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NCERT ANSWERS
CHAPTER 14

1. Which of the following examples represent periodic motion?

(a) A swimmer completing one (return) trip from one bank of a river to the other and back.

(b) A freely suspended bar magnet displaced from its N-S direction and released.

(c) A hydrogen molecule rotating about its center of mass.

(d) An arrow released from a bow.

Ans.

Case	Y/N	Reason
(a) A swimmer completing one (return) trip from one bank of a river to the other and back.	N	Time may not be same.
(b) A freely suspended bar magnet displaced from its N-S direction and released.	Y	Magnet oscillates about its mean position with definite time period
(c) A hydrogen molecule rotating about its center of mass.	Y	It comes to the mean position after equal intervals of time
(d) An arrow released from a bow.	N	Arrow only moves in forward direction

2. Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?

(a) the rotation of earth about its axis.

(b) motion of an oscillating mercury column in a U-tube.

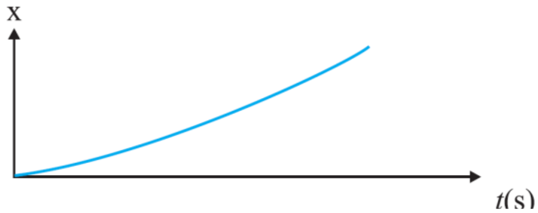
(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.

(d) general vibrations of a polyatomic molecule about its equilibrium position.

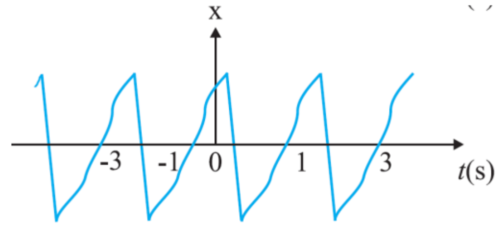
Ans.

Case	P/SHM	Reason
(a) the rotation of earth about its axis.	P	No SHM because earth does not have to and fro motion about its axis.
(b) motion of oscillating mercury column in U-tube.	SHM	To and fro motion with definite time period
(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lower most point.	SHM	To and fro motion with definite time period
(d) general vibrations of a polyatomic molecule about its equilibrium position	P	Vibration of a polyatomic molecule is the superposition of individual SHMs of different molecules.

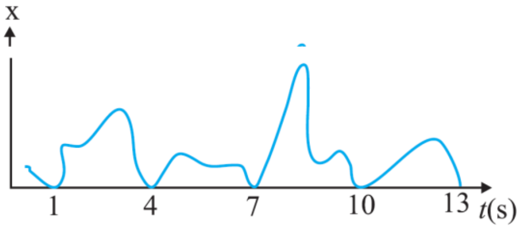
3. Figure depicts four *x*-*t* plots for linear motion of a particle. Which of the plots



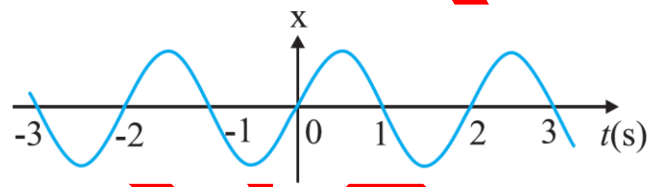
(a)



(b)



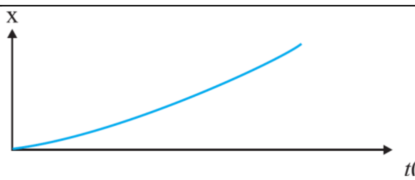
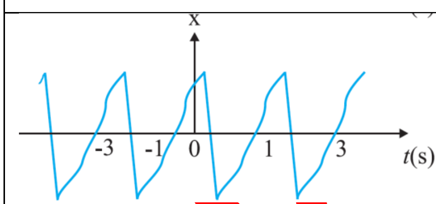
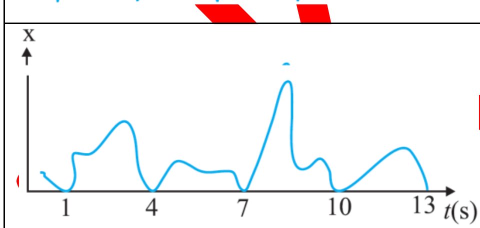
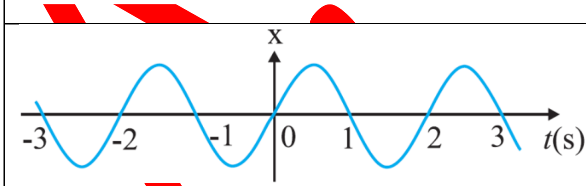
(c)



(d)

represent periodic motion? What is the period of motion (in case of periodic motion)?

Ans.

	• No repetition of motion. • Not a periodic motion. • It is a unidirectional, linear uniform motion.
	• Motion of the particle repeats itself after 2 s. • Periodic motion • Time period = 2 s.
	• Particle repeats the motion in one position only. • Not a periodic motion.
	• Motion of the particle repeats itself after 2 s. • Periodic motion • Time period = 2 s.

4. Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):

(a) $\sin \omega t - \cos \omega t$ (b) $\sin^3 \omega t$ (c) $3 \cos (\pi/4 - 2\omega t)$

(d) $\cos \omega t + \cos 3\omega t + \cos 5\omega t$ (e) $\exp(-\omega^2 t^2)$ (f) $1 + \omega t + \omega^2 t^2$

Ans.

(a) $\sin\omega t - \cos\omega t = \sqrt{2}\left[\frac{1}{\sqrt{2}}\sin\omega t - \frac{1}{\sqrt{2}}\cos\omega t\right] = \sqrt{2}\left[\cos\frac{\pi}{4}\sin\omega t - \sin\frac{\pi}{4}\cos\omega t\right] = \sqrt{2}\sin\left(\omega t - \frac{\pi}{4}\right)$

- Function represents SHM as it is similar to $a\sin(\omega t + \phi)$
- Period : $\frac{2\pi}{\omega}$

(b) $\sin^3\omega t = \frac{1}{2}[3\sin\omega t - \sin 3\omega t]$

- Superposition of two SHMs ($\sin\omega t$, $\sin 3\omega t$) is periodic motion not SHM.

(c) $3\cos\left[\frac{\pi}{4} - 2\omega t\right] = 3\cos\left[2\omega t - \frac{\pi}{4}\right]$

- Function represents SHM as it is similar to $a\cos(\omega t + \phi)$
- Period : $\frac{2\pi}{2\omega} = \frac{\pi}{\omega}$

(d) $\cos\omega t + \cos 3\omega t + \cos 5\omega t$

- Superposition of three SHMs ($\cos\omega t$, $\cos 3\omega t$, $\cos 5\omega t$) is periodic motion not SHM.

(e) $e^{-\omega^2 t^2}$

- Non- periodic as exponential functions do not repeat themselves.

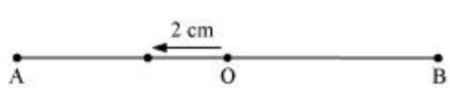
(f) $1 + \omega t + \omega^2 t^2$

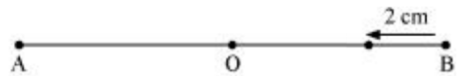
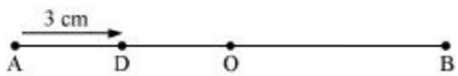
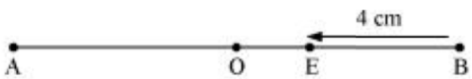
- Non-periodic

5. A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as positive direction and give the signs of velocity, acceleration & force on the particle when it is

(a) at the end A,
(b) at the end B,
(c) at the mid-point of AB going towards A,
(d) at 2 cm away from B going towards A,
(e) at 3 cm away from A going towards B, and
(f) at 4 cm away from B going towards A.

Ans.

Case	velocity	Acceleration	Force
(a) at the end A	Zero Velocity is zero at ends	Positive Directed from A to B	Positive Directed from A to B
(b) at the end B	Zero Velocity is zero at ends	Negative Directed from B to A	Negative Directed from B to A
(c) at the mid-point of AB going towards A 	Negative Directed from B to A	Zero Acc. is zero at mid-pt.	Zero

(d) at 2 cm away from B going towards A 	Negative	Negative	Negative
	direction of motion is opposite to the conventional +ve direction		
(e) at 3 cm away from A going towards B 	Positive	Positive	Positive
	direction of motion is same as the conventional +ve direction		
(f) at 4 cm away from B going towards A 	Negative	Negative	Negative
	direction of motion is opposite to the conventional +ve direction		

6. Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?

(a) $a = 0.7x$ (b) $a = -200x^2$ (c) $a = -10x$ (d) $a = 100x^3$

Ans. (c) because it is equivalent to the SHM relation $a = -\frac{k}{m}x$

7. The motion of a particle executing simple harmonic motion is described by the displacement function, $x(t) = A\cos(\omega t + \varphi)$. If the initial ($t = 0$) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is π s^{-1}.
If instead of the cosine function, we choose the sine function to describe the SHM: $x = B\sin(\omega t + \alpha)$, what are the amplitude and initial phase of the particle with the above initial conditions.

Ans. (At t = 0, x = 1, v = ω cm/s, ω = π rad/s)

Case I

As $x = A\cos(\omega t + \phi)$

$$1 = A\cos(\omega(0) + \phi)$$

$$A\cos\phi = 1 \longrightarrow (1)$$

Also $v = \frac{dx}{dt}$

$$\omega = \frac{d}{dt}(A\cos(\omega t + \phi))$$

$$\omega = -A\omega\sin(\omega t + \phi)$$

$$1 = -A\sin(\omega(0) + \phi)$$

$$A\sin\phi = -1 \longrightarrow (2)$$

$$(1)^2 + (2)^2 \Rightarrow (A\cos\phi)^2 + (A\sin\phi)^2 = (1)^2 + (-1)^2$$

$$A^2 = 2$$

$$A = \sqrt{2}cm$$

Case II

As $x = B\sin(\omega t + \alpha)$

$$1 = B\sin(\omega(0) + \alpha)$$

$$B\sin\alpha = 1 \longrightarrow (1)$$

Also $v = \frac{dx}{dt}$

$$\omega = \frac{d}{dt}(B\sin(\omega t + \alpha))$$

$$\omega = B\omega\cos(\omega t + \alpha)$$

$$1 = B\cos(\omega(0) + \alpha)$$

$$B\cos\alpha = 1 \longrightarrow (2)$$

$$(1)^2 + (2)^2 \Rightarrow (B\sin\alpha)^2 + (B\cos\alpha)^2 = (1)^2 + (1)^2$$

$$B^2 = 2$$

$$B = \sqrt{2}cm$$

$(2)\div(1) \Rightarrow \quad \frac{A\sin\phi}{A\cos\phi} = \frac{-1}{1}$

$$\tan\phi = -1$$

$$\phi = \frac{7\pi}{4}$$

$(1)\div(2) \Rightarrow \quad \frac{B\sin\alpha}{B\cos\alpha} = \frac{1}{1}$

$$\tan\phi = 1$$

$$\phi = \frac{\pi}{4}$$

8. A spring balance has a scale that reads from 0-50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

Ans. Spring constant, $k = \frac{F}{l} = \frac{50\times 9.8}{0.2} = 2450\,Nm^{-1}$

Time period, $T = 2\pi\sqrt{\frac{m}{k}}$

$$T^2 = \frac{4\pi^2 m}{k}$$

$$0.6\times 0.6 = \frac{4\times 3.14\times 3.14\times m}{2450}$$

$$m = 22.36\,kg$$

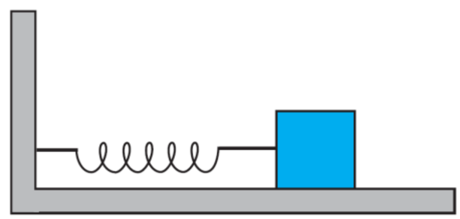
Weight, $W = mg = 22.36\times 9.8 = 219.2\,N$

9. A spring having with a spring constant 1200 N m^{-1} is mounted on a horizontal table as shown in Fig. A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released. Determine

(i) the frequency of oscillations,

(ii) maximum acceleration of the mass, and

(iii) the maximum speed of the mass.



Ans. $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}} = 20\,rad.s^{-1}$

(a) $\nu = \frac{\omega}{2\pi} = \frac{20}{2\times 3.14} = 3.18\,Hz$

(b) $a = \omega^2 A = 400\times 0.02 = 8\,ms^{-2}$

(c) $v = \omega A = 20\times 0.02 = 0.4\,ms^{-1}$

10. In Q9, let us take the position of mass when the spring is unstreched as x = 0, and the direction from left to right as the positive direction of x-axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is

(a) at the mean position,

(b) at the maximum stretched position, and

(c) at the maximum compressed position.

In what way do these functions for SHM differ from each other, in frequency, in amplitude or the initial phase?

Ans. $Given: \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1200}{3}} = 20\, rad.s^{-1},\ A = 2cm$

Displacement equation : $x = A\sin(\omega t + \phi)$

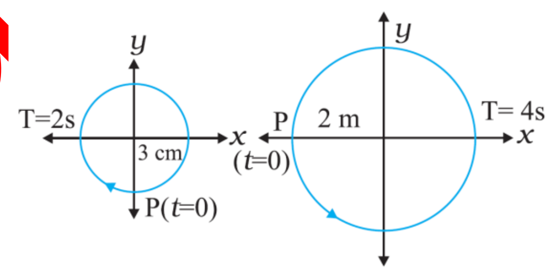
(a) At mean position $\phi = 0$, $x = 2\sin(20t + 0) = 2\sin 20t$

(b) At maximum stretched position, $\phi = 90^0$ (as mass is towards extreme right), $x = 2\sin\left(20t + \frac{\pi}{2}\right) = 2\cos 20t$

(c) At maximum compressed position $\phi = \frac{3\pi}{2}$ (as mass is towards extreme left), $x = 2\sin\left(20t + \frac{3\pi}{2}\right) = -2\cos 20t$

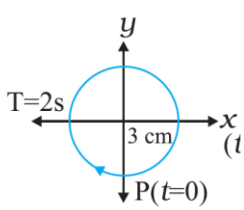
- The functions have the same frequency $\left(\nu = \frac{\omega}{2\pi} = \frac{20}{2\times 3.14} = 3.18\, s^{-1}\right)$ and amplitude (2 cm), but different initial phases $\left(\phi = 0, \frac{\pi}{2}, \frac{3\pi}{2}\right)$

11. Figures correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure. Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.



(a) (b)

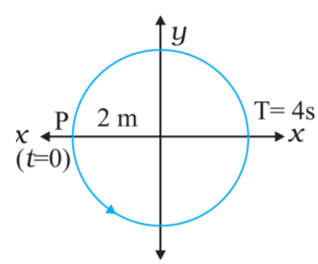
Ans. Displacement equation : $x = A\cos(\omega t + \phi) = A\cos\left(\frac{2\pi}{T}t + \phi\right)$



$Given\ :\ T = 2s,\quad A = 3cm$

Phase angle, $\phi = \frac{\pi}{2}$ [∵ At t = 0, radius vector OP makes $\frac{\pi}{2}$ angle with +X − axis]

$x = 3\cos\left(\frac{2\pi}{2}t + \frac{\pi}{2}\right) = (-3\sin \pi t)\, cm$



$Given\ :\ T = 4s,\quad A = 2cm$

Phase angle, $\phi = \pi$ [∵ At t = 0, radius vector OP makes π angle with X − axis]

$x = 2\cos\left(\frac{2\pi}{4}t + \pi\right) = (-2\cos\frac{\pi}{2}t)\, m$

12. Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t =0) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).

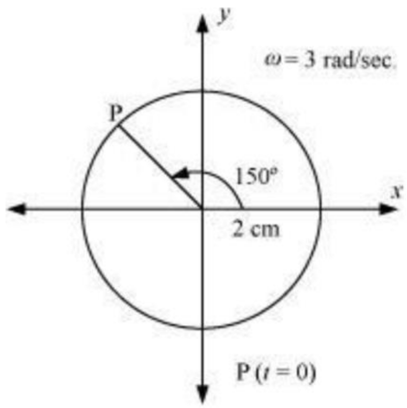
(a) $x = -2 \sin (3t + \pi/3)$ (b) $x = \cos (\pi/6 - t)$

(c) $x = 3 \sin (2\pi t + \pi/4)$ (d) $x = 2 \cos \pi t$

Ans. Displacement equation : $x = A\cos(\omega t + \phi) = A\cos\left(\frac{2\pi}{T}t + \phi\right)$

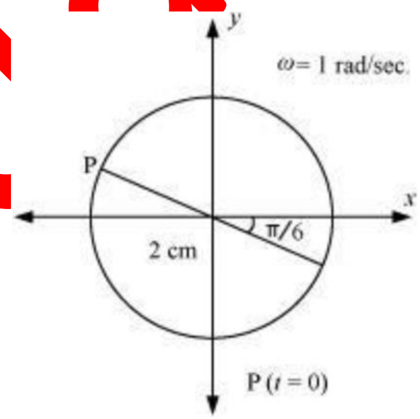
(a) $x = -2\sin\left(3t + \frac{\pi}{3}\right) = 2\cos\left(3t + \frac{\pi}{3} + \frac{\pi}{2}\right) = 2\cos\left(3t + \frac{5\pi}{6}\right)$

So, A = 2cm, $\phi = \frac{5\pi}{6} = 150^0$, $\omega = 3\ rad.s^{-1}$



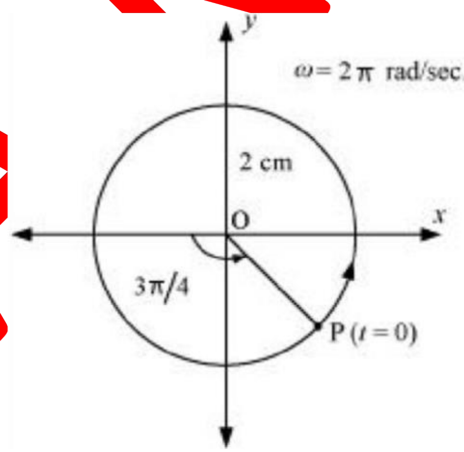
(b) $x = \cos\left(\frac{\pi}{6} - t\right) = \cos\left(t - \frac{\pi}{6}\right)$

So, A = 1cm, $\phi = -\frac{\pi}{6} = -30^0$, $\omega = 1\ rad.s^{-1}$



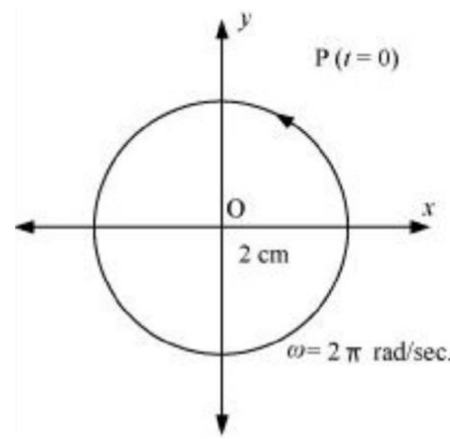
(c) $x = 3\sin\left(2\pi t + \frac{\pi}{4}\right) = -3\cos\left(2\pi t + \frac{\pi}{4} + \frac{\pi}{2}\right) = -3\cos\left(2\pi t + \frac{3\pi}{4}\right)$

So, A = 3cm, $\phi = \frac{3\pi}{4} = 135^0$, $\omega = 2\pi\ rad.s^{-1}$



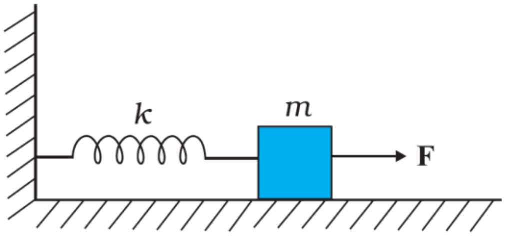
(d) $x = 2\cos \pi t$

So, A = 2cm, $\phi = 0^0$, $\omega = \pi\ rad.s^{-1}$

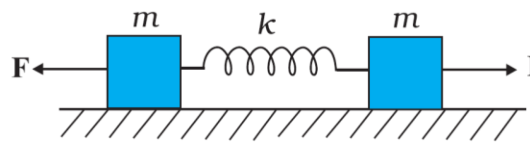


13. Figure (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force **F** applied at the free end stretches the spring. Figure (b) shows the same spring with both ends free and

attached to a mass *m* at either end. Each end of the spring in Fig. (b) is stretched by the same force **F**.



(a)

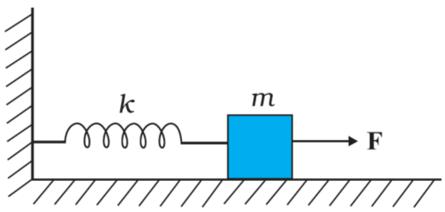


(b)

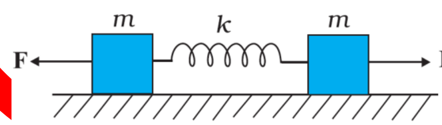
(a) What is the maximum extension of the spring in the two cases?

(b) If the mass in Fig. (a) & the two masses in Fig. (b) are released, what is the period of oscillation in each case ?

Ans. (a) Extension produced



(a)



(b)

$F = kx$

Here, $x = l$, $F = kl$

Extension, $l = \dfrac{F}{k}$

$F = kx$

Here, $x = l/2$, $F = 2k\dfrac{l}{2}$

Extension, $l = \dfrac{F}{k}$

(b) Period of oscillation

$F = -kx$

$$m\frac{d^2x}{dt^2} = -kx$$

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

So, $\omega = \sqrt{\dfrac{k}{m}}$

Time period, $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{k}}$

$F = -2kx$

$$m\frac{d^2x}{dt^2} = -2kx$$

$$\frac{d^2x}{dt^2} = -\frac{2k}{m}x$$

$$\frac{d^2x}{dt^2} = -\omega^2 x$$

So, $\omega = \sqrt{\dfrac{2k}{m}}$

Time period, $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{m}{2k}}$

14. The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min, what is its maximum speed?

Ans.
$$\text{Given} : A = \frac{1}{2} = 0.5m, \quad \omega = 200\ rad.\text{min}^{-1}$$
$$\text{To find} : v_{max} = ?$$
$$\text{Sol}^n : v_{max} = A\omega = 0.5 \times 200 = 100\ rad.\text{min}^{-1}$$

15. The acceleration due to gravity on the surface of moon is 1.7 m s^{-2}. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (g =9.8 m s^{-2})

Ans. For earth, $T = 3.5s, \quad g = 9.8ms^{-2}$

Time period on earth, $T = 2\pi\sqrt{\frac{l}{g}}$

$$3.5 = 2 \times 3.14\sqrt{\frac{l}{9.8}}$$
$$l = \frac{3.5 \times 3.5 \times 9.8}{4 \times 3.14 \times 3.14} = 3.04\ m$$

For moon, $l = 3.04m, \quad g = 1.7ms^{-2}$

Time period on moon, $T' = 2\pi\sqrt{\frac{l}{g}} = 2 \times 3.14 \times \sqrt{\frac{3.04}{1.7}} = 8.4s$

16. Answer the following questions :

(a) Time period of a particle in SHM depends on the force constant k and mass m of the particle: $T = 2\pi\sqrt{\frac{m}{k}}$. A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?

(b) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that T is greater than $2\pi\sqrt{\frac{l}{g}}$. Think of a qualitative argument to appreciate this result.

(c) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?

(d) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?

Ans. (a) For a simple pendulum, k is expressed in terms of mass m, $k\ \alpha\ m$ or $\frac{m}{k} = const.$

Hence, the time period T, of a simple pendulum is independent of the mass of the bob.

(b) $F = -mg\sin\theta$

$$g = -\frac{F}{m\sin\theta}$$

For large θ, sinθ > θ. This decreases the effective value of g. Hence, time period increases as $T = 2\pi\sqrt{\frac{l}{g}}$

(c) Motion in the wristwatch depends on spring action and is independent of acceleration due to gravity.

(d) During free fall g = 0 $\longrightarrow$ Time period, $T = 2\pi\sqrt{\frac{l}{0}} = \infty$ $\longrightarrow$ Frequency, $\nu = \frac{1}{T} = \frac{1}{\infty} = 0$

17. A simple pendulum of length l and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

Ans. Acceleration experienced by simple pendulum due to gravity, a = g

Acceleration experienced by simple pendulum due to circular motion, $a' = \frac{v^2}{R}$

Resultant acceleration, $a_{res} = \sqrt{a^2 + (a')^2} = \sqrt{g^2 + \left(\frac{v^2}{R}\right)^2} = \sqrt{g^2 + \frac{v^4}{R^2}}$

Time period, $T = 2\pi\sqrt{\frac{l}{a_{res}}} = 2\pi\sqrt{\frac{l}{\sqrt{g^2 + v^4/R^2}}}$

18. A cylindrical piece of cork of density of base area A and height h floats in a liquid of density ρ_l. The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period $T = 2\pi\sqrt{\frac{h\rho}{\rho_l g}}$ where ρ is the density of cork. (Ignore damping due to viscosity of the liquid).

Ans. In equilibrium, Weight of cork = Weight of liquid displaced by cork

Let the cork be depressed slightly by x. As a result, some extra water of a certain volume is displaced. Hence, an extra up-thrust acts upward and provides the restoring force to the cork.

Restoring force, $F = -mg$

$$kx = -\rho_l V g$$
$$kx = -\rho_l A x g$$
$$k = -\rho_l A g$$

Time period of oscillation of the cork, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{Ah\rho}{Ag\rho_l}} = 2\pi\sqrt{\frac{h\rho}{g\rho_l}}$

19. One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is

removed, the column of mercury in the U-tube executes simple harmonic motion.

Ans. Restoring force, $F = -mg$

$$kx = -\rho V g$$
$$kx = -\rho A(2x)g$$
$$k = -2\rho A g$$

Time period of oscillation of the cork, $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{Al\rho}{2\rho A g}} = 2\pi\sqrt{\frac{l}{2g}}$

Hence, the mercury column executes simple harmonic motion with time period $T = 2\pi\sqrt{\frac{l}{2g}}$