A.K. Yadav

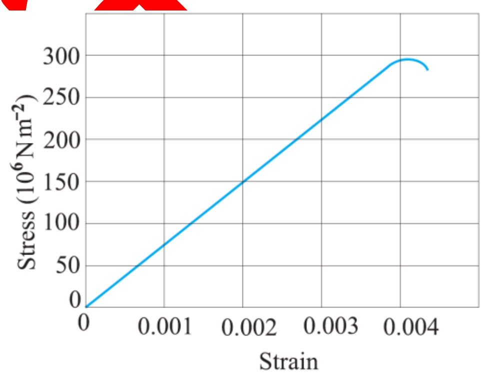
NCERT ANSWERS

CHAPTER 9

1. A steel wire of length 4.7 m and cross-sectional area 3 x 10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of 4 x 10^{-5} m^2 under a given load. What is the ratio of the Young's modulus of steel to that of copper?

Ans.
$$\frac{Y_S}{Y_C}=\frac{\dfrac{F_S L_S}{A_S \Delta L_S}}{\dfrac{F_C L_C}{A_C \Delta L_C}}=\frac{F_S L_S}{A_S \Delta L_S}\times\frac{A_C \Delta L_S}{F_C L_C}=\frac{F\times 4.7}{3\times10^5\times\Delta L}\times\frac{4\times10^5\times\Delta L}{F\times 3.5}=1.79:1$$

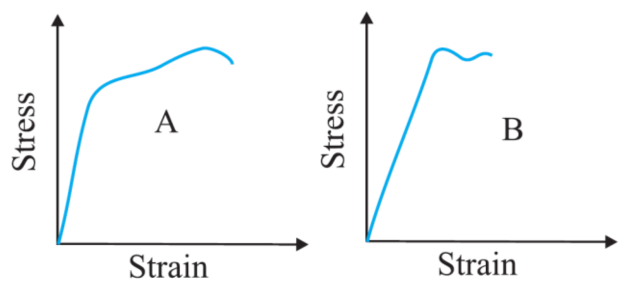
2. Figure shows the strain-stress curve for a given material. What are
(a) Young's modulus and
(b) approximate yield strength for this material?



Ans.
(a) $Y=\dfrac{stress}{strain}=\dfrac{150\times10^6}{0.002}=7.5\times10^{10}\,Nm^{-2}$

(b) Yield strength (max. stress that the material can sustain without crossing the elastic limit): 300× 10^6 Nm^{-2}

3. The stress-strain graphs for two materials *A* and *B* are shown in Fig.
The graphs are drawn to the same scale.
(a) Which of the materials has the greater Young's modulus?
(b) Which of the two is the stronger material?



Ans.
(a) $Y=\dfrac{stress}{strain}$

For a given strain, $Y \,\alpha\, stress$

Here stress A > Stress B hence, $Y_A > Y_B$

(b) A is stronger than B as its fracture point is more than the fracture point of B.

4. Read the following two statements below carefully and state, with reasons, if it is true or false.

(a) The Young's modulus of rubber is greater than that of steel;

(b) The stretching of a coil is determined by its shear modulus.

Ans. (a) False

Reason: for a constant stress, $Y \alpha \dfrac{1}{strain}$

Strain in rubber > strain in steel

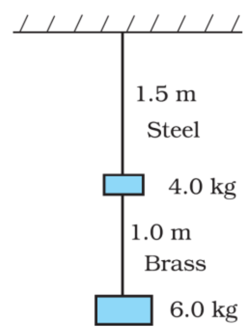
So, $Y_S > Y_R$

(b) True

Reason: $G = \dfrac{tangential\ stress}{shearing\ strain}$

Stretching a body changes the shape i.e. shearing strain which in turn depends on shear modulus.

5. Two wires of diameter 0.25 cm, one made of steel and the other made of brass are loaded as shown in Fig. The unloaded length of steel wire is 1.5 m and that of brass wire is 1m. Compute the elongations of the steel and the brass wires.



Ans.

For steel wire	For brass wire
Young's modulus, $Y_S = 2\times10^{11}\, Nm^{-2}$	Young's modulus, $Y_B = 0.91\times10^{11}\, Nm^{-2}$
Diameter of wire, $D_S = 0.25cm = 25\times10^{-4}\, m$	Diameter of wire, $D_S = 0.25cm = 25\times10^{-4}\, m$
Radius of wire, $r_S = 0.125cm = 12.5\times10^{-4}\, m$	Radius of wire, $r_B = 0.125cm = 12.5\times10^{-4}\, m$
Length of wire, $L_S = 1.5m$	Length of wire, $L_B = 1m$
Force on wire, $F_S = (4+6)\times9.8 = 98N$	Force on wire, $F_B = 6\times9.8 = 58.8N$
$\Delta l_S = \frac{F_S L_S}{A_S Y_S} = \frac{98\times1.5}{3.14\times12.5\times12.5\times10^{-8}\times2\times10^{11}} = 1.49\times10^{-4}\, m$	$\Delta l_B = \frac{F_B L_B}{A_B Y_B} = \frac{58.8\times1}{3.14\times12.5\times12.5\times10^{-8}\times0.91\times10^{11}} = 1.3\times10^{-4}\, m$

6. The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?

Ans. $\text{Shearing stress} = \frac{F}{A} = \frac{100 \times 9.8}{0.1 \times 0.1} = 9800N$

$\text{Shearing strain} = \frac{\Delta l}{L} = \frac{\Delta l}{0.1}$

$$G = \frac{tangential\ stress}{shearing\ strain}$$

$$25 \times 10^9 = \frac{9800}{\frac{\Delta l}{0.1}}$$

$$\frac{\Delta l}{0.1} = \frac{9800}{25 \times 10^9}$$

$$\Delta l = 3.92 \times 10^{-7} m$$

7. Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 and 60 cm respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column.

Ans. Force on one column, $F = \frac{50000}{4} \times 9.8 = 122500N$

Area of column, $A = \pi\left(R^2 - r^2\right) = 3.14\left(0.6^2 - 0.3^2\right) = 0.8478m^2$

Stress on column, $stress = \frac{122500}{0.8478} = 144491.62N$

$$Strain = \frac{stress}{modulus\ of\ elasticity} = \frac{144491.62}{2 \times 10^{11}} = 7.22 \times 10^{-7}$$

8. A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N force, producing only elastic deformation. Calculate the resulting strain?

Ans. Force, $F = 44500N$

Area, $A = lb = 15.2 \times 10^{-3} \times 19.1 \times 10^{-3} = 2.9 \times 10^{-4} m^2$

$$stress = \frac{44500}{2.9 \times 10^{-4}} = 15344.8 \times 10^4 N$$

$$Strain = \frac{stress}{modulus\ of\ elasticity} = \frac{144491.62}{2 \times 10^{11}} = 7.22 \times 10^{-7}$$

9. A steel cable with a radius of 1.5 cm supports a chairlift at a ski area. If the maximum stress is not to exceed 108 N m^{-2}, what is the maximum load the cable can support?

Ans. $Stress = \frac{Force}{Area}$

Max. Force, $F = 10^8 \times 3.14 \times 0.015 \times 0.015 = 7.065 \times 10^4 N$

10. A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratios of their diameters if each is to have the same tension.

Ans.
- Extension in each wire is same as same tension is acting on all the wires.
- Strain is same as all the wires are of same length

So, $Y = \dfrac{\frac{F}{A}}{strain} = \dfrac{4F}{\pi d^2}$ becomes $Y \;\alpha\; \dfrac{1}{d^2}$

$$\frac{d_1}{d_2} = \sqrt{\frac{Y_2}{Y_1}} = \sqrt{\frac{110\times10^9}{190\times10^9}} = \sqrt{\frac{11}{19}} = 1:1.31$$

11. A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path.

Ans. Total force, $F = mg + ml\omega^2 = 14.5\times9.8 + 14.5\times1\times2^2 = 200.1N$

$$Y = \frac{FL}{A\Delta L}$$

$$2\times10^{11} = \frac{200.1\times1}{0.065\times10^{-4}\times\Delta L}$$

$$\Delta L = 1.539\times10^{-4}m$$

12. Compute the bulk modulus of water from the following data: Initial volume = 100 litre, Pressure increase = 100 atm (1 atm = 1.013 × 10^5 Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

Ans.
- Change in volume, $\Delta V = V_2 - V_1 = 100.5\times10^{-3} - 100\times10^{-3} = 0.5\times10^{-3}m^3$

$$B_w = \frac{PV}{\Delta V} = \frac{100\times1.013\times10^5\times100\times10^{-3}}{0.5\times10^{-3}} = 2.026\times10^9\,Pa$$

$$B_a = 10^5\,Pa$$

$$\frac{B_w}{B_a} = \frac{2.026\times10^9}{10^5} = 2.026\times10^4$$

- Ratio is very high because air is more compressible than water.

13. What is the density of water at a depth where pressure is 80 atm, given that its density at the surface is 1.03 × 10^3 kg m^{-3}?

Ans. Let the given depth be *h*.

Pressure at the given depth, *p* = 80.0 atm = 80 × 1.01 × 10_5 Pa

Density of water at the surface, $\rho_1 = 1.03 \times 10^3$ kg m^{-3}

Let ρ_2 be the density of water at the depth h.

Let V_1 be the volume of water of mass m at the surface and V_2 be the volume of water at the depth h.

Change in volume, $\Delta V = V_1 - V_2 = \frac{m}{\rho_1} - \frac{m}{\rho_2} = m\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$

Volumetric strain, $\frac{\Delta V}{V_1} = \frac{m\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)}{\frac{m}{\rho_1}} = \rho_1\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right) = 1 - \frac{\rho_1}{\rho_2}$

Now, $B = \frac{PV_1}{\Delta V}$

$$0.2\times10^{10} = \frac{80\times1.013\times10^5}{1-\frac{\rho_1}{\rho_2}}$$

$$1-\frac{\rho_1}{\rho_2} = 4.05\times10^{-3}$$

$$\frac{\rho_1}{\rho_2} = 1-4.05\times10^{-3}$$

$$\frac{1.03\times10^3}{\rho_2} = 0.99595$$

$$\rho_2 = 1034 kgm^{-3}$$

14. Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.

Ans.

$$B = \frac{PV}{\Delta V}$$

$$\frac{\Delta V}{V} = \frac{P}{B} = \frac{10\times1.013\times10^5}{37\times10^9} = 2.73\times10^{-5}$$

15. Determine the volume contraction of a solid copper cube, 10 cm on an edge, when subjected to a hydraulic pressure of 7×10^6 Pa.

Ans.

$$B = \frac{PV}{\Delta V}$$

$$B = \frac{Pl^3}{\Delta V}$$

$$140\times10^9 = \frac{7\times10^6\times(0.1)^3}{\Delta V}$$

$$\Delta V = 5\times10^{-2} cm^{-3}$$

16. How much should the pressure on a litre of water be changed to compress it by 0.10%?

Ans.

$$\frac{\Delta V}{V} = \frac{\frac{0.1}{100}}{1} = 0.001$$

$$B = \frac{PV}{\Delta V}$$

$$P = B\frac{\Delta V}{V} = 2.2 \times 10^9 \times 0.001 = 2.2 \times 10^6\, Nm^{-2}$$