Class XI

Oscillations and waves worksheet 4

1.	What is the relation between uniform circular motion and S.H.M?	1
Ans.	Uniform circular motion can be thought of as two S.H.M operating at right angle to each other.	
2.	What is the minimum condition for a system to execute S.H.M?	1
Ans.	The minimum condition for a body to posses S.H.M. is that it must have elasticity and inertia.	
3.	A particle executing S.H.M. along a straight line has a velocity of u m/s when its displacement from mean position is 3 m and 3 m / s when displacement is 4m. Find the time taken to travel 2.5 m from the positive extremity of its oscillation?	1
Ans.	$v_1^2 = \omega^2(a^2 - y_1^2)$ $16 = \omega^2(a^2 - 3^2) \longrightarrow (1)$ $v_2^2 = \omega^2(a^2 - y_2^2)$ $9 = \omega^2(a^2 - 4^2) \longrightarrow (2)$ $(1) \div (2)$ $\frac{16}{9} = \frac{\omega^2(a^2 - 9)}{\omega^2(a^2 - 16)}$ $16(a^2 - 16) = 9(a^2 - 9)$ $16a^2 - 256 = 9a^2 - 81$ $16a^2 - 9a^2 = -81 + 256$ $7a^2 = 175$ $a^2 = 25$ $a = 5m$ $from (1)$ $16 = \omega^2(5^2 - 3^2)$ $16 = \omega^2 \times 16$ $\omega = 1\ rad.s^{-1}$ When the particle is 2.5m from the positive extreme position, its displacement from the mean position, y = 5 – 2.5 = 2.5m. Since the time is measured when the particle is at extreme position $y = a\cos\omega t$ $2.5 = 5\cos t$ $\frac{1}{2} = \cos t$ $\frac{\pi}{3} = t$ $\frac{3.14}{3} = t$ $t = 1.047s$	

4.	Springs is spring constant K, 2K, 4K, ----- are connected in series. A mass M Kg is attached to the lower end of the last spring and system is allowed to vibrate. What is the time period of oscillation?	1
Ans.	$\frac{1}{K}=\frac{1}{k_1}+\frac{1}{k_2}+----$ $\frac{1}{K}=\frac{1}{k}+\frac{1}{2k}+\frac{1}{4k}+\frac{1}{8k}+----$ $\frac{1}{K}=\frac{1}{k}\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+---\right]$ $\frac{1}{K}=\frac{1}{k}\left[\frac{1}{1-\frac{1}{2}}\right]$ $\frac{1}{K}=\frac{2}{k}$ $K=\frac{k}{2}$ $T=2\pi\sqrt{\frac{M}{K}}=2\pi\sqrt{\frac{M}{\frac{k}{2}}}=2\pi\sqrt{\frac{2M}{k}}$	
5.	A particle is moving with SHM in a straight line. When the distance of the particle from mean position has values x_1 and x_2 the corresponding values of velocities are v_1 and v_2. Show that the time period of oscillation is given by: $T=2\pi\left[\frac{x_2^2-x_1^2}{v_1^2-v_2^2}\right]^{1/2}$	2

Ans.	$v_1^2 = \omega^2(a^2 - x_1^2) \longrightarrow (1)$ $v_2^2 = \omega^2(a^2 - x_2^2) \longrightarrow (2)$ $(1)-(2)$ $v_1^2 - v_2^2 = \omega^2(a^2 - x_1^2) - \omega^2(a^2 - x_2^2)$ $= \omega^2\left[a^2 - x_1^2 - a^2 + x_2^2\right]$ $= \omega^2\left(x_2^2 - x_1^2\right)$ $\omega^2 = \frac{\left(v_1^2 - v_2^2\right)}{\left(x_2^2 - x_1^2\right)}$ $\omega = \left[\frac{\left(v_1^2 - v_2^2\right)}{\left(x_2^2 - x_1^2\right)}\right]^{\frac{1}{2}}$ $\frac{2\pi}{T} = \left[\frac{\left(v_1^2 - v_2^2\right)}{\left(x_2^2 - x_1^2\right)}\right]^{\frac{1}{2}}$ $T = 2\pi\left[\frac{\left(x_2^2 - x_1^2\right)}{\left(v_1^2 - v_2^2\right)}\right]^{\frac{1}{2}}$	
6.	Show that the total energy of a body executing SHM is independent of time?	2
Ans.		
7.	A particles moves such that its acceleration 'a' is given by a = -b x where x = displacement from equilibrium position and b is a constant. Find the period of oscillation?	2
Ans.	$a = -bx$ As $a \,\alpha\, x$ and is directed towards mean position so the particle executes S. H. M $a = bx \;\; (magnitude)$ $\frac{a}{x} = \frac{1}{b}$ $T = 2\pi\sqrt{\frac{x}{a}} = 2\pi\sqrt{\frac{1}{b}}$	
8.	A particle is S.H.M. is described by the displacement function: $x = a\cos(wt + \phi)$ If the initial (t = 0) position of the particle is 1 cm, angular frequency is π/s and its initial velocity is π cm|s, What are its amplitude and phase angle?	3

Ans.	$t=0$ $x=1$ $v=\pi\ ms^{-1}$ $x=a\cos(\omega t+\phi)$ $1=a\cos(\omega(0)+\phi)$ $1=a\cos\phi \longrightarrow (1)$ $v=\frac{dx}{dt}$ $v=-a\omega\sin(\omega t+\phi)$ $\pi=-a(\pi)\sin(\omega(0)+\phi)$ $-1=a\sin\phi \longrightarrow (2)$ $(1)^2+(2)^2$ $1+1=a^2(\cos^2\phi+\sin^2\phi)$ $a^2=2$ $a=\sqrt{2}cm$ $(2)\div(1)$ $\tan\phi=-1$ $\phi=\tan^{-1}(-1)$	
9.	Determine the time period of a simple pendulum of length = l when mass of bob = m Kg?	3
Ans.	Prove $T=2\pi\sqrt{\frac{l}{g}}$	