**Absolute Error, Relative Error and Percentage Error**

**Absolute Error**- It is the magnitude of the difference between the true value & the individual

 measured value of the quantity.

Let a physical quantity is measured n times and the measured values are a1, a2, a3 ….an. Now the true value (i.e. the arithmetic mean) is

$$a\_{m}=\frac{a\_{1}+a\_{2}+...…a\_{n}}{n}$$

So, the absolute error for first observation is Δa1=am-a1

 the absolute error for second observation is Δa2=am-a2 and so on

\*The absolute errors can be both positive and negative

**Mean absolute error** - It is the arithmetic mean of the magnitudes of absolute errors.

$$∆a\_{mean}=\frac{\left|∆a\_{1}\right|+\left|∆a\_{2}\right|+…\left|∆a\_{n}\right|}{n}$$

\*While taking arithmetic mean we ignore the sign ( $\pm $) of absolute errors.

**Relative Error**-It is defined as the ratio of mean absolute error to the mean value of the quantity measured.

$$Relative error=\frac{mean absolute error}{mean value}=\frac{∆a\_{mean}}{a\_{m}}$$

**Percentage Error** - Relative error expressed in percentage

$$Percentage error=\frac{mean absolute error}{mean value}x 100\%=\frac{∆a\_{mean}}{a\_{m}}x 100\%$$

**Propagation or combination of errors**

1. **Error in a sum**

Suppose X = a + b

Let Δa – absolute error in measurement of a

Δb – absolute error in measurement of b

ΔX – absolute error in measurement of X

So, X $\pm $ ΔX = ( a$ \pm $ Δa )+ (b$ \pm $ Δb)

 X $\pm $ ΔX =( a + b) $\pm $ Δa $\pm $ Δb

X $\pm $ ΔX =X $\pm $ Δa $\pm $ Δb

ΔX = $\pm $ Δa$ \pm $ Δb

So, the maximum value is ΔX = $\pm $ (Δa + Δb)

Q. The lengths of 2 cylinders are measured to be l1 = (5.62 $\pm $ 0.01) cm and l2 = (4.34 $\pm $ 0.02) cm.

 Calculate the total length with error limits

Ans. L = l1 + l2 =5.62 + 4.34 = 9.96cm

 ΔL = $\pm $ (Δ l1 + Δ l2)= $\pm $ (0.01 + 0.02)=$ \pm $0.03

Total length is (9.96 $\pm $ 0.03)cm

1. **Error in a difference**

Suppose X = a - b

Let Δa – absolute error in measurement of a

Δb – absolute error in measurement of b

ΔX – absolute error in measurement of X

So, X $\pm $ ΔX = ( a$ \pm $ Δa )- (b$ \pm $ Δb)

 X $\pm $ ΔX =( a - b) $\pm $ Δa ∓Δb

X $\pm $ ΔX =X $\pm $ Δa ∓ Δb

ΔX = $\pm $ Δa ∓ Δb

So, the maximum value is ΔX = $\pm $ (Δa + Δb)

Q. The lengths of 2 cylinders are measured to be l1 = (5.62 $\pm $ 0.01) cm and l2 = (4.34 $\pm $ 0.02) cm.

 Calculate the difference in length with error limits

Ans. L = l1 - l2 =5.62 - 4.34 = 1.28cm

 ΔL = $\pm $ (Δ l1 + Δ l2)= $\pm $ (0.01 + 0.02)=$ \pm $0.03

Total length is (1.28 $\pm $ 0.03)cm

1. Error in difference