the tension in the string is $T = mg = 40 \times 10 = 400$ N. ½

(iv) As the monkey falls down the rope nearly under gravity, the tension in the string is given by $T = m(g - a) = m(g - g) = 0$ ½

Since the string can withstand a maximum tension of 600 N, hence the rope will break only in the first case (i) ½

24. **(a)** Let a constant force $\vec{F}$ be applied to a body moving with initial velocity $\vec{u}$, so that its velocity becomes $\vec{v}$ along the direction of force when s is its displacement. Using Newton's second law of motion we get magnitude of force, $F = ma$ and from equation of motion, we get $v^2 - u^2 = 2as$, where a is the acceleration of the body. **1**

Multiplying both sides by m, we get

$$mv^2 - mu^2 = 2mas$$

i.e., $\quad \dfrac{1}{2}mv^2 - \dfrac{1}{2}mu^2 = mas$

i.e., $\quad \dfrac{1}{2}mv^2 - \dfrac{1}{2}mu^2 = Fs = W \qquad (\because ma = F)$

i.e., $\quad K.E._{(f)} - K.E._{(i)} = W$ **1**

where $K.E._{(f)}$ is final kinetic energy and $K.E._{(i)}$ is initial kinetic energy.

Thus work done on a body by a net force is equal to the change in kinetic energy of the body. **1**

(b) Suppose, ΔW be the amount of work done in a small time interval Δt, when P_{av} be the average power, then

$$P_{av} = \frac{\Delta W}{\Delta t} \qquad \text{....(i)} \quad ½$$

When P be the instantaneous power, then by definition 'S'.

$$P = \underset{\Delta t \to 0}{Lt} P_{av}$$

$$= \underset{\Delta t \to 0}{Lt} \frac{\Delta W}{\Delta t} \qquad ½$$

$$\Rightarrow \quad P = \frac{dW}{dt} = \frac{d}{dt}(W) \qquad \text{....(ii)}$$

Now $\quad W = \vec{F}.\vec{s} \qquad$ ....(iii) ½

Where, F = constant force producing a displacements

$\therefore$ From equations (ii) and (iii), we get

$$P = \frac{d}{dt}(\vec{F}.\vec{S})$$

$$= \vec{F}\frac{d\vec{S}}{dt} = \vec{F}.\vec{v} \qquad ½$$

Or

(a) **(i)** In physics work is not said to be done, if (a) the applied force (F) is zero. A body moving with uniform velocity on a smooth surface has some displacement but no external force, so in this case work done is zero. **1**

(ii) The displacement (s) is zero. A labourer standing with a load on his head does no work. ½

(iii) The angle between force and displacement (θ) is $\pi/2$ rad or 90°. Then $\cos \theta = \cos 90° = 0$. Thus work done is also zero.

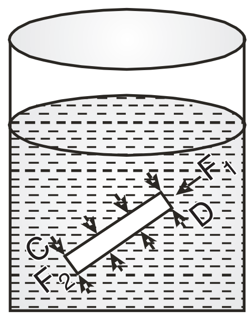
In circular motion, instantaneous work done is always zero because of this reason. **1**

(iv) The change in kinetic energy (ΔKE) is zero. ½

(b) If roads were to go straight up, the slope (θ) would have been large, the frictional force ($\mu mg \cos \theta$) would be small. The wheels of the vehicle would slip. Also for going up a large slope, a greater power shall be required. **2**

25. It states that if gravity effect is neglected, the pressure at every point of liquid in equilibrium or rest is same. ½

Proof : Consider two points C and D inside the liquid in a container which is in equilibrium or rest. Imagine a right circular cylinder with axis CD of uniform cross-section area A such that points C and D lie on float faces of the cylinder in figure. **1**



½

The liquid inside the cylinder is in equilibrium under the action of forces exerted by the liquid outside the cylinder. These forces are acting every where ⊥ to the surface of the cylinder. Thus force on the flat faces of the cylinder at C and D will be ⊥ to the forces on the curved surface of the cylinder. Since the liquid is in equilibrium therefore, the sum of forces acting on the curved surface of the cylinder must be zero. If P_1 and P_2 are the pressure at points C and D and F_1 and F_2 are the forces acting on the flat faces of the cylinder due to liquid, then ½

$$F_1 = P_1 A$$

and
$$F_2 = P_2 A$$

Since the liquid is in equilibrium,

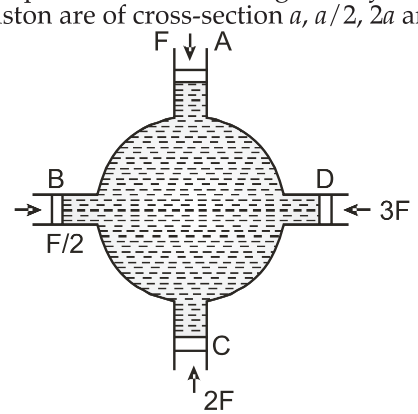
$$F_1 = F_2$$

or
$$P_1 A = P_2 A$$

or
$$P_1 = P_2$$

It means the pressure at C and D are the same. ½

Experimental Proof : Consider a spherical vessel having four cylindrical tubes A, B, C and D each fitted with air tight frictionless piston are of cross-section a, $a/2$, $2a$ and $3a$ respectively.



½

Fill the vessel with an incompressible liquid so that no air gap is left inside the vessel and piston fitted in the various cylindrical tubes. Push the piston A with force F. The pressure developed on the liquid $= F/a = P$ (say) ½

It is seen that all other pistons will be pushed outwards. To keep the pistons at their respective original positions, the force F/2, 2F and 3F respectively required to be applied on pistons of tubes B, C and

D respectively to hold them. Now pressure developed on liquid in tubes B, C and D are F/a, 2F/2a, 3F/3a *i.e.*, each equal to F/a. This indicates that the pressure applied is transmitted equally to all parts of liquid. This proves Pascal's law. **1**

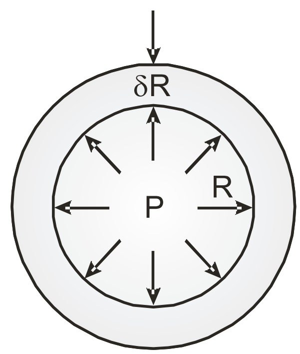
Or

(i) Excess Pressure nside a liquid drop : Consider a liquid drop of radius R. The molecules lying on the surface of liquid drop, due to surface tension will experience a resultant force acting inward ⊥ to the surface.

Let S = Surface tension of liquid drop

P = excess pressure inside the drop

Due to excess of pressure, let there be an increase in the radius of the drop by a small quanity δR, as shown in figure.



½

Then work done by the excess pressure,

$$W = \text{force} \times \text{displacement}$$
$$= (\text{excess pressure} \times \text{area} \times \text{increase in radius})$$
$$= P \times 4\pi R^2 \times \delta R \quad \text{....(i)} \quad ½$$

Increase in surface area of the drop

$$= \text{final surface area} - \text{initial surface area}$$
$$= 4\pi(R + \delta R)^2 - 4\pi R^2$$
$$= 4\pi[R^2 + 2R(\delta R) + (\delta R)^2 - R^2]$$
$$= 8\pi R.\delta R \text{ [Neglecting, } (\delta R)^2 \text{ being very very small]} \quad ½$$

∴ Increase in surface energy= increase in surface area × surface tension

$$= 8\pi R\,(\delta R) \times S \quad \text{....(ii)} \quad ½$$

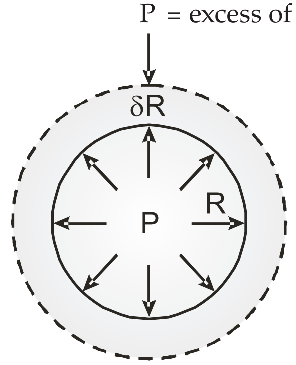
As the increase in surface energy is at the cost of work done by the excess pressure, therefore from (i) and (ii),

$$P \times 4\pi R^2 \times \delta R = 8\pi R \delta R \times S \quad ½$$
$$P = \frac{2S}{R}$$

(ii) Inside a liquid bubble : Consider a soap bubble of radius R, the molecules lying on the surface of liquid bubble will experience a resultant force acting on water ⊥ to the surface due to the surface tension.

Let S = surface tension of the soap solution

P = excess of pressure inside the bubble.



½

Due to it, let there be an increase in the radius of the bubble by a small amount δR, as shown in fig., then work done, W = force ×placement

$= (\text{excess pressure} \times \text{area}) \times \text{increase in radius}$

$= P \times 4\pi R^2 \times \delta R$ ....(iii) ½

The soap bubble has two free surfaces one outside the bubble and one inside the bubble, when soap solution and air are in contact.

∴ The effective increase in surface area of the bubble

$= 2\,[\text{final S.A} - \text{initial S.A.}]$

$= 2\,[4\pi(R + \delta R)^2 - 4\pi R^2]$

$= 2 \times 4\pi[R^2 + 2R(\delta R) + (\delta R)^2 - R^2]$

$= 8\pi \times 2R(\delta R)$, [Neglecting $(dR)^2$, being very small]

$= 16\pi R.\delta R$ ½

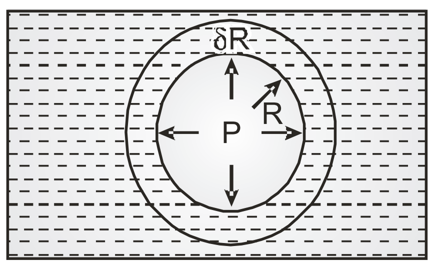
∴ Increase in surface energy is as the cost of work done by the excess pressure therefore from (iii) and (iv), we get

$$P \times 4\pi R^2 \times (\delta R) = 16\pi R\,(\delta R) \times S$$

Or $$P = \frac{4S}{R}$$ ½

(iii) Gross Pressure inside a bubble in liquid : Consider an air bubble of radius R. Just inside a liquid of surface tension S. The air bubble will have only one free surface as shown in figure. It can be shown that the pressure inside the air bubble is given by,

$$P = \frac{2S}{R}$$ ½



½

26. (a) Assumptions of Kinetic theory :

(i) Size of a molecule of a gas is very small as compared to intermolecular distance.

(ii) Behaviour of the molecules of the gas is perfectly elastic.

(iii) Molecules do not exert any force on each other except in collision.

(iv) Velocities of the molecules are in all directions.

(v) Molecules are in random motion.

(vi) The duration of collision is instantaneous.

(vii) The molecules move in a straight line between two successive collisions.

(viii) Collisions of the molecules with one another and the walls of the container are perfectly elastic. **2**

(b) Boyle's law :

Using the relation $$P = \frac{1}{3}\rho v^2 = \frac{1}{3}\frac{M}{V}v^2$$

$$PV = \frac{1}{3}Mv^2$$ **1**

Here M is fixed and $v^2 \propto T$

If temperature T is fixed then PV = constant

Or $P \propto 1/V$

i.e., It temperature of a given mass of a gas is kept constant, its pressure is inversely proportional to its volume.

Charle's law :

Using the relation $P = \frac{1}{3}\rho v^2 = \frac{1}{3}\frac{M}{V}v^2$

$$V = \frac{1}{3}\frac{M}{P}v^2$$ **1**

$V^2 \propto T$ and M is fixed

$V \propto T$

If pressure of a given mass of gas is kept constant its volume is directly proportional to the temperature of the gas. **1**

Or

(a) Pressure exerted by one mole of gas, $P = \frac{1}{3}\rho v^2 = \frac{1}{3}\frac{M}{V}v^2$

Or $PV = \frac{1}{3}Mv^2$

But $PV = RT$ **1**

$\therefore$ $\frac{1}{3}Mv^2 = RT$

Or $\frac{1}{2}Mv^2 = \frac{3}{2}RT$

Or $\frac{1}{2}(Nm)v^2 = \frac{3}{2}RT$ $(\because M = Nm)$

Or $\frac{1}{2}mv^2 = \frac{\frac{3}{2}RT}{N} = \frac{3}{2}KT$ (Where $\frac{R}{K}$ = K) **1**

$\therefore$ Total random K.E. for one mole $= \frac{3}{2}RT$

and average K.E. per molecule $= \frac{3}{2}KT$. **1**

(b) Root mean square velocity is defined as the square root of the average of the squares of the individual velocities of the gas molecules *i.e.*, ½

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + v_3^2 + ... + v_n^2}{n}}$$

$$= \sqrt{\overline{v}^2}$$ ½

where $v_1, v_2, v_3, ..., v_n$ are individual velocities

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{Nm}}$$ *i.e.*,

$v_{rms} \propto \sqrt{T}$. **1**

❑❑❑

S O L U T I O N S

SAMPLE
QUESTION PAPER - 8

Self Assessment______________________________

Time : 3 Hours **Maximum Marks : 70**

1. 1 kWh = 1000 × 3600 Ws = $3{\cdot}6 \times 10^6$ J. **1**

2. The normal force exerted by a liquid on any surface is called thrust. **1**

3. **(a)** PV^{γ} = constant or TV^{r-1} = constant. **½**

(b) $W = -nC_v (T_f - T_i)$. **½**

4. The strong smelling molecules of a gas move rapidly to intermingle with air molecules through the surrounding. This process takes little longer time because of collision of gas and air molecules. This keeps the odour persisting for some time. **1**

5. Condition **(i)** is not sufficient, because it gives no reference of the direction of acceleration, whereas in S.H.M. the acceleration is always in a direction opposite to that of the displacement. **1**

6. It is not necessary that a precise measurement has to be more accurate.

Let true value of certain length be 3 cm. This length is measured with a measuring instrument of limit of resolution of 0·1 cm and the measured value is obtained as 3·1 cm. This length is again measured with another measuring instrument of resolution 0·01 cm and the length is measured as 2·8 cm. **1**

In this case first measurement has more accuracy because it is closer to the true value but less precision because the resolution is only 0·1 cm whereas in the second it was 0·01 cm. **1**

7. This graph can be for a ball dropped vertically from a height *d*. It hits the ground with some downward velocity and bounces upto height *d*/2 where its upward velocity becomes zero. **2**

8. As the vectors $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ are perpendicular to each other, therefore **½**

$$(\vec{A}+\vec{B}).(\vec{A}-\vec{B}) = 0$$

$$\vec{A}.\vec{A}-\vec{A}.\vec{B}+\vec{B}.\vec{A}-\vec{B}.\vec{B} = 0$$

Or $$A^2 - B^2 = 0$$ **½**

$\Rightarrow$ $$A = B, \quad (\because \vec{A}.\vec{B} = \vec{B}.\vec{A})$$ **1**

Or

Static friction (f_s) comes into play when the horse applies force to start the motion in the cart, on the other hand, kinetic friction (f_k) comes into play when the cart is moving $f_s > f_k$. **2**

9. **(a)** The inability of a body to change its state of uniform rotation about an axis is called rotational inertia or M.I. of the body. **½**

It plays the same role in rotatory motion as is played by the mass in translatory motion, *i.e.*, it is rotational analogue of mass. ½

(b) M.I. of a body about a fixed axis of rotation is defined as the sum of the products of the masses and square of the perpendicular distance of various constituent particles from the axis of rotation. **1**

10. When a simple harmonic system oscillates with a constant amplitude which does not change with time, its oscillations are called undamped oscillations. When a simple harmonic system oscillates with a decreasing amplitude, with time its oscillations are called damped oscillations. **2**

11. Let the measured values be :

Mass of block, $(m) = 39{\cdot}3$ g
Length of block, $(l) = 5{\cdot}12$ cm
Breadth of block, $(b) = 2{\cdot}56$ cm
Thickness of block, $(t) = 0{\cdot}37$ cm

The density of the block is given by

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{m}{l \times b \times t}$$

$$= \frac{39{\cdot}3\,\text{g}}{5{\cdot}12\,\text{cm} \times 2{\cdot}56\,\text{cm} \times 0{\cdot}37\,\text{cm}}$$

$$= 8{\cdot}1037 \text{ g cm}^{-3}$$

Now, uncertainty in $m = \pm 0{\cdot}01$ g
uncertainty in $l = \pm 0{\cdot}01$ cm
uncertainty in $b = \pm 0{\cdot}01$ cm
uncertainty in $t = \pm 0{\cdot}01$ cm **1**

Maximum relative error in the density value is, therefore given by

$$\frac{\Delta\rho}{\rho} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta t}{t} + \frac{\Delta m}{m}$$

$$= \frac{0{\cdot}01}{5{\cdot}12} + \frac{0{\cdot}01}{2{\cdot}56} + \frac{0{\cdot}01}{0{\cdot}37} + \frac{0{\cdot}1}{39{\cdot}3}$$

$$= 0{\cdot}0019 + 0{\cdot}0039 + 0{\cdot}027 + 0{\cdot}0024$$

$$= 0{\cdot}0358$$ **1**

Hence, $\rho = 0{\cdot}0358 \times 8{\cdot}1037 \approx 0{\cdot}3 \text{ g cm}^{-3}$.

We cannot, therefore, report the calculated value of ρ (= $8{\cdot}1037$ gm^{-3}) upto the fourth decimal place. Since $\rho = 0{\cdot}3$ g cm^{-3} the value of ρ can be regarded as accurate upto the first decimal place only. Hence the value of ρ must be rounded off as $8{\cdot}1$ g cm^{-3} and the result of measurements should be reported as $\rho = (8{\cdot}1 + 0{\cdot}3)$ g cm^{-3}. **1**

12. **(a)** Bold As the ball is moving under the effect of gravity, the direction of acceleration due to gravity remain vertically downwards. ½

(b) If the ball is at the highest point of its motion, its velocity becomes zero and the acceleration is equal to the acceleration due to gravity = $9{\cdot}8$ ms^{-2} in vertically downward direction. ½

(c) If the highest point is chosen as the location for $x = 0$ and $t = 0$ and vertically downward direction to be the positive direction of x-axis.

For upward motion, sign of position is negative, sign of velocity is negative and the sign of acceleration is positive, *i.e.*, $v < 0$, $a > 0$.

For downward motion, sign of position is positive, sign of velocity is positive and the sign of acceleration is also positive, *i.e.*, $v > 0$, $a > 0$. ½

(d) Suppose, t = time taken by the ball to reach the highest point.

H = height of the highest point from the ground.

During vertically upward motion of the ball,

∴ Initial velocity, $u = -29{\cdot}4 \text{ ms}^{-1}$

$a = g = 9{\cdot}8 \text{ ms}^{-2}$.

Final velocity $v = 0$, $s = \text{H} = ?$, $t = ?$

Applying the relation, $v^2 - u^2 = 2as$, we get

$$0^2 - (29{\cdot}4)^2 = 2 \times 9{\cdot}8 \text{ H}$$

Or $\quad H = -\dfrac{29 \cdot 4 \times 29 \cdot 4}{2 \times 9 \cdot 8} = -44 \cdot 1 \text{ m}$ ½

where negative sign indicates that the distance is covered in upward direction.

Using relation $\quad v = u + at$, we get

$$0 = -29 \cdot 4 + 9 \cdot 8 \times t$$

$\therefore \quad t = \dfrac{29 \cdot 4}{9 \cdot 8} = 3 \text{ s.}$

i.e., Time of ascent = 3 s. ½

It is also well known that when the object moves under the effect of gravity alone, the time of ascent is always equal to the time of descent.

$\therefore$ Total time after which the ball returns to the player's hand. ½

$$= 2t = 2 \times 3 = 6 \text{ s.}$$

13. **Step I :** Using $\quad H = \dfrac{u^2 \sin^2 \theta}{2g}$ ½

when $\quad H = 25$ m, $u = 40$ m/s

and $\quad g = 9 \cdot 8$ m/s^2

$25 = \dfrac{40^2 \sin^2 \theta}{2 \times 9 \cdot 8}$ ½

i.e., $\quad \sin^2 \theta = \dfrac{490}{40^2}$

$$\sin \theta = \frac{\sqrt{490}}{40} = 0 \cdot 5534$$

i.e., $\quad \theta = 33 \cdot 6°$ **1**

Step 2 : $\quad R = \dfrac{u^2 \sin 2\theta}{g}$

$$R = \frac{40^2 \sin 2\,(33 \cdot 6)}{9 \cdot 8}$$

$$= \frac{40^2 \sin 67 \cdot 2}{9 \cdot 8} = \frac{40^2 \times 0 \cdot 9219}{9 \cdot 8}$$

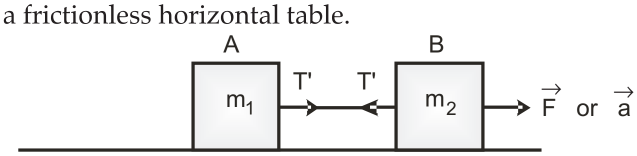
$= 150 \cdot 514$ **1**

14. Given, $\quad F = 600$ N

Suppose $\quad m_1 = 10$ kg

and $\quad m_2 = 20$ kg

be the masses lying on a frictionless horizontal table.



Suppose T be tension in the string and '*a*' be the acceleration of the system, in the direction of force applied.

(a) If force is applied on the heavier mass.

Then equation of motion of A and B are

$m_1 a = T$ ....(i)

$m_2 a = F - T$ ....(ii)

Dividing equation (ii) by equation (i), we get

$$\frac{m_2}{m_1} = \frac{F-T}{T} = \frac{F}{T} - 1 \qquad ½$$

$$\Rightarrow \quad \frac{20}{10} = \frac{F}{T} - 1$$

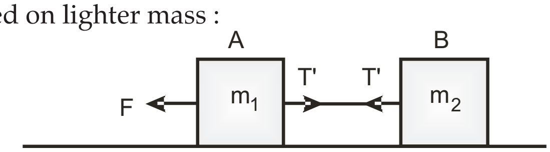
$$\Rightarrow \quad \frac{F}{T} = 2 + 1 = 3$$

$$\Rightarrow \quad T = \frac{F}{3}$$

$$= \frac{600}{3}$$

$$\Rightarrow \quad = 200 \text{ N} \qquad ½$$

(b) If the force is applied on lighter mass :



½

Suppose T be the tension in the string in this case, then equations of motion of A and B are,

$$F - T' = m_1 a \qquad ....(iii)$$

and
$$T' = m_2 a \qquad ....(iv) \quad ½$$

Equations (iii) and (iv) give

$$\frac{F-T'}{T'} = \frac{m_1 a}{m_2 a}$$

$$\Rightarrow \quad \frac{F}{T'} - 1 = \frac{m_1}{m_2} = \frac{10}{20} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{F}{T'} = 1 + \frac{1}{2} = \frac{3}{2}$$

$$\Rightarrow \quad T' = \frac{3}{2}F = \frac{2}{3} \times 600$$

$$= 400 \text{ N} \qquad ½$$

$$\Rightarrow \quad T' = \frac{2}{3} \times 600 = 400 \text{ N} \qquad ½$$

15. If m is mass of the gas molecule and M is the mass of the wall. The K.E. of the molecule before collision

$$E_1 = \frac{1}{2} m(200)^2$$

$$= 2 \times 10^4 \text{ m J} \qquad \mathbf{1}$$

The total K.E. after collision

$$E_2 = (100^2 + 100^2)\text{m}$$
$$= (2 \times 100^2)\text{m}$$
$$= (2 \times 10^4)\text{m} \qquad \mathbf{1}$$

Hence, the total kinetic energy of the molecule remains conserved during the collision. Therefore the collision is elastic. **1**

16. Using equation for position vector, $\vec{r} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$

Let $\vec{r} = 0$, **1**

then $m_1 \vec{r}_1 + m_2 \vec{r}_2 = 0$

i.e., $m_2 \vec{r}_2 = -\, m_1 \vec{r}_1$

i.e., $$r_2 = -\frac{m_1}{m_2}\vec{r_1}$$ **1**

Clearly, $\vec{r_2}$ and $\vec{r_1}$ are opposite to each other. It indicates when centre of mass lies on origin, m_1 lies on the left side of origin and m_2 on the right side, *i.e.,* they lie on a straight line. **1**

17. As per Kepler's third law, $$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$ **½**

Let 1 denotes the planet and 2 denotes the Earth

$$R_1^3 = \frac{T_1^2}{T_2^2}\times R_2^3$$ **1**

As the planet is revolving twice as fast as the Earth,

$$T_1 = \frac{T_2}{2}$$

$$\Rightarrow \quad R_1^3 = \frac{\left(\frac{T_2}{2}\right)^2}{T_2^2}\times(R_2)^3$$

$$\Rightarrow \quad R_1^3 = \frac{1}{4}(R_2)^3$$

$$R_1 = \left(\frac{1}{4}\right)^{1/3} R_2$$

$$R_1 = \frac{1}{\sqrt[3]{4}}R_2$$

$$R_1 = 0{\cdot}62996\ R_2\ \text{A.U.} = 0{\cdot}63\ R_2\ \text{A.U.}$$ **1**

This result can be stated as the orbital radius of the planet is 0·63 times the orbital radius of the Earth. **½**

18. If the temperature changes from 30°C to 10°C, then change in length of the wire is

$$\Delta l = l\alpha\,\Delta t = l \times 1.7 \times 10^{-5} \times 20$$
$$= 3.4 \times l \times 10^{-4}\ \text{m.}$$

Given :

$\rho = 9 \times 10^3$ kgm^{-3}, V = 1.4×10^{11} Nm^{-2}, $\Delta t = 30 - 10 = 20$ °C,

$$\alpha = 1.7 \times 10^{-5}\ °\text{C}^{-1}$$

Now $$Y = \frac{F/a}{\Delta l/l}$$

or $$F = Y a\frac{\Delta l}{l}$$ **1**

where a = area of cross-section of the wire

or $$F = \frac{1.4\times10^{11}\times a\times l\times3.4\times10^{-4}}{l}$$

$$\Rightarrow \quad F = 4.76 \times 10^7 \times a\text{N.}$$

Now $$v = \sqrt{\frac{T}{m}}$$ **1**

where $$m = a,\ \rho = 9 \times 10^3\ \text{kg m}^{-3}$$

$$\Rightarrow \quad v = \sqrt{\frac{4.76\times10^7\times a}{a\times9\times10^3}}$$

$$v = 72.72$$

So, $$v = 72\ \text{ms}^{-1}$$ **1**

Or

Node (N) is a point where the amplitude of oscillation is zero (and pressure is maximum).
Antinode (A) is a point where the amplitude of oscillation is maximum (and pressure is minimum).
These nodes and antinodes do not coincide with pressure nodes and antinodes. **1**
In fact, N coincides with pressure antinode and A coincides with pressure node, as is clear from the definitions. **1**
The separation between a node and the nearest antinode is $\lambda/4$.
As the phase difference between two points separated by A is 2π radian, therefore phase difference between two points separated by $\lambda/10$ is $2\pi/10 = \pi/5$ radian. **1**

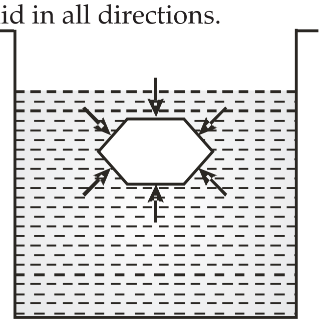
19. It states, that change in pressure is produced in any part of an enclosed fluid, the same is transmitted equally to all points of liquid in all directions. **1**

Or

Ignoring the effect of gravity, pressure in a fluid at rest is the same at all points.

Or

Equal pressure is exerted by a liquid in all directions.

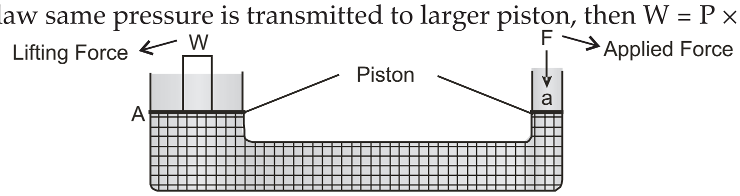


½

Applications : This principle is used to manufacture hydraulic lift. It consist of two cylinders, one of larger area of cross-section A and the other that of smaller area '*a*'. Force is applied to smaller piston to produce a pressure.

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$ **½**

As per Pascal's law same pressure is transmitted to larger piston, then W = P × A.



Clearly large area A is producing more lifting force W.
Hydraulic brakes are also based upon Pascal's law where force is pressure is transmitted to brake drum.
The large force then operates the brake shoe. **1**

20. (a) Streamline flow of a liquid is that flow in which every particle of the liquid follows exactly the path of its preceding particle and has the same velocity in magnitude and direction as that of its preceding particle while crossing through that point.

The streamline flow is accompanied by streamlines.

A streamline is the actual path followed by the procession of particles in a steady flow, which may be straight or curved such that tangent to it at any point indicates the direction of flow of liquid at the point. **1½**

(b) Important properties of streamlines :

(i) In a streamline flow, no two streamlines can cross each other. If they do so, the particles of the liquid at the point of intersection will have two different directions for their flow, which will destroy the steady nature of the liquid flow.

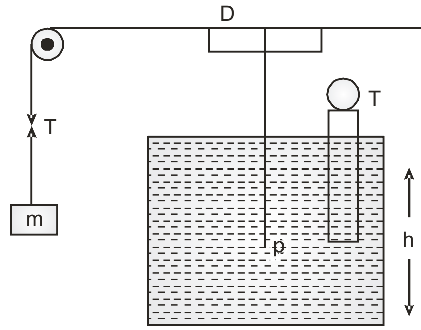
(ii) The greater is the crowding of streamlines at a place, the greater is the velocity of liquid particles at that place and vice-versa. **1½**

21. Joule observed that amount of mechanical work W when disappeared will appears in the form of heat, *i.e.*,

$$W \propto H$$
$$W = JH$$

Or $$J = W/H = 4{\cdot}18 \text{ Joule/calorie}$$ **1**

He made two equal weights to fall through some heights. This makes the drum D rotate and thus paddle P will push the water inside the calorimeter. **½**



½

This makes the water heated up. The rise in temperature is noted with thermometer T. The height of fall of weights helps in working out the mechanical work W. The temperature helps in calculating the heat energy H. It was observed that,

$$W \propto H.$$ **1**

22. **(a)** **Kelvin's statement :** It is not possible to get continuous work done by cooling a body to a temperature lower than that of coldest of its surroundings. **1**

Celsius statement : It is not possible to transfer heat from a body at lower temperature to another at higher temperature without the help of some external energy. **1**

(b) It is defined as the ratio of the amount of heat removed in a cycle from the refrigerator to the work done by some external agency to help the removal of this heat, *i.e.*,

$$\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$$ **½**

Also, $$\beta = \frac{T_2}{T_1 - T_2}$$

For higher efficiency of refrigerator β should be higher. **½**

23. **(a)** Sabita is sincere and hard working and having a scientific temper. **1**

(b) The restoring force F_1 acting on the mass m due to the spring having force constant k_1 will be,

$$F_1 = k_1 x_1$$

Here x_1 is the elongation.

The restoring force F_2 acting on the mass m due to the spring having force constant k_2 will be,

$$F_2 = k_2 x_2$$

Here x_2 is the elongation.

The tension in the two springs will be same.

So, $$k_1 x_1 = k_2 x_2$$

But the total extension is $x_1 + x_2$.

Thus the effective spring constant k of the combination will be,

$$k = \frac{F}{x}$$
$$= \frac{F}{(x_1 + x_2)}$$
$$= \frac{1}{[x_1/F + x_2/F]}$$
$$= \frac{1}{[1/k_1 + 1/k_2]}$$

$$= \frac{k_1 k_2}{k_1 + k_2}$$ **1**

Thus the time period of an object of mass m on a spring executes a simple harmonic motion is given by, $$T = 2\pi\sqrt{\frac{m}{k}}$$ ½

So frequency f on the oscillation would be,

$$f = \frac{1}{T}$$
$$= \frac{1}{\left(2\pi\sqrt{\frac{m}{k}}\right)}$$
$$= \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$ (Since, $T = 2\pi\frac{\sqrt{m}}{k}$) **1**

To find out the frequency of oscillation f, substitute $\frac{k_1 k_2}{k_1 + k_2}$ for the spring constant k in the equation

we get,

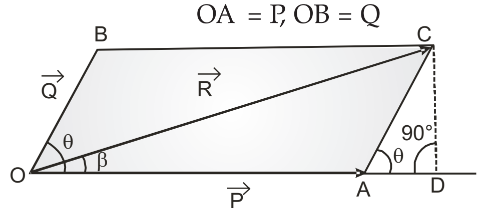
$$f = \frac{1}{2\pi}\sqrt{\frac{m}{k}},$$
$$f = \frac{1}{2\pi}\sqrt{\frac{m}{k}},$$
$$= \frac{1}{2\pi}\frac{\sqrt{\left(\frac{k_1 k_2}{k_1 + k_2}\right)}}{m}$$
$$T = 2\pi\sqrt{\frac{m(k_1 + k_2)}{k_1 k_2}}$$ **1**

24. It states that, if two vectors can be represented completely (*i.e.*, both in magnitude and direction) by the two adjacent sides of a parallelogram drawn from a point then their resultant is represented completely by its diagonal drawn from the same point. ½

Proof : Let $\vec{P}$ and $\vec{Q}$ be the two vectors represented completely by the adjacent sides OA and OB of the parallelogram OACB *s.t.*, such that,

$$\vec{OA} = \vec{P}, \vec{OB} = \vec{Q}$$

Or $$OA = P, OB = Q$$



½

$$\theta = \angle AOB$$

If R be their resultant, then it will be represented completely by the diagonal OC through point O such that, $$\vec{OC} = \vec{R}.$$

Magnitude of $\vec{R}$: Draw CD ⊥ to OA produced

$$\therefore \quad \angle DAC = \angle AOB$$
$$= \theta$$

Now in right angled triangle ODC,

$$OC^2 = OD^2 + DC^2$$
$$= (OA + AD)^2 + DC^2$$
$$= OA^2 + AD^2 + 2OA.AD + DC^2$$
$$= OA^2 + (AD^2 + DC^2) + 2OA.OD \quad ....(i)$$

Also in right angled triangle ADC,

$$AC^2 = AD^2 + AC^2 \qquad ....(ii)$$

Also, $$\frac{AD}{AC} = \cos\theta$$

Or $$AD = AC\cos\theta \qquad ....(iii)$$

and $$\frac{DC}{AC} = \sin\theta$$

Or $$DC = AC\sin\theta \qquad ....(iv)$$

∴ From eqns. (i), (ii), (iii), we get

$$OC^2 = OA^2 + AC^2 + 2OA.AC\cos\theta$$

Or $$OC = \sqrt{OA^2 + AC^2 + 2OA.AC\cos\theta} \qquad ....(v)$$

As $$OC = R, OA = P, AC = OB = Q \qquad ....(vi)$$

∴ From equations (v) and (vi), we get

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \qquad ....(vii)$$

Equation (iii) gives the magnitude of $\vec{R}$. **1**

Direction of R : Let β the angle made by $\vec{R}$ with $\vec{P}$.

∴ In right angled triangle ODC,

$$\tan\beta = \frac{DC}{OD} = \frac{DC}{OA + AD}$$

$$= \frac{AC\sin\theta}{OA + AC\cos\theta}, \qquad \text{[by using (iii) and (iv)]}$$

$$\tan\beta = \frac{Q\sin\theta}{P + Q\cos\theta} \qquad ....(viii)$$

Special cases : (a) When two vectors are acting in same direction, then

$$\theta = 0°$$

∴ $$R = \sqrt{(P+Q)^2}$$

$$= P + Q$$

and $$\tan\theta = \frac{Q.0}{P+Q} = 0$$

Or $$\beta = 0°$$

Thus, the magnitude of the resultant vector is equal to the sum of the magnitudes of the two vectors acting in the same direction and their resultant acts in the direction of P and Q. **1**

(b) When two vectors act in the opposite direction, then

$$\theta = 180°$$

∴ $$\cos\theta = -1 \text{ and } \sin\theta = 0$$

∴ $$R = \sqrt{P^2 + Q^2 + 2PQ(-1)}$$

$$= \sqrt{P^2 + Q^2 - 2PQ}$$

$$= \sqrt{(P-Q)^2} \text{ or } \sqrt{(Q-P)^2}$$

$$= (P - Q) \text{ or } (Q - P)$$

and $$\tan\beta = \frac{Q\times 0}{P + Q(-1)} = 0$$

$$= \tan 0 \text{ or } \tan 180°$$

∴ $$\beta = 0° \text{ or } 180°.$$

Thus, the magnitude of the resultant of two vectors acting in the opposite direction is equal to the difference of the magnitude of two vectors and it acts in the direction of bigger vector. **1**

(c) If θ = 90°, *i.e.,* if $\vec{P} \perp \vec{Q}$, then cos 90° = 0

and $$\sin 90° = 1$$

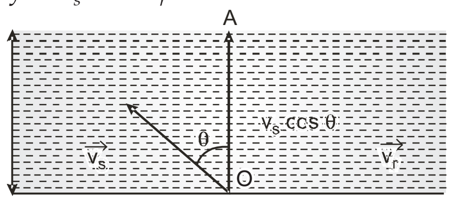
$$\therefore \qquad R = \frac{1}{\sqrt{P^2+Q^2}}$$

and $$\tan\beta = \frac{0}{P}$$ **1**

Or

(a) Let $\vec{v_s}$ and $\vec{v_r}$ be the velocities of swimmer and river respectively.

Let $\vec{v}$ = resultant velocity of v_s an d v_r.



½

(i) Let the swimmer begins to swim at an angle θ with the line OA where OA is ⊥ to the flow of river. If t = time taken to cross the river, then,

$$t = \frac{l}{v_s \cos\theta}$$ ½

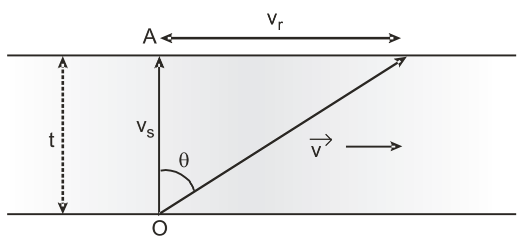
where l = breadth of river

For t to be minimum, cos θ should be maximum,

i.e., $\cos\theta = 1.$

This is possible, if $\theta = 0°.$

Thus, we conclude that the swimmer should swim in a direction ⊥ to the direction of flow of river.



(ii) $$v = \sqrt{v_s^2 + v_r^2}$$ ½

(iii) $$\tan\theta = \frac{v_r}{v_s} = \frac{X}{l}$$

or $$X = l\frac{v_r}{v_s}$$ ½

(iv) $$t = \frac{l}{v_s}$$ ½

(b) It is defined as a vector having zero magnitude and acting in the arbitrary direction. It is denoted by $\vec{0}$. ½

Properties of null vector :

(i) The addition or subtraction of zero vector from a given vector is again the same vector.

i.e., $$\vec{A} + 0 = \vec{A} - 0 = \vec{A}$$

(ii) The multiplication of zero vector by a non-zero real number is again the zero vector.

i.e., $$\pi 0 = 0$$ ½

(iii) If $n_1\vec{A} = n_2\vec{B}$, where n_1 and n_2 are non-zero real numbers, then the relation will hold good.

If $$\vec{A} = \vec{B} = 0$$

i.e., both $\vec{A}$ and $\vec{B}$ are null vectors. ½

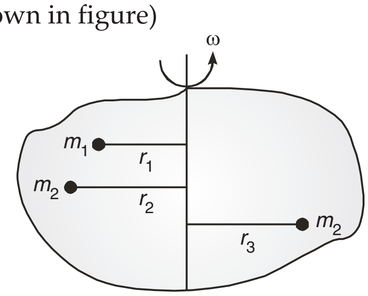
Physical significance of null vector : It is useful in describing the physical situation involving vector quantities.

e.g. $\vec{A} - \vec{A} = 0$

$\vec{0} \times \vec{A} = 0$ ½

25. **Torque and moment of inertia :** Consider a rigid body rotating about a given axis with a uniform angular acceleration a, under the action of a torque. ½

Let the body consist of particles of masses m_1, m_2, m_3, ..., mn at ⊥ distances r_1, r_2, r_3,...., r_n respectively from the axis of rotation (as shown in figure) **1**



As the body is rigid, angular acceleration 'a' of all the particles of the body is the same. However, the linear accelerations of the particles from the axis. differ. If a_1, a_2, a_3,..., a_n are the respective linear accelerations of the particle, then

$$a_1 = r_1\alpha,\ a_2 = r_2\alpha,\ a_3 = r_3\alpha,\ ....$$ **1**

Force on particle of mass m, is

$$f_1 = m_1 a_1 = m_1 r_1 \alpha$$

Moment of this force about the axis of rotation

$$f_1 \times r_1 = (m_1 r_1 \alpha) \times r_1 = m_1 r_1^2 \alpha$$ **1**

Similarly, moment of forces on other particles about the axis of rotation are $m_2r_2^2\alpha$, $m_3r_3^2\alpha$, ..., $m_nr_n^2\alpha$.

∴ Torque acting on the body,

$$\tau = m_1r_1^2\alpha + m_2r_2^2\alpha + m_3r_3^2\alpha,....,\ m_nr_n^2\alpha$$

$$\tau = (m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + m_nr_n^2)\alpha$$

So, $$\tau = \left(\sum_{i=1}^{i=n} m_i r_i^2\right)\alpha$$

$$\tau = I\alpha$$ **1**

where $m_1r_1^2$ = I = moment of inertia of this body about the given axis of rotation.

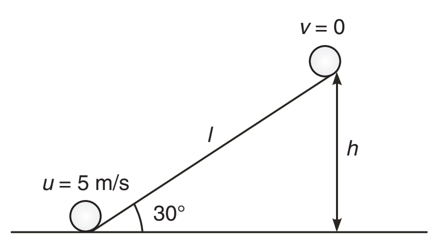
If $\alpha = 1,\ \tau = I$

$$\vec{\tau} = I\vec{\alpha}.$$ **1**

Or

If 'a' is the acceleration (*i.e.*, deceleration) in the present case

$$a = \frac{g \sin 30°}{\left(1 + \frac{K^2}{r^2}\right)}$$ **1**



In case of a solid cylinder

$$k = \frac{1}{\sqrt{2}}r$$ ½

$$a = \frac{g\sin 30°}{\left(1+\frac{\frac{1}{2}r^2}{r^2}\right)}$$

$$= \frac{g\frac{1}{2}}{1+\frac{1}{2}} = \frac{g}{3}$$ ½

From $v^2 = u^2 + 2as$

$$0 = (5)^2 + 2\left(\frac{-g}{3}\right)l$$

$$0 = 25 - \frac{2g}{3}l$$

$$l = \frac{25\times 3}{2g} = \frac{75}{2\times 9.8}$$

$= 3{\cdot}8$ m **1**

Now time for going up and returning must be equal.

From $s = ut + \frac{1}{2}at^2$,

we get, $l = 0 \times t + \frac{1}{2}\left(\frac{-g}{3}\right)t^2$

$$t^2 = \frac{6l}{g} = \frac{6\times 3.8}{9.8}$$

$$t = \sqrt{\frac{6\times 3.8}{9.8}}$$

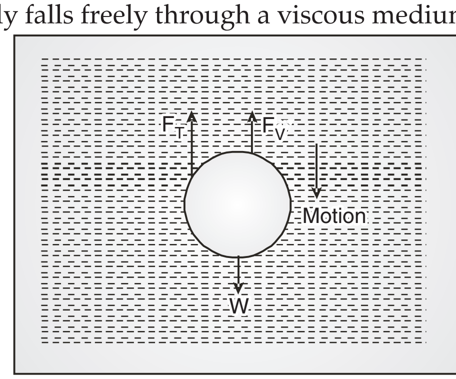
$= 1{\cdot}5$ sec. **1**

Total time for going up and coming down $= 2t$

$= 2 \times 1{\cdot}5 = 3$ s **1**

26. **Terminal velocity :** It is the maximum constant velocity acquired by a body while falling freely in a viscous medium.

When a small spherical body falls freely through a viscous medium, three forces act on it.



(i) Weight of the body acting vertically downwards. ½

(ii) Upward thrust due to buoyancy equal to weight of liquid displaced. ½

(iii) Viscous drag acting in the direction opposite to the motion of body.

According to Stoke's law,

$$F \propto v,$$

i.e., the opposing viscous drag goes on increasing with the increasing velocity of the body. ½

As the body falls through a medium, its velocity goes on increasing due to gravity. Therefore, the opposing viscous drag which acts upwards also goes on increasing. A stage reaches when the true weight of the body is just equal to the sum of the upward thrust due to buoyancy and the upward viscous drag. At this stage, there is no net force to accelerate the body. Hence it starts falling with a constant velocity, which is called terminal velocity. **1**

Let ρ be the density of the material of the spherical body of radius r and ρ_0 be the density of the medium.

$\therefore$ True weight of the body,

$$W = \text{volume} \times \text{density} \times g = \frac{4}{3}\pi r^3 \rho g$$

Upward thrust due to buoyancy,

F_T = weight of the medium displaced

$\therefore$ F_T = volume of the medium displaced × density × g = $\frac{4}{3}\pi r^3 \rho_0 g$ **1**

If v is the terminal velocity of the body, then according to Stoke's law.

$$\text{upward viscous drag } F_V = 6\pi\eta r v$$

When body attains terminal velocity, then

$$F_T + F_V = W$$

$\therefore$
$$\frac{4}{3}\pi r^3 \rho_0 g + 6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g$$

Or
$$6\pi\eta r v = \frac{4}{3}\pi r^3 (\rho - \rho_0) g$$

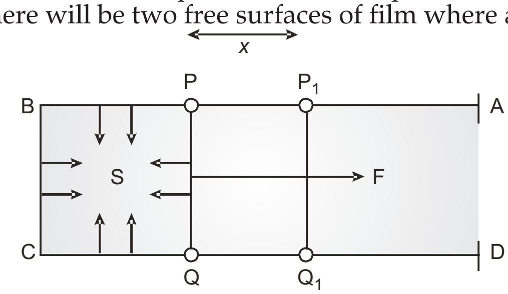
Or
$$v = \frac{2r^2(\rho - \rho_0)g}{9\eta}$$ **1**

Factors it depend upon : The terminal velocity varies directly as the square of the radius of the body and inversely as the coefficient of viscosity of the medium. It also depends upon densities of the body and the medium. ½

Or

Surface energy is defined as the amount of the work done against the force of surface tension, in forming the liquid surface of a given area at a constant temperature. **1**

To obtain a expression for surface energy, take a rectangular frame ABCD having a wire PQ which can slide along the sides AB and CD. Dip the frame in soap solution and form a soap film BCQP on the rectangular frame. There will be two free surfaces of film where air and soap are in contact. ½



Let S = surface tension of the soap solution.

l = length of the wire PQ. ½

Since there are two free surfaces of the film and surface tension acts on both of them, hence total inward force on the wire PQ is

$$F = S \times 2l$$ ½

To increase the area of the soap film, we have to pull the sliding wire PQ outwards with a force F. Let the film be stretched by displacing wire PQ through a small distance x to the position P_1Q_1.

The increase in area of film PQQ_1 $P_1 = a$

$= 2\,(l \times x)$ ½

$\therefore$ Work done in stretching film is,

$$\begin{aligned} E &= \text{force applied} \times \text{distance moved} \\ &= (S \times 2l) \times x \\ &= S \times (2lx) \\ &= S \times a, \end{aligned}$$ ½

where $2lx$ = a = increase in area of the film in both sides. ½

If temperature of the film remains constant in this process, this work done is stored in the film as its surface energy. Thus surface energy = Surface tension × increase in area. **1**

❑❑❑

SOLUTIONS

SAMPLE QUESTION PAPER - 9

Self Assessment

Time : 3 Hours **Maximum Marks : 70**

1. The value of g on the moon is small, therefore, the weight of the moon travellers will also be small. **1**

2. Since there is no water under the block to exert an upward force on it, therefore, there is no buoyant force. **1**

3. At the boiling point, vapour pressure of a liquid is equal to the atmospheric pressure. So, when the atmospheric pressure on the surface of the liquid increases, the liquids boil at higher temperature to generate greater vapour pressure. **1**

4. Since there is no acceleration in the body at the mean position, hence the resultant force on it will be zero, *i.e.*, the force applied by the spring will be exactly equal to the weight of the body. **1**

5. Yes, it is applicable to electromagnetic waves. **1**

6. The dimensions of L.H.S.,

$$\text{F. S.} = [MLT^{-2}].\,[L]$$
$$= [ML^2T^{-2}] \qquad \tfrac{1}{2}$$

The dimensions of R.H.S.,

$$\frac{1}{2}mv^2 \text{ or } \frac{1}{2}mu^2 = [M]\,[LT^{-1}]^2$$
$$= [ML^2T^{-2}] \qquad \tfrac{1}{2}$$

So, dimensions of L.H.S. = R.H.S

So, the given equation is dimensionally correct. **1**

7.
$$1\text{ N} = 1\text{ kg} \times 1\text{m/s}^2$$
$$= 1000\text{ gm} \times 100\text{ cm/s}^2$$
$$= 10^5\text{ gm-cm/s}^2$$
$$= 10^5\text{ dyne.} \qquad 1$$

The dyne is a unit of force specified in the centimeter-gram-second (cgs) system of units. One dyne is equal to exactly 10^{-5} newtons. Further, the dyne can be defined as " the force required to accelerate a mass of one gram at a rate of one centimeter per second squared. **1**

Or

$$|\vec{a}-\vec{b}|^2 - (|\vec{a}|+|\vec{b}|)^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}| \qquad \tfrac{1}{2}$$

$$\cos\theta - |\vec{a}|^2 - |\vec{b}|^2 - 2|\vec{a}||\vec{b}|$$

$$= 4|\vec{a}||\vec{b}|\cos^2\frac{\theta}{2}$$ **1**

= a negative quantity

Hence $|\vec{a}-\vec{b}| < |\vec{a}|+|\vec{b}|$ **½**

8. It's the power measured at some instant of time. Since power is energy per unit of time, it follows that

$$\text{Instantaneous power} = \frac{dE}{dt}.$$ **2**

9. Put the given graph for a stress of 150×10^6 Nm^{-2}, the strain is 0·002.

(a) From the formula, Young's modulus of the material (Y) is given by

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{150\times10^6}{0{\cdot}002} = \frac{150\times10^6}{2\times10^{-3}}$$

$$= 75 \times 10^9 \text{ Nm}^{-2}$$

$$= 7{\cdot}5 \times 10^{10} \text{ Nm}^{-2}.$$ **1**

(b) Yield strength of a material is defined as the maximum stress it can sustain without crossing the elastic limit.

∴ From the graph, the approximate yield strength of the given material

$$= 300 \times 10^6 \text{ Nm}^{-2} = 3 \times 10^8 \text{ Nm}^{-2}.$$ **1**

10. The P. E. of a particle executing S.H.M. is given by :

$$E_P = \frac{1}{2}m\omega^2 y^2.$$ **½**

E_P is maximum when $y = r$ = amplitude of vibration *i.e.*, the particle is passing from the extreme position and is minimum when $y = 0$, *i.e.*, the particle is passing from the mean position. **½**

The K.E. of a particle executing S.H.M. is given by,

$$E_K = m\omega^2(r^2 - y^2)$$

E_K is maximum when y = 0, *i.e.*, the particle is passing from the mean position and E_K is minimum when $y = r$, *i.e.*, the particle is passing from the extreme position. **1**

11. Size or diameter of a planet can be measured with the help of parallax method. Distance D of the planet from the Earth is measured with the help of relation $D = h/\theta$, where h is the distance between two observatories on the Earth and θ is the angle between two directions along which the planet was viewed from two observations. **1**

Let d be the diameter of planet and α the angle subtended by the diameter d at a point E on the Earth, then α being very small, $d/D < 1$. Let AB be an arc (of length d) of a circle with centre E, then distance

$$AB = d = D\alpha$$ **1**

Or $$\alpha = \frac{d}{D}$$

∴ Diameter, $$d = \alpha D$$ **1**

12. (i) Velocity at $t = 0$, $u = 0$

Velocity at $t = 2$ sec, $v = 20$ m/s

So, from $v = u + at$

We get $a = 10$ m/s^2

So distance covered between 0 to 2 sec.

$$s = ut + \frac{1}{2}at^2$$

$$= 0\times2 + \frac{1}{2}\times10(2)^2$$

$$= 20 \text{ m}$$

From 2 sec to 5 sec, velocity is same 20 m/s.

So, distance travelled = 20 × 3 = 60 m **1**

$\therefore$ Total distance covered between 0 to 5 sec

$$= 20 + 60 = 80 \text{ m}$$ **1**

(ii) At $t = 5$ sec, $u = 20$ m/s,

At $t = 10$ sec, $v = 0$ m/s

So, from $\quad v = u + at$

$\Rightarrow \quad 0 = 20 + a \times 5$

$\Rightarrow \quad a = -4 \text{ m/s}^2$

So, distance covered from $v^2 = u^2 + 2as$,

$$(0)^2 = (20)^2 - 2 \times 4 \times s$$

$\Rightarrow \quad s = \dfrac{400}{8} = 50$ m

So, the total distance covered $= 80 + 50$ **1**

$= 130$ m.

13. The nature of a vector may or may not be changed when it is multiplied, by a physical quantity **1**
For example, when a vector is multiplied by a pure number like 1, 2, 3,...., etc., then the nature of the vector does not change. **1**
On the other hand, when a vector is multiplied by a scalar physical quantity, then the nature of the vector changes. For example, when acceleration (vector) is multiplied by mass (scalar) of a body, then it gives force (a vector quantity) whose nature is different than acceleration. **1**

14. Given, F = 50 N, $m_1 = 5$ kg, $m_2 = 10$ kg, $m_3 = 15$ kg.

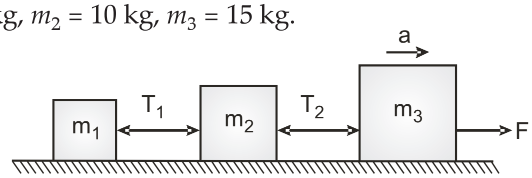


Fig. (a)

Since, the three blocks move with an acceleration 'a'.

So, $\quad a = \dfrac{F}{m_1 + m_2 + m_3}$

$\Rightarrow \quad a = \dfrac{50}{5+10+15} = \dfrac{50}{30}$

$= \dfrac{5}{3} \text{ ms}^{-2}$. **1**

To determine T_2 : Imagine the free body diagram (a).

Here $\vec{F}$ and $\vec{T_2}$ act towards right and left respectively.

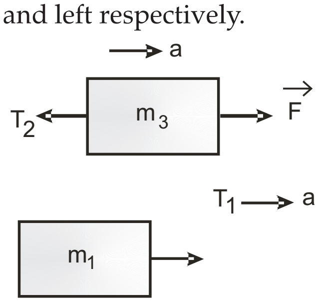


Fig. (b)

Since, the motion is towards the right side, so according to Newton's Second law of motion :

$$F - T_2 = m_3 a$$

$\Rightarrow \quad 50 - T_2 = 15 \times \dfrac{5}{3} = 25$

$\Rightarrow \quad T_2 = 50 - 25 = 25$ N. **1**

To determine T_1 : Consider the free body diagram (b).

Here $m_1 a = T_1$

Or $T_1 = m_1 a = 50 \times \frac{5}{3} = \frac{25}{3}$

$= 8.33$ N. **1**

15. Following are some important points about uniform motion :'

(i) The velocity in uniform motion does not depend upon the time interval $(t_2 - t_1)$. ½

(ii) The velocity in uniform motion is independent of the choice of origin. ½

(iii) No net force acts on the object having uniform motion. **1**

(iv) Velocity is taken to be positive when the object moves towards right of the origin and it is taken – ve if object moves toward left of the origin. **1**

16. The vehicle stops when its kinetic energy is spent in working against the force or friction between the tyres and the road. This force of friction varies directly with the weight of the vehicle. **1**

As the K.E. = Work done = Force of friction × distance

$E = f \times d$

Or $d = E/f$

Where, E = Kinetic energy

f = Force **1**

For given kinetic energy, distance d will be smaller, where F is larger, such as in the case of truck. Thus truck stops earlier. **1**

17. Using theorem of perpendicular axes,

$I_z = I_x + I_y$

As I_x and I_y are along the two diameters of disc so using symmetry, **1**

$I_x = I_y$.

So,zz $I_z = 2I_x$, But $I_2 = \frac{MR^2}{2}$ **1**

So, $I_x = \frac{I_z}{2} = \frac{MR^2}{4}$. **1**

Or

Momentum conservation and motion of the centre of the mass.

When a system of n particles is under the action of a total force f, then according to Newton's second law, ½

$$\vec{f} = \sum_{i=1}^{i=n} \frac{d}{dt}(m_i \vec{v_i}) \quad \text{....(1)}$$

$$\vec{f} = \frac{d}{dt}(m_1\vec{v_1} + m_2\vec{v_2} + ... + m_n\vec{v_n}) \quad ½$$

If no external force acts on the system, then total force, $\vec{f} = 0$. From (1) we obtain,

$$\frac{d}{dt}(m_1\vec{v_1} + m_2\vec{v_2} + ... + m_n\vec{v_n}) = 0$$

$$(m_1\vec{v_1} + m_2\vec{v_2} + ... + m_n\vec{v_n}) = k \quad \text{....(2)} \ ½$$

If M is the total mass of the system concentrated at the centre of mass, whose position vector is r, then

$$M\frac{d^2\vec{r}}{dt^2} = \vec{f}$$

Or

$$M\frac{d}{dt}\left(\frac{d\vec{r}}{dt}\right) = \vec{f}$$

But

$$\frac{d\vec{r}}{dt} = v_{c.m.}$$

$$= \text{velocity of centre of mass of the system}$$

$$\therefore \quad M\frac{d}{dt}(\vec{v}_{c.m.}) = \vec{f} \qquad \text{....(3)} \ \tfrac{1}{2}$$

If no external force acts on the system then $\vec{f} = 0$

From eqn. (3), we get

$$M\frac{d}{dt}(\vec{v}_{c.m.}) = 0 \qquad \tfrac{1}{2}$$

$$M \neq 0$$

$$\therefore \quad \frac{d}{dt}(\vec{v}_{c.m.}) = 0$$

$$\Rightarrow \quad \vec{v}_{c.m.} = \text{constant.} \qquad \tfrac{1}{2}$$

18. The minimum energy required for a satellite to leave its orbit around the Earth and escape to infinity is called the binding energy.

Expression :

A satellite revolving around the Earth has potential energy (P.E.) as well as kinetic energy (k.E.) P.E. due to gravitational field of the Earth and K.E. because of its motion.

$$\Rightarrow \quad \text{P.E.} = -\frac{GM_e m}{R_e} \qquad \text{...(i)} \ \tfrac{1}{2}$$

$$\text{K.E.} = \frac{1}{2}\left(\frac{-GM_e m}{R_e}\right) \qquad \text{...(ii)} \ \tfrac{1}{2}$$

$$\text{Total energy (T.E.)} = \text{P.E.} + \text{K.E.},$$

$$= \frac{-GM_e m}{R_e} + \frac{1}{2}\frac{GM_e m}{R_e} \qquad \mathbf{1}$$

$$= \frac{-1}{2}\frac{GM_e m}{R_e} \qquad \mathbf{1}$$

19. **(i)**

$$v_{\max} = A\omega.$$

If amplitude A is doubled, then value of the maximum velocity becomes double. **1**

(ii) Total energy,

$$E = \frac{1}{2}m\omega^2 A^2 .$$

If A is doubled, then E becomes four times. **1**

(iii) Time period, $T = 2\pi\sqrt{\frac{m}{k}}$,

since m and k do not change with the change in amplitude of oscillation, hence period of oscillator remains unchanged with change in amplitude of oscillations. **1**

20. Here,

$$y = 2\cos 2\pi(10t - 0{\cdot}0080x + 0{\cdot}35)$$

$$= 2\cos [2\pi(10\,t - 0{\cdot}0080x) + 2\pi\,(0{\cdot}35)]$$

$$= 2\cos\left[2\pi\times 0.0080\left(\frac{10}{0.0080}t - x\right) + 2\pi 0.35\right] \qquad \tfrac{1}{2}$$

Standard equation for a travelling wave is

$$y = r\cos\left[\frac{2\pi}{\lambda}(vt - x) + \phi\right]$$

Here

$$\phi = \frac{2\pi}{\lambda}\,x = 2\pi \times 0{\cdot}008x \qquad \left[\because \frac{2\pi}{\lambda} = 0.008\right] \ \tfrac{1}{2}$$

(a) When $x = 4$ m = 400 cm,

$$\phi = 2\pi \times 0{\cdot}008 \times 400$$
$$= 6{\cdot}4\ \pi \text{ rad.} \qquad \tfrac{1}{2}$$

(b) When $x = 0{\cdot}5$ m = 50 cm,

$$\phi = 2\pi \times 0{\cdot}008 \times 50$$
$$= 0{\cdot}8\ \pi \text{ rad.} \qquad \tfrac{1}{2}$$

(c) When $x = \dfrac{\lambda}{2}$,

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi \text{ rad.} \qquad \tfrac{1}{2}$$

(d) When $x = \dfrac{3}{4}\lambda$,

$$\phi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3}{2}\ \pi \text{ rad.} \qquad \tfrac{1}{2}$$

21. **(a)** The height of the blood column in the human body is more at the feet than at the brain. That is why, the blood exerts more pressure at the feet than at the brain ($\because$ pressure = $h\rho g$). **1**

(b) We know that the density of air is maximum near the surface of earth and decreases rapidly with height and at a height of about 6 km, it decreases to nearby half its value at the sea level. Beyond 6 km height, the density of air decreases very slowly with height. Due to this reason, the atmospheric pressure at the height of about 6 km decreases to nearly half of its value at sea level. **1**

(c) Since, due to applied force on liquid, the pressure is transmitted equally in all directions inside the liquid. That is why there is no fixed direction for pressure due to liquid. Hence hydrostatic pressure is a scalar quantity. **1**

22. **(a)** The gas would rush from A to B. The change in pressure or volume will take place under adiabatic conditions. The final pressure in the two cylinders would be 0·5 atm. **½**

(b) The change in internal energy of the gas will be zero. **½**

(c) The change in temperature will be zero. **1**

(d) Since the process is rapid, the intermediate states are not equilibrium states and hence do not satisfy the gas equation. So, the intermediate states of the system do not lie on the P – V – T surface. **1**

23. **(a)** Radha takes care of things and has concern for others. Practical in finding the solutions to problems. **2**

(b) When the wheel is rolling, the angular momentum is conserved. However, due to frictional force, it continues to decrease. Thus, the wheel can stay upright on its rim only for a certain interval of time. In the stationary position, the wheel falls due to unstable equilibrium. **2**

24. **(a)** Using the relation for kinetic energy,

$$\text{K.E.} = \frac{1}{2}\ mv^2$$

we get rate of change of K.E. with respect to time as

$$\frac{d}{dt}(\text{K.E.}) = \frac{d}{dt}\left[\frac{1}{2}mv^2\right]$$
$$= \frac{1}{2}\ m.2\ \frac{dv}{dt}\ v$$
$$= \frac{mdv}{dt}v$$

But $$\frac{mdv}{dt} = ma = \text{F}.$$ **1**

where a is acceleration and F is force.

$$\therefore \quad \frac{d}{dt}\text{ K.E.} = \text{F}.v.$$

$$= \text{F}\frac{dx}{dt}$$

Or $$d(\text{K.E.}) = \text{F}dx$$ **1**

Integrating between the intial and final energies, *i.e.*, K.E. and K.E.$_f$ and also positions, *i.e.*, x_i and x_f respectively, we get

$$\int_{\text{K.E}f}^{\text{K.E.}_i} d\text{.K.E.} = \int_{x_i}^{x_f} \text{F}.dx$$

$$\therefore \quad \text{K.E.}_f - \text{K.E.}_i = \text{W}.$$ **1**

The work energy theorem is verified.

(b) Potential energy will increase. This is because in bringing two protons closer, work has to be done against the force of repulsion. This is stored up in the form of potential energy. **1**

However, the potential energy will decrease when a proton and an electron are brought nearer. Work will be done by the force of attraction between them. **1**

Or

(a) Let mass of the block = m

After breaking, $$m_1 = \frac{2}{5}m \text{ and } m_2 = \frac{3}{5}m$$

Initial momentum, $$\text{P}_f = m_1\vec{v}_1 + m_2\vec{v}_2$$ **½**

According to law of conservation of momentum

$$\text{P}_f = \text{P}_i$$

$$\Rightarrow \quad m_1\vec{v}_1 + m_2\vec{v}_2 = 0$$ **1**

$$\vec{v}_1 = \text{velocity of smaller part}$$

$$\vec{v}_2 = \text{velocity of bigger part}$$

$$\Rightarrow \quad \frac{2}{5}m(8\hat{i}+6\hat{j})+\frac{3}{5}m(\vec{v}_2) = 0$$ **½**

$$\Rightarrow \quad \frac{3}{5}m\vec{v}_2 = -\frac{1}{5}m(16\hat{i}+12\hat{j})$$

$$\vec{v}_2 = -\left(\frac{16}{3}\hat{i}+4\hat{j}\right)$$ **1**

(b) As energy associated with discharge of a single neuron is 10^{-10} J, therefore total energy in a nerve impulse, where 10^5 neurons are fired is $10^{-10} \times 10^5$ J = 10^{-5} J. **2**

25. (a) When we blow over a piece of paper, velocity of air above the paper becomes more than that below it. Since, K.E. of air above the paper increases, so in accordance with Bernoulli's theorem $\text{P}+\frac{1}{2}\rho v^2$ = constant), its pressure energy and hence its pressure decreases. Because of greater value of pressure below the piece of paper – atmospheric pressure, it remains horizontal and does not fall.

While we blow under the paper, the pressure on the lower side decreases. The atmospheric pressure above the paper will therefore bend the paper downward. So the paper will not remain horizontal. **1**

(b) This can be cleared from the equation of continuity, *i.e.*, $a_1v_1 = a_2v_2$. We try to close a water tap with our fingers, the area of cross-section of the outlet of water jet is reduced considerably to the openings between our fingers provide constriction (*i.e.*, regions of smaller area). Hence, velocity of water increases greatly and fast jets of water come through the openings between our fingers. **1**

(c) From Bernoulli's theorem, we know that

$$P + \frac{1}{2}\rho v^2 + gh = \text{constant} \qquad \text{....(i)}$$

Here the size of the needle controls the velocity of flow and the thumb pressure controls pressure.

Now P occurs with power 1 and velocity (v) occuring with power 2 in equation (i) therefore, the velocity has more influence. That is why the needle of syringe has a better control over the flow rate. **1**

(d) If a fluid is flowing out of a small hole in a vessel, it acquires a large velocity and hence possesses large momentum. Since no external force is acting on the system, a backward velocity must be attained by the vessel (according to the law of conservation of momentum). As a result of it, an impulse (backward thrust) is experienced by the vessel. **1**

(e) This is because of Magnus effect, let a ball moving to the right be given a spin at the top of the ball. The velocity of air at the top is higher than the velocity of air below the ball. So according to Bernoulli's theorem, the pressure above the ball is less than the pressure below the ball. Thus there is a net upward force on the spinning ball, so the ball follows a curved path. This effect is known as magnus effect. **1**

Or

Given, Atmospheric pressure, P_0 = 76 cm of Hg.

(a) In figure (a) pressure head,

h_1 = 20 cm of Hg.

$\therefore$ Absolute pressure (P) of the gas is greater than the P_0,

i.e, $P = P_0 + h_1\rho g$

= 76 cm of Hg + 20 of Hg

= 96 cm of Hg. **1**

Gauge pressure is the difference between the absolute pressure and the atmospheric pressure. ½

It means, Gauge pressure $= P - P_0$

= 96 cm of Hg – 76 cm of Hg

= 20 cm of Hg

In figure (b), pressure head,

h_2 = – 18 cm of Hg. ½

$\therefore$ The absolute pressure of the gas (is lesser than the atmospheric pressure) is given by

$P = P_0 + h_2\rho g$

= 76 cm of Hg + (– 18 cm) of Hg

= 58 cm of Hg

Gauge pressure = absolute pressure – atmoshperic pressure ½

= 58 cm of Hg – 76 cm of Hg

= – 18 cm of Hg

It means, gauge pressure is simply equal to h cm of Hg. ½

(b) Given 13·6 cm of water added in the right limb is equivalent to $\frac{13.6}{13.6}$ = 1 cm of Hg column.

i.e., h= 1 cm of Hg column, which can be calculated as follows,

h_w = 13·6 cm of water

Suppose h_m = height of Hg column equivalent to 13·6 cm of water, thus equilibrium,

$$h_m\rho_m g = h_w\rho_w g \qquad ½$$

Or
$$h_m = h_w \frac{\rho_w}{\rho_m} = \frac{h_w}{\left(\frac{\rho_m}{\rho_w}\right)}$$
$$= \frac{13.6}{13.6} = 1 \text{ cm of Hg.}$$ ½

The mercury will rise in the left limb such that the difference in the height of Hg column in the two limbs

$$= (20 - 1) \text{ cm}$$
$$= 19 \text{ cm of Hg column.}$$ **1**

26. (a) It states that if the pressure remains constant, then the volume of a given mass of a gas increases or decreases by its volume at 0°C for each 1°C rise or fall of temperature.

Let V_0 be the volume of the given mass of gas at °C. According to Charle's law its volume at 1°C, **1**

$$V_1 = V_0 + \frac{V_0}{273.15}$$
$$V_0 = \left[1 + \frac{1}{273.15}\right]$$ **1**

Volume of the gas at 2°C,

$$V_2 = V_0\left[1+\frac{2}{273.15}\right]$$

$\therefore$ Volume of the gas at t°C,

$$V_1 = V_0\left[1+\frac{t}{273.15}\right]$$
$$= V_0\left[\frac{273.15 + t}{273.15}\right]$$ **1**

If T_0 and T are temperatures on kelvin scale corresponding to 0°C and t°C, then

$$T_0 = 273{\cdot}15 + 0 = 273{\cdot}15$$
$$T = 273{\cdot}15 + t$$
$$V_t = V_0 \frac{T}{T_0}$$
$$\frac{V_t}{T} = \frac{V_0}{T_0}$$
$$\frac{V}{T} = \text{constant}$$

i.e., $V \propto T.$ **1**

(b) This equation give the relation between pressure P, volume V and absolute temperature T of a gas,

$$PV = nRT.$$ ½

Derivation. According to Boyle's law,

$$V \propto \frac{1}{P} \qquad ....(1)$$

According to Charle's law,

$$V \propto T \qquad ....(2)$$

Comparing (1) and (2), we have

$$\frac{PV}{T} = \text{constant}$$

As $PV = RT.$

For n moles of gas $\quad PV = nRT$ ½

This is perfect or ideal gas equation.

Or

(a) Numerical value of R : Consider one mole of a gas at S.T.P., then

$$R = \frac{P_0 V_0}{T_0}$$ ½

Standard pressure, $\quad P_0 = 0{\cdot}76$ m of Hg column

$$= 0{\cdot}76 \times 13{\cdot}6 \times 10^3 \times 9{\cdot}8 \text{ N/m}^2$$

Standard temperature $= T_0 = 273{\cdot}15$ K

Volume of one mole of gas at $= 22{\cdot}4 \times 10^{-3}$ m^3

$$R = \frac{0.76 \times 13.6 \times 10^3 \times 9.8 \times 22.4 \times 10^{-3}}{273.15}$$

$$= 8{\cdot}01 \text{ J mole}^{-1} \text{ K}^{-1}.$$ **1**

In the C.G.S. system,

$$R = \frac{8.31}{4.2} \text{ cal mole}^{-1}\ °\text{C}^{-1}$$

$$= 1{\cdot}98 \text{ cal mole}^{-1}\ °\text{C}^{-1}.$$ ½

Numerical value of K_B : We know that

$$K_B = \frac{R}{NA}$$

$$K_B = \frac{8.31 \text{ J mole}^{-1} \text{ K}^{-1}}{6.02 \times 10^{23} \text{ mole}^{-1}}$$ ½

$$= 1{\cdot}38 \times 10^{-23} \text{ J/K}.$$ ½

(b) Suppose a polyatomic gas molecule has n degrees of freedom.

Total energy associated with the gram molecule of gas,

$$E = n \times \frac{1}{2} RT = \frac{n}{2} RT$$

i.e.,

As $\quad C_v = \frac{dE}{dT}$

$\therefore \quad C_v = \frac{d}{dT}(E)$ ½

$$= \frac{d}{dT}\left(\frac{n}{2} RT\right)$$

$$= \frac{n}{2} R$$

As $\quad C_P = C_V + R$

$$C_P = \frac{n}{2} R + R$$

$$= \left(\frac{n}{2} + 1\right) R$$ ½

As $\quad \gamma = \frac{C_P}{C_V}$

$$\gamma = \frac{\left(\frac{n}{2} + 1\right) R}{\frac{n}{2} R} = \frac{2}{n}\left(\frac{n}{2} + 1\right)$$

$$\gamma = 1 + \frac{2}{n}.$$ **1**

❑❑❑

SOLUTIONS

SAMPLE
QUESTION PAPER - 10

Self Assessment

Time : 3 Hours **Maximum Marks : 70**

1. Its value remains same as $6{\cdot}67 \times 10^{11}$ Nm^2 kg^{-2} as it is a universal constant and it does not depend on value of g. **1**

2. Compressibility is the reciprocal of the bulk modules, *i.e.*, compressibility = 1/K. **1**

3. **(a)** PV = constant, since $\Delta T = 0$ **½**

(b) $W = 2{\cdot}303\, n\mathrm{RT} \log\left(\frac{V_f}{V_i}\right)$ **½**

4. It gives an evidence for the existence of atoms. **1**

5. Yes, when a ball is dropped from a height on a perfectly elastic plane surface, the motion of a ball is oscillatory but not simple harmonic. **1**

6. The conventional rules are :

(i) If the insignificant digit to be dropped is more than 5 the preceeding digit is increased by 1 but if it is less than 5 then proceeding digits is not changed, *e.g.*, 1·748 is rounded off to 3 significant figures as 1·75 and 1·742 as 1·74. **1**

(ii) If the insignificant digit to be dropped is 5 then this digit is simply dropped, if the prceeding digit is even but if odd then the preceeding digit is increased by 1.

e.g., the number 1·845 rounded off to three significant digits is 1·84 but for number 1·875 it is 1·88. **1**

7. In one dimensional motion, the initial instant of time or zero time (t = 0) is considered at the beginning of the observation of motion. It may not be the instant of beginning of motion. Suppose a ball is thrown from a height of 500 m. It becomes visible say at a height of 100 m. Then origin or initial instant of time or zero time (t = 0) is this instant only and not the instant when motion actually began. **1**

Thus, if zero instant is considered as present then times, before the instant is past and time after this instant is future. **1**

8. Let θ is the angle between vectors $\vec{A}$ and $\vec{B}$

$\therefore$ $\vec{A} \cdot \vec{B} = AB\cos\theta$ ....(i) **½**

and $\vec{B} \cdot \vec{A} = BA\cos\theta$ ....(ii) **½**

From eqns. (i) and (ii), we get

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$ **1**

Or

In this case force $\vec{F}$ refers to the total external force, on the system (internal forces in the system are not included in $\vec{F}$) and $\vec{a}$ refers to the acceleration of the whole system, *i.e.*, acceleration of the centre of mass of the system. **2**

9. (a) The rotating wheels of a bicycle possess angular momentum. In the absense of an external torque, neither the magnitude nor the direction of angular momentum can change. The direction of angular momentum is along the axis of the wheel. So, the bicycle does not get tilted. **1**

(b) The cycle wheel is constructed in such a way so as to increase the M.I. of the wheel with minimum possible mass, which can be achieved by using spokes and the M.I. is increased to ensure the uniform speed. **1**

10. (i)

$$\sin \omega t - \cos \omega t = \sqrt{2}\left[\sin \omega t \cos \frac{\pi}{4} - \cos \omega t \sin \frac{\pi}{4}\right]$$

$$= \sqrt{2} \sin\left(\omega t - \frac{\pi}{4}\right) \qquad \frac{1}{2}$$

This function represents S.H.M. and periodic having period

$$T = \frac{2\pi}{\omega} \text{ and initial phase} = -\frac{\pi}{4} \text{ rad.} \qquad \frac{1}{2}$$

(ii)

$$\sin^2 \omega t = \frac{1}{2}(1 - \cos 2\omega t)$$

$$= \frac{1}{2} - \frac{1}{2}\cos 2\omega t. \qquad \frac{1}{2}$$

The function is periodic, having a period,

$$T = \frac{2\pi}{2\omega} = \frac{\pi}{\omega}$$

but does not represent S.H.M. ½

11. It is the indirect method of measuring the distances of the order of 10^{-10} m which is the size of an atom, *i.e.*, small distances. An atom is a tiny sphere. When such atoms lie packed in any substance, empty spaces are left in between. According to Avogadro's hypothesis, volume of all atoms in one gram of substance is 2/3 of the volume occupied by one gram of the substance.

$$V' = \frac{2}{3} V \qquad \text{....(i)}$$

V = Actual volume of one gram mass.

V′ = Volume occupied by atoms in 1 gram mass.

ρ = Density of the substance

$$V = \frac{1}{\rho} \qquad \text{....(ii)} \quad \mathbf{1}$$

Let M = atomic weight of the substance

N = Avogadro's no.

∴ No. of atoms in 1 gm of the substance $= \frac{N}{M}$

If r be the radius of each atom, then

V′ = no. of atoms in gm × volume of each atom

$$V' = \frac{N}{M} \times \frac{4}{3}\pi r^3 \qquad \text{....(iii)} \quad \mathbf{1}$$

∴ From eqns. (i), (ii) and (iii), we get

$$\frac{N}{M} \times \frac{4}{3}\pi r^3 = \frac{2}{3} \times \frac{1}{\rho}$$

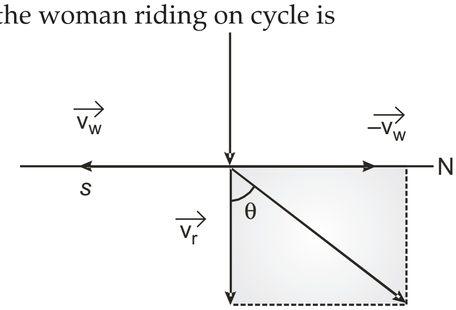
$$r = \left(\frac{M}{2\pi N\rho}\right)^{1/3} \qquad \mathbf{1}$$

12. Figure (a) does not represent one dimensional motion of particle because the particle has two different positions at the same instant which is not the case of one dimensional motion. Figure (d) also does not represent one dimensional motion of the particle because here the total path length is shown to decrease with time which is not possible in one dimensional motion. **1**

Graph (b) does not represent one dimensional motion because at the same instant a particle cannot have positive and negative velocity if the motion is one dimensional. **1**

Graph (c) does not represent one dimensional motion because speed cannot be negative as shown in graph. **1**

13. Velocity of rain relative to the woman riding on cycle is



1

$$\vec{v}_{rw} = \vec{v}_r - \vec{v}_w$$

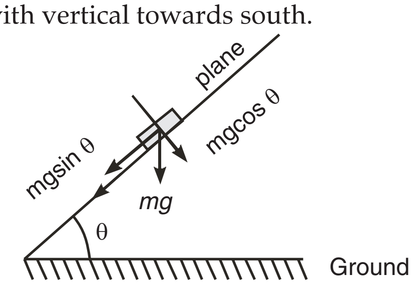
$$\tan\theta = \frac{v_w}{v_r} = \frac{10}{30} = \frac{1}{3} \qquad \mathbf{1}$$

$$= 0{\cdot}3333$$

$$\theta = 18°26' \qquad \tfrac{1}{2}$$

∴ She should hold her umbrella with vertical towards south. ½

14.



½

Given :

Angle of sliding $= 20°$

μ = coefficient of friction

$= \tan 20°$

$= 0.3647$

$s = 1.2$ m

Suppose, a is the acceleration when the inclination is increased to 30° and F be the value of limiting friction, then

$$ma = mg\sin\theta - F \qquad \mathbf{1}$$

Now $\quad F = \mu R = \mu mg\cos\theta$

∴ From equation (1),

$$ma = mg\sin\theta - \mu mg\cos\theta \qquad \tfrac{1}{2}$$

or $\quad a = g\sin\theta - \mu\, g\cos\theta$

⇒ $\quad a = 9.8\,(\sin 30° - 0.3647 \times \cos 30°)$

⇒ $\quad a = 9.8 \times \left(\frac{1}{2} - 0.3647 \times \frac{\sqrt{3}}{2}\right)$

$\Rightarrow \qquad a = 1.81 \text{ ms}^{-2}$

$\therefore$ Downward force,

$$F' = ma$$
$$= 10 \times 1.81 = 0.81 \text{ N}$$

Work done, $\quad W = Fs = 1.81 \times 1.2$
$$= 21.72 \text{ J.}$$ **1**

15. We are given that

T_{max} = maximum tension in the string so that it does not break

$= 2 \text{ kg } \omega t = 2 \times 10 \text{ N} = 20 \text{ N}$

Let T_1 be the tension in the string when the stone is in its lowest position of its circular path. We know that

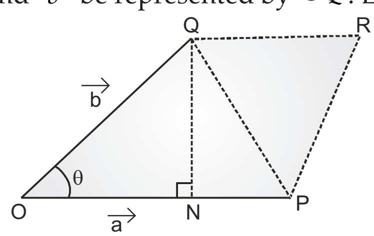
$$T_1 = \frac{mv_1^2}{l} + mg.$$ **1**

T_1 would have its minimum value when v_1 equal its minimum value = $\sqrt{5gl}$, needed by the stone, to complete its vertical circular path. **1**

Hence, $\quad (T_1) \text{ min} = mv_{\text{min}}^2 + mg$
$$= 6mg$$
$$= 6 \times 0{\cdot}4 \times 10$$
$$= 24 \text{ N}$$

We thus see that $(T_1)_{min}$ is more than the breaking strength of the string. Hence the particle cannot describe the vertical circle. **1**

16. Let $\vec{a}$ be represented by $\overrightarrow{OP}$ and $\vec{b}$ be represented by $\overrightarrow{OQ}$. Let $\angle POQ = \theta$



Complete the parallelogram OPRQ. Join PQ and draw QN ⊥ OP.

In ΔQNO, $\quad \sin\theta = \frac{QN}{OQ} = \frac{QN}{b}$

$$QN = b \sin\theta$$

Now, by definition,

$$|\vec{a} \times \vec{b}| = ab \sin\theta$$
$$= (OP)(QN)$$
$$= \frac{2(OP)(QN)}{2}$$
$$= 2 \times \text{area of } \Delta OPQ$$

$\therefore \qquad \text{Area of } \Delta OPQ = \frac{1}{2}|\vec{a} \times \vec{b}|$ **1**

17. **(a)** The linear speed ($v = \omega R$) changes because the distance, *i.e.*, (R) of the comet from the Sun changes due to it elliptical orbit around the Sun. ½

(b) The angular speed of the comet also changes because it covers different angle in equal intervals of time. ½

(c) The angular momentum of the comet is same throughout due to the conservation of angular momentum in the absence of any torque. ½

(d) Kinetic energy changes because linear speed is different at different points. ½

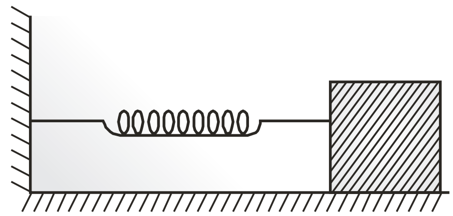
(e) The potential energy at different points is different because the comet is not at the same distance from the Sun (the orbit is not circular). ½

(f) The total energy of comet remains the same throughout the motion. ½

18. (i) Using, $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{M}}$, we get

$$f = \frac{1}{2\pi}\sqrt{\frac{1200}{3}}$$

$$= 3{\cdot}18 \text{ s}^{-1}$$



(ii) Maximum acceleration $= r\omega^2$ **1**

$$= r(2\pi f)^2$$
$$= 4\pi^2 f^2 r$$
$$= 4\pi^2 (3{\cdot}18)^2 \times 0{\cdot}02$$
$$= 7{\cdot}98 \text{ ms}^{-2}.$$ **1**

(iii) Maximum speed $= r\omega$

$$= r \times 2\pi f$$
$$= 2\pi \times 3{\cdot}18 \times 0{\cdot}02$$
$$= 0{\cdot}04 \text{ ms}^{-1}$$ **1**

Or

According to Newton's law, speed of sound,

$$v = \sqrt{\frac{K_{isothermal}}{\rho}} \quad ...(i)$$

where $K_{isotermal}$ = pressure P for isothermal change

$$v = \sqrt{\frac{P}{\rho}} \quad ...(ii)$$ **1**

where $P = 76$ cm of Hg

$= 76 \times 13{\cdot}6 \times 980$ dyne cm^{-2}

and $\rho = 1{\cdot}293 \times 10^{-3}$ g cm^{-3}

∴ v at N.T.P. $= 280$ ms^{-1} (from ii)

Laplace suggested that $K_{isothermal}$ should be replaced with $K_{adiabitic}$.

$$v = \sqrt{\frac{K_{adiabitic}}{\rho}}$$

Here PV^{γ} = constant **1**

Differentiating, we have

$$P\gamma V^{\gamma-1}\, dV + V^{\gamma}\, dP = 0$$

i.e., $\gamma P dV = -V dP$

Or $\dfrac{dV}{V} = -\dfrac{dP}{\gamma P}$

But $K_{adiabitic} = \dfrac{dp}{dv/v} = \gamma P$

∴ $v = \sqrt{\dfrac{\gamma P}{\rho}}$

$$v_{\text{N.T.P.}} = \sqrt{\frac{(1{\cdot}41)(76\times13{\cdot}6\times980)}{1{\cdot}293\times10^{-3}}}$$

$= 333\ ms^{-1}$. **1**

19. Given, $Y = 12{\cdot}5\times10^{11}$ dyne cm^{-2}

$= 12{\cdot}5\times10^{10}\ Nm^{-2}$

Diameter, $D = 2{\cdot}5$ mm $= 2{\cdot}5\times10^{-3}$ m

$\therefore$ Radius, $r = \frac{D}{2} = 1{\cdot}25\times10^{-3}$ m.

$F = 100$ kgf

$= 100\times9{\cdot}8$ N

$= 980$ N

$$\frac{\Delta L}{L}\times100 = ?$$

$$A = \pi r^2 = \pi\,(1{\cdot}25\times10^{-3})^2\ m^2.$$

$$Y = \frac{F/A}{\Delta L/L}$$ **1**

$$\%\ \text{increase in length} = \frac{\Delta L}{L}\times100$$

$$= \frac{F}{AY}\times100$$

$$= \frac{F}{\pi r^2 Y}\times100$$

$$= \frac{980}{3{\cdot}142\times(1{\cdot}25\times10^{-3})^2\times12{\cdot}5\times10^{10}}\times100$$

$= 15{\cdot}96\times10^{-2}$

$= 0{\cdot}16\%$ **1**

20. Let a be the area of cross-section of the rod, V be the volume of mass attached, when rod is floating on water, then

weight of float = weight of water displaced

$= (a\times3 + V)\times1\times g$ ....(1) ½

When rod is floating in a liquid.

Weight of liquid displaced $= (a\times3{\cdot}15 + V)\ 0{\cdot}9\times g$ ....(2)

Or $3a + V = 3{\cdot}15\,a + 0{\cdot}9\,V$

Or $V = 1{\cdot}5\,a$ ½

Let x be the depth of the rod immersed in a liquid of sp. gravity 1·2, then weight of liquid displaced

$= (xa + V)1{\cdot}2\times g$ **1**

$\therefore$ $(xa + V)\ 1{\cdot}2\times g = (a\times3 + V)\ g$

$(xa + 1{\cdot}5\,a)\times1{\cdot}2 = 3\,a + 1{\cdot}5\,a = 4{\cdot}5\,a$

Or $1{\cdot}2\,x + 1{\cdot}8 = 4{\cdot}5$

$x = 2{\cdot}25$ cm. **1**

21. Using Newton's Law of Cooling :

Rate of loss of heat = (K)* (difference in temperature of surrounding and body)

Taking average of 80 °C and 50 °C *i.e.,* 65 ° C as the temperature of body.

$$\frac{(80-50)}{5\,\text{min}} = K\times(65\times20)$$

$$6 = K\times45$$

This gives the value of $K = \frac{6}{45}$ **1**

Now assume that the second change takes about "t min."

And we see that the avg. temp of body now is $\frac{60+30}{2} = \frac{90}{2} = 45$

from Newton's Law,

$\Rightarrow$ $$\frac{(60-30)}{t} = \frac{6}{45} \times (45-20)$$

Solving the above for t gives t = 9 min. **1**

22. For using the internal energy of sea water to operate the engine of a ship, the internal energy of the sea water has to be converted into mechanical energy. The whole of the internal energy cannot be converted into mechanical energy, a part has to be rejected to a colder body (sink). Since, no such body is available, the internal energy of the sea water cannot be used to operate the engine of the ship. **3**

23. (a) Adarsh is hardworking, thinks logically, having scientific temper, able to find solutions with patience. **2**

(b) Since, length of the pendulum l is proportional to g, the length of the pendulum on the surface of the moon will be 1/6 m. **2**

24. Proof. When we can show that Newton's first and third laws are contained in the second law, then we can say that it is the real law of motion.

(i) First law is contained in second law : According to Newton's second law of motion,

$$\vec{F} = m\vec{a} \quad \text{....(1)}$$

where m = mass of the body on which an external force $\vec{F}$ is applied and a is acceleration produced in it.

When no external force is applied on the body, *i.e.*,

When $\vec{F}$ = 0, then from equation (1), we get

$$m\vec{a} = 0, \text{ but as } m \neq 0$$

$\therefore$ $$\vec{a} = 0$$ ½

It means that there will be no acceleration in the body if no external force is applied. This represents that a body at rest will remain at rest and a body in uniform motion will continue to move along the same straight line in the absence of an external force. This corresponds to Newton's first law of motion. So, first law of motion is contained in second law of motion. ½

(ii) Third law is contained in second law : Consider an isolated system of two bodies A and B. Let they interact enternally.

Suppose,

F_{AB} = force applied on body A by body B.

and F_{BA} = force applied on body B by body A.

When $\frac{d\vec{P}_A}{dt}$ = rate of change of momentum of body A.

and $\frac{d\vec{P}_B}{dt}$ = rate of chage of momentum of body B.

Then, from Newton's second law of motion,

$$F_{AB} = \frac{d\vec{P}_A}{dt} \quad \text{....(2)}$$

$$F_{BA} = \frac{d\vec{P}_B}{dt} \quad ....(3) \quad \mathbf{1}$$

Equations (2) and (3) give,

$$\vec{F}_{AB} + \vec{F}_{BA} = \frac{d}{dt}(\vec{P}_A) + \frac{d}{dt}(\vec{P}_B) = \frac{d}{dt}(\vec{P}_A + \vec{P}_B) \quad \mathbf{1}$$

Since no external force acts on the system ($\because$ it is isolated), therefore according to Newton's second law of motion,

$$\frac{d}{dt}(\vec{P}_A + \vec{P}_B) = 0$$

Or
$$\vec{F}_{AB} + \vec{F}_{BA} = 0$$

Or
$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Action force = – Reaction force. **1**

It means that action and reaction are equal and opposite. It is the statement of Newton's third law of motion. Thus, 3rd law is contained in the second law of motion.

As both first and third laws are contained in second law, so second law is the real law of motion. **1**

Or

(a) Impulse-momentum theorem states that the impulse of force on a body is equal to the change in momentum of the body.

i.e.,
$$\vec{J} = \vec{F}\,t = \vec{P}_2 - \vec{P}_1 \quad \mathbf{1}$$

Proof. According to Newton's second law of motion, we know that

$$\vec{F} = \frac{d\vec{P}}{dt}$$

Or
$$\vec{F}\,dt = d\vec{P} \quad ....(i) \quad \tfrac{1}{2}$$

When $\vec{F}$ = constant force acting on the body.

Suppose $\vec{P}_1$ and $\vec{P}_2$ be the linear momentum of the body at time $t = 0$ and t respectively.

$\therefore$ Integrating equation (i) within these limits, we get

$$\int_0^t \vec{F}\,dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{P} \quad \tfrac{1}{2}$$

$\Rightarrow$
$$F\int_0^t dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{P}$$

$$\vec{F}\,[t]_0^t = [P]_{p_1}^{p_2}$$

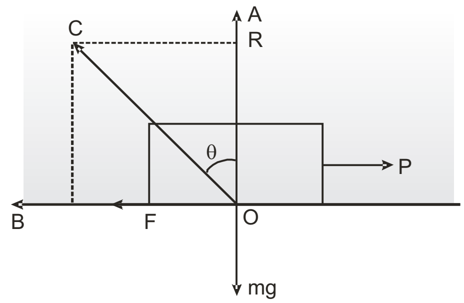
$$\vec{F}\,t = \vec{P}_2 - \vec{P}_1$$

$$\vec{J} = \vec{P}_2 - \vec{P}_1 \quad . \quad \mathbf{1}$$

(b) The coefficient of friction between any two surfaces in contact is defined as the ratio of the force of limiting friction and normal reaction between them.

$$\mu = \frac{F}{R} \quad \tfrac{1}{2}$$

Angle which the resultant of force of limting friction F and normal reaction R makes with the direction of normal reaction R is the angle of reaction. ½



Relation in Δ AOC

$$\tan\theta = \frac{AC}{OA} = \frac{OB}{OA} = \frac{F}{R} = \mu$$

Hence $\mu = \tan\theta$. **1**

25. Suppose two balls A and B of masses m_1 and m_2 are moving initially along the same striaght line with velocities u_1 and u_2 respectively.

When $u_1 > u_2$, relative velocity of approach before collision,

$$= u_1 - u_2$$ ½

When the two balls collide, let the collision be perfectly elastic. After collision, suppose v_1 is velocity of A and v_2 is velocity of B along the same straight line, When $v_2 - v_1$, relative velocity seperation after collision,

$$= v_2 - v_1$$ ½

Linear momentum of the two balls before collision,

$$= m_1u_1 + m_2u_2$$ ½

Linear momentum of the two balls after collision,

$$= m_1v_1 + m_2v_2$$ ½

As linear momentum is conserved in an elastic collision, therefore ½

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2 \quad \text{....(i)}$$

Or

$$m_2(v_2 - u_2) = m_1(u_1 - v_1) \quad \text{....(ii)}$$

Total K.E. of the two balls before collision

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \quad \text{....(iii)}$$

Total K.E. of the two balls after collision

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{...(iv)}$$ ½

As K.E. is also conserved in an elastic collision, therefore from (iii) and (iv),

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1u_2^2 + \frac{1}{2}m_2u_2^2$$ ½

Or

$$\frac{1}{2}m_2(v_2^2 - u_2^2) = \frac{1}{2}m_1(u_1^2 - v_1^2)$$

Or

$$m_2(v_2^2 - u_2^2) = m_1(u_1^2 - v_1^2) \quad \text{....(v)}$$ ½

Dividing (v) by (ii), we get

$$\frac{m_2(v_2^2 - u_2^2)}{m_2(v_2 - u_2)} = \frac{m_1(u_1^2 - v_1^2)}{m_1(u_1 - v_1)}$$

$$\frac{(v_2 + u_2)(v_2 - u_2)}{(v_2 - u_2)} = \frac{(u_1 + v_1)(u_1 - v_1)}{(u_1 - v_1)}$$

Or $v_2 + u_2 = u_1 + v_1$

Or $v_2 - v_1 = u_1 - u_2$ ....(vi) ½

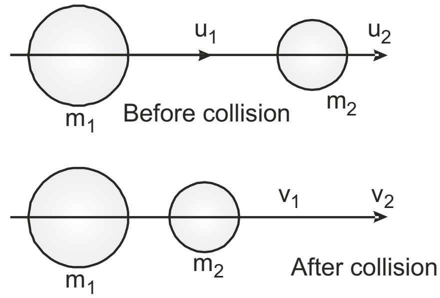
Hence in one dimensional elastic collision, relative velocity of separation after collision is equal to relative velocity of approach before collision.

From, $$\frac{v_2 - v_1}{u_1 - u_2} = 1$$

By definition, $$\frac{v_2 - v_1}{u_1 - u_2} = e = 1.$$ ½

Or

Figure shows two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 respectively, along a single axis. They collide involving some loss of kinetic energy. Therefore, the collision is inelastic. Let v_1 and v_2 be the velocities of the two bodies after collision. ½



As the two bodies form one system, which is closed and isolated, we can write the law of conservation of linear momentum for the two body system as :

Total momentum before the collision (P_i) = Total momentum after the collision (P_f)

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$ ½

For perfectly inelastic collision between two bodies of masses m_1 and m_2 : The body of mass m_2 happen to be initially at rest ($u_2 = 0$) we refer to this body as the target. The incoming body of mass m_1, moving with intial velocity u_1 is referred to as the projectile. After the collision, the two bodies move together with a common velocity v. The collision is perfectly inelastic. As the total linear momentum of the system cannot change, therefore $P_i = P_f$. ½

i.e., $m_1u_1 + m_2u_2 = (m_1 + m_2)v$

Or $m_1 u_1 = (m_1 + m_2)v$, $(\because \boldsymbol{u}_2 = 0)$

Or $$v = \frac{m_1u_1}{m_1 + m_2}$$ ...(i) **1**

Knowing m_1, m_2 and u_1, we can calculate the final velocity v. As the mass ratio

$$\frac{m_1}{m_1 + m_2} < 1,$$

$\therefore$ therefore $v < u_1$. **1**

We can calculate loss of K.E. in this collision.

Total K.E. before collision, $$E_1 = \frac{1}{2}m_1u_1^2$$

Total K.E. after collision, $$E_2 = \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2}(m_1 + u_2)\left(\frac{m_1u_1}{m_1 + m_2}\right)^2$$ Using (i)

$$= \frac{m_1^2u_1^2}{2(m_1 + m_2)}$$ ½

Loss of K.E. $= E_1 - E_2$

$$= \frac{1}{2}m_1u_1^2 - \frac{m_1^2u_1^2}{2(m_1+m_2)}$$

$$= \frac{m_1^2u_1^2 + m_1m_2u_1^2 - m_1^2u_1^2}{2(m_1+m_2)}$$

$$= \frac{m_1m_2u_1^2}{2(m_1+m_2)}.$$

1

26. Consider an incompressible non-viscous liquid entering the cross-section A_1 at A with a velocity v_1 and coming out at a height h_2 at B with velocity v_2.

The P.E. and K.E. increase since h_2 and v_2 are more than h_1 and v_1 respectively. These are done by the pressure doing work on the liquid. If P_1 and P_2 are the pressures at A and B and a small displacement at A and B.

The work done on the liquid $\quad A = (P_1A_1)$

$\Delta x_1 = P_1A_1v_1\Delta t$

The work done by the liquid at B $= -(P_2A_2)$

$\Delta x_2 = -P_1A_2v_2\,\Delta t$ **1**

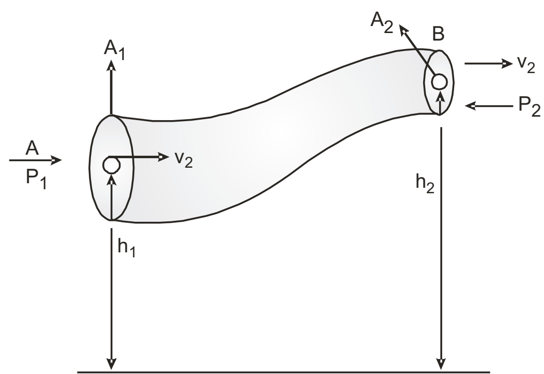
(Considering a small time Δt so that area may be same.)

Net work done by pressure $= (P_1 - P_2)Av\Delta t,$ $\quad [\because A_1v_1 = A_2v_2]$

From the conservation of energy,

$(P_1 - P_2)Av\Delta t$ = change in (K.E. + P.E.) **1**

$(P_1 - P_2)Av\Delta t = Av\rho\Delta tg(h_2 - h_1) + \frac{1}{2}Av\rho\Delta tg(v_2^2 - v_1^2)$ **½**



$$(P_1 - P_2) = \rho g(h_2 - h_1) + \frac{\rho}{2}(v_2^2 - v_1^2)$$ **½**

i.e., $$P_1 + \rho gh_1 + \frac{\rho}{2}v_1^2 = P_2 + \rho hg_2 + \frac{\rho}{2}v_1^2$$

i.e., $$\frac{P}{\rho g} + h + \frac{v^2}{2g} = \text{constant.}$$ **1**

Limitations of bernoulli's theorem :

(a) While deriving Bernoulli's theorem, it is assumed that velocity of every particle of liquid across any cross section of tube is uniform. Practically it is incorrect.

(b) The viscous drag of the liquid which comes into play when liquid is in motion has not been taken into account.

(c) While deriving the equation, it is assumed that there is no loss of energy when liquid is in motion. **1**

Or

Refer Ans 25, Sample Question Paper 7.

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